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Critical States of Superconductivity and Their Crossover in Multilayer Superconductor/Ferromagnet Structures

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In order to calculate the critical temperature of multilayer S/F structures (where S is a superconductor and F is a ferromagnet), a matrix method for solving linearized Usadel equations has been proposed. The spectrum of critical temperatures $T^{(k)}$ for the F/$N_{\rm bl}$(S/F) structure has been obtained in the single-mode approximation. Eigenfunctions describing the spatial distribution of superconducting correlations in the direction perpendicular to the S–F interfaces have been calculated for each $T^{(k)}$ value. It has been found that dependences of $T^{(k)}$ on the thickness of F layers have a jump near the transition from 0 to π-state; any of the calculated $T^{(k)}$ values can be implemented in the region of jumps. It has been shown that the crossover of eigenstates is characterized by the suppression of superconductivity in outer S layers and by induced countercurrents in F layers. The possibility of the experimental implementation of a state corresponding to a given value from the spectrum of $T^{(k)}$ has been discussed.

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Singlet superconductivity was found in three-layer S/F/S structures (where S is a superconductor and F is a ferromagnet) and superlattices can be implemented in a 0 or π-state with the critical temperature T_{c0} or $T_{c\pi}$, respectively, depending on the thickness of the F layer $d_{\rm F}$ [1–5]. These states are not only possible states in a multilayer structure. In particular, as was shown in [6], the superconducting condensate in the F/3(S/F) structure below the critical temperature T_c can be in π0π and 0π0 states. The order parameter in a periodic structure with alternating 0 and π contacts is characterized by the phase difference $\varphi \in (-\pi, \pi)$ between neighboring S layers [7].

In this work, the complete set of the eigenstates of superconductivity with the eigenvalues of the temperature $T^{(k)}$ in the F/$N_{\rm bl}$(S/F) structures is determined by solving the boundary value problem for linearized Usadel equations [8].

We show that an unusual behavior of the superconducting condensate, which is manifested in the shape of Usadel functions and in the distribution of the transport current calculated in the single-mode approximation, can be observed at thicknesses belonging to the vicinity of the intersection of $T^{(k)}(d_{\rm F})$ curves (vicinity of crossover). It was shown earlier that the vicinity of 0–π crossover for three-layer S/F/S structure is characterized by the broadening of the resistive characteristic caused by the interaction of the measuring transport current with spontaneous closed currents [9]; the transport current in multilayer structures induces countercurrents flowing through F layers. Finally, on the basis of solutions obtained for the F/$N_{\rm bl}$(S/F), we demonstrate that the critical state of an arbitrary type can be specified by varying the thicknesses of given S or F layers.

To prove the statements formulated above, we consider a F/$N_{\rm bl}$(S/F) structure that is infinite in the X and Y directions and consists of alternating superconducting and monodomain ferromagnetic layers with the same direction of the magnetization vector. Let the Z axis be perpendicular to the surface of the layers and the XY plane coincide with the mirror-symmetry plane of the structure. In this case, the linearized Usadel equations describing the proximity effect in the system under investigation can be represented in the form

$$-\frac{D_{\rm S}}{2}\Phi''_{\pm,n} + \omega_n\Phi_{\pm,n} = 2\delta_{\pm}\pi T\lambda\sum_{m=0}^{n_{\rm D}}\Phi_{+,m} \qquad (1)$$

for the S layers and

$$-\frac{D_{\rm F}}{2}\Phi''_{\pm,n} + \omega_n\Phi_{\pm,n} + iE_{\rm ex}\Phi_{\mp,n} = 0 \qquad (2)$$

for the F layers. Here, $D_{\rm S(F)}$ is the diffusion coefficient of the superconductor (ferromagnet); $\omega_n = \pi T(2n+1)$ are the Matsubara frequencies for $n = 0, 1, \ldots, n_{\rm D}$; $n_{\rm D}$ is the integer part of the expression $\omega_{\rm D}/2\pi T - 0.5$; $\omega_{\rm D}$ is the Debye frequency; λ is the effective electron–

electron coupling constant; $\delta_+ = 1$; $\delta_- = 0$; E_{ex} is the energy of the exchange interaction; and $\Phi_{\pm,n}(z) = (F_n(z) \pm F_{-n-1}(z))/2$, where $F_{\pm n}(z)$ are the quasiclassical anomalous Green's functions averaged over the Fermi surface.

Equations (1) and (2) should be supplemented by the following matching conditions at the S–F interfaces $z = z_i$ ($i = 1, \ldots, 2N_{bl}$) [10]:

$$\frac{1}{\rho(z_i+0)}\Phi'_{\pm,n}(z_i+0) = \frac{1}{\rho(z_i-0)}\Phi'_{\pm,n}(z_i-0), \qquad (3)$$

$$\Phi_{\pm,n}(z_i+0) = \Phi_{\pm,n}(z_i-0) + \gamma_b\xi_F\frac{\rho_F}{\rho(z_i-0)}\Phi'_{\pm,n}(z_i-0). \qquad (4)$$

Here, $\rho(z) = \rho_S$ and $\rho(z) = \rho_F$ for the S and F layers, respectively; $\rho_{S(F)}$ is the low-temperature resistivity of the superconducting (ferromagnetic) material; γ_b is the parameter characterizing the quantum-mechanical transparency of the S–F interface [10]; $\xi_{F(S)} = \sqrt{D_{F(S)}/2\pi T_S}$ is the coherence length in the ferromagnetic (superconducting) material; and T_S is the critical temperature of the bulk superconductor (determined by the expression $T_S = 1.14\omega_D\exp(-1/\lambda)$ [11]).

The outer boundary conditions have the form

$$\Phi'_{\pm,n}(-L/2) = \Phi'_{\pm,n}(L/2) = 0. \qquad (5)$$

Let us write the solution of the system of Eqs. (1)–(4) in the matrix form [12–14]

$$\mathbf{Y}(z) = \hat{\mathbf{R}}(z)\mathbf{Y}(-L/2). \qquad (6)$$

Here, $\mathbf{Y}(z) = \Phi_+(z) \oplus \Phi'_+(z) \oplus \Phi_-(z) \oplus \Phi'_-(z)$, where $\Phi_\pm = (\Phi_{\pm,0}, \Phi_{\pm,1} \ldots \Phi_{\pm,n_D+1})^{tr}$ are the $(n_D + 1)$-dimensional vector functions (superscript tr marks transposition) and $\hat{\mathbf{R}}(z)$ is the principal matrix solution of the system of equations (1)–(4); i.e., the matrix of the fundamental system of solutions that satisfies the condition $\hat{\mathbf{R}}(-L/2) = \hat{\mathbf{1}}$, where $\hat{\mathbf{1}}$ is the identity matrix.

The substitution of Eq. (6) to Eq. (5) yields the following system of linear homogeneous algebraic equations:

$$\hat{\mathbf{R}}_{24,13}(L/2)\Phi(-L/2) = \mathbf{0}. \qquad (7)$$

Here, $\Phi = \Phi_+ \oplus \Phi_-$ and

$$\hat{\mathbf{R}}_{24,13} = \begin{pmatrix} \hat{\mathbf{R}}_{2,1} & \hat{\mathbf{R}}_{2,3} \\ \hat{\mathbf{R}}_{4,1} & \hat{\mathbf{R}}_{4,3} \end{pmatrix},$$

where $\hat{\mathbf{R}}_{\alpha,\beta}$ ($\alpha, \beta = 1, 2, 3, 4$) are the matrix blocks of the dimension $(n_D + 1) \times (n_D + 1)$ of the matrix $\hat{\mathbf{R}}$.

The condition of the existence of a nontrivial solution of system (7) provides the characteristic equation

$$\det[\hat{\mathbf{R}}_{24,13}(L/2)] = 0. \qquad (8)$$

The principal matrix solution $\hat{\mathbf{R}}$ is obtained in the explicit form and is expressed in terms of the product of the principal matrix solutions $\hat{\mathbf{S}}$ and $\hat{\mathbf{M}}$ of the S and F layers, respectively, and matrices $\hat{\Gamma}_{SF}$ and $\hat{\Gamma}_{FS}$ of matching conditions (3) and (4). In particular, the principal matrix solution relating the vectors $\mathbf{Y}(-L/2)$ and $\mathbf{Y}(L/2)$ for the F/N_{bl}(S/F) structures has the form

$$\hat{\mathbf{R}}(L/2) = \hat{\mathbf{M}}(d_F)(\hat{\Gamma}_{FS}\hat{\mathbf{S}}(d_S)\hat{\Gamma}_{SF}\hat{\mathbf{M}}(d_F))^{N_{bl}}, \qquad (9)$$

where d_S is the thickness of the S layer.

The $\hat{\mathbf{S}}$ matrix is represented as

$$\hat{\mathbf{S}}(z) = \begin{pmatrix} \hat{\mathscr{C}}\hat{\mathbf{S}}_+(z)\hat{\mathscr{C}}^{tr} & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \hat{\mathbf{S}}_-(z) \end{pmatrix}, \qquad (10)$$

where

$$\hat{\mathbf{S}}_\pm(z) = \begin{pmatrix} \mathrm{diag}[\cosh(k_n^\pm z)] & \mathrm{diag}[(k_n^\pm)^{-1}\sinh(k_n^\pm z)] \\ \mathrm{diag}[k_n^\pm\sinh(k_n^\pm z)] & \mathrm{diag}[\cosh(k_n^\pm z)] \end{pmatrix}. \qquad (11)$$

Here, diag[a_n] is the diagonal matrix with the elements $a_0, a_1, \ldots, a_{n_D}$ of the principal diagonal, $k_n^+ = \xi_S^{-1}\sqrt{-2T\mu_n/T_S}$, and $k_n^- = \xi_S^{-1}\sqrt{(2n+1)T/T_S}$, where $\mu_n \equiv \mu_n(T)$ are the root of the equation

$$\psi\left(\frac{\omega_D}{2\pi T} + \mu + 1\right) - \psi\left(\frac{1}{2} + \mu\right) = \psi\left(\frac{\omega_D}{2\pi T_S} + 1\right) - \psi\left(\frac{1}{2}\right),$$

where $\psi(t)$ is the digamma function. The matrix $\hat{\mathscr{C}}$ in Eq. (10) has the form

$$\hat{\mathscr{C}} = \begin{pmatrix} \hat{\mathbf{C}} & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \hat{\mathbf{C}} \end{pmatrix},$$

where $\hat{\mathbf{C}}$ is the orthogonal matrix with the elements

$$c_{nm} = \frac{2s_m}{2n + 1 + 2\mu_m},$$

where the normalization coefficients are given by the expression

$$s_m = \left(\sum_{l=0}^{n_D} \frac{4}{(2l+1+2\mu_m)^2}\right)^{-1/2}.$$

It is easy to obtain the following formula for the principal matrix solution for the F layer:

$$\hat{\mathbf{M}}(z) = \begin{pmatrix} \mathrm{Re}[\hat{\mathfrak{m}}(z)] & \imath\mathrm{Im}[\hat{\mathfrak{m}}(z)] \\ \imath\mathrm{Im}[\hat{\mathfrak{m}}(z)] & \mathrm{Re}[\hat{\mathfrak{m}}(z)] \end{pmatrix}, \qquad (12)$$

where

$$\hat{\mathfrak{m}}(z) = \begin{pmatrix} \mathrm{diag}[\cosh(\kappa_n z)] & \mathrm{diag}[(\kappa_n)^{-1}\sinh(\kappa_n z)] \\ \mathrm{diag}[\kappa_n \sinh(\kappa_n z)] & \mathrm{diag}[\cosh(\kappa_n z)] \end{pmatrix}. \qquad (13)$$

The characteristic exponents κ_n in Eq. (13) are given by the formula

$$\kappa_n = \frac{1}{\xi_F}\sqrt{\frac{\imath E_{ex} + \omega_n}{\pi T_S}}. \qquad (14)$$

Finally, it is easy to obtain the following expression from Eqs. (3) and (4):

$$\hat{\Gamma}_{FS(SF)} = \begin{pmatrix} \hat{\mathbf{P}}_{FS(SF)} & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \hat{\mathbf{P}}_{FS(SF)} \end{pmatrix}, \qquad (15)$$

where

$$\hat{\mathbf{P}}_{FS} = \begin{pmatrix} \hat{\mathbf{1}} & \gamma_b \xi_F p^{-1}\hat{\mathbf{1}} \\ \hat{\mathbf{0}} & p^{-1}\hat{\mathbf{1}} \end{pmatrix}, \quad \hat{\mathbf{P}}_{SF} = \begin{pmatrix} \hat{\mathbf{1}} & \gamma_b \xi_F \hat{\mathbf{1}} \\ \hat{\mathbf{0}} & p\hat{\mathbf{1}} \end{pmatrix}. \qquad (16)$$

Here, $p = \rho_S/\rho_F$ and the dimension of the identity and zero matrices is $n_D + 1$.

Thus, Eqs. (6)–(16) provide the complete solution of the problem given by Eqs. (1)–(5). The set of eigenvalues for the temperature $T^{(k)}$, which correspond to eigenvectors $\Phi^{(k)}(-L/2)$ determined from Eq. (7) is obtained from characteristic equation (8); then, the vector eigenfunctions $\Phi^{(k)}(z)$ are obtained using Eq. (6). The maximum eigenvalue among $T^{(k)}$ is the actual critical temperature T_c of the multilayer structure. The vector functions of nondegenerate states of the structures under investigation are obviously symmetric or antisymmetric.

To obtain information on T_c in experiments, the measuring transport current is directed into the structure along the S–F interfaces, for example, along the X direction. Since this current density J_x is low, its spatial distribution in the Z direction can be determined in the first approximation neglecting the effects of the suppression of the functions $\Phi_\pm$ by this current, omitting the terms responsible for these effects in Eqs. (1) and (2), and assuming that the only consequence of the presence of J_x is that the functions $\Phi_\pm$ contain the same phase factor $\exp(ikx)$, in which the wave vector k is proportional to the superfluid velocity of the condensate.

The substitution of such solutions into the expression for the current density [3–5] written in the matrix form

$$\mathbf{J} = \frac{4\pi T}{ie\rho(z)}(\Phi_+^{*\mathrm{tr}}\nabla\Phi_+ - \Phi_-^{*\mathrm{tr}}\nabla\Phi_- - \mathrm{c.c.}) \qquad (17)$$

gives the expression

$$J_x(z) = \frac{8k\pi T}{e\rho(z)}(\Phi_+^{*\mathrm{tr}}(z)\Phi_+(z) - \Phi_-^{*\mathrm{tr}}(z)\Phi_-(z)). \qquad (18)$$

Since the critical temperature chosen from the set $T^{(k)}$ corresponds to the vector eigenfunction $\Phi^{(k)}(z)$, it is characterized by the "own" current distribution $J_x^{(k)}(z)$. As will be shown below, in view of the presence of the component $\Phi_-(z)$ (which is due to the exchange interaction in a ferromagnet and identically vanishes for the superconductor/normal metal structures), the sign of $J_x(z)$ for certain $T^{(k)}$ values can alternate with variation of z; i.e., countercurrents can appear in the structures under investigation.

NUMERICAL EXAMPLES

In what follows, we use the single-mode approximation. In this case, the state functions become effectively two-component: $\Phi \longrightarrow (\Phi_+ \ \Phi_-)^{\mathrm{tr}}$. The components Φ_+ and Φ_- are real and imaginary, respectively, and can be considered as the real and imaginary parts of the condensate wavefunction.

For example, we consider a F/5(S/F) structure with the parameters close to the parameters of the Nb/PdNi system [15]: $p = 0.29$, $\gamma_{b\zeta} \equiv \gamma_b\xi_F/\zeta_F = 0.28$, $\zeta_F = 0.5\xi_S$, where $\zeta_F \equiv \sqrt{D_F/E_{ex}}$. The single-mode approximation is applicable in this case, because the thickness of the S layers is accepted to be sufficient large: $d_S = 4.7\xi_S$ [16]. Figure 1 shows the $T^{(k)}(d_F)$ plots, as well as $T_{c0}(d_F)$ and $T_{c\pi}(d_F)$ curves calculated with the same material parameters for the superlattice, i.e., the F/N_{bl}(S/F) structure with infinite number of S/F bilayers (N_{bl}). The eigenvalues are enumerated according to the number of zeros of the vector state eigenfunctions $\Phi^{(k)}(z)$.

It is seen that all curves $T^{(k)}(d_F)$ (for $k = 0, \ldots, 4$) intersect in a very narrow vicinity of the point of the 0–π crossover $d_F^* \approx 1.35\zeta_F$, as well as in the vicinity of the π–0 crossover. At $d_F < d_F^*$, we have $T^{(k+1)} < T^{(k)}$ and

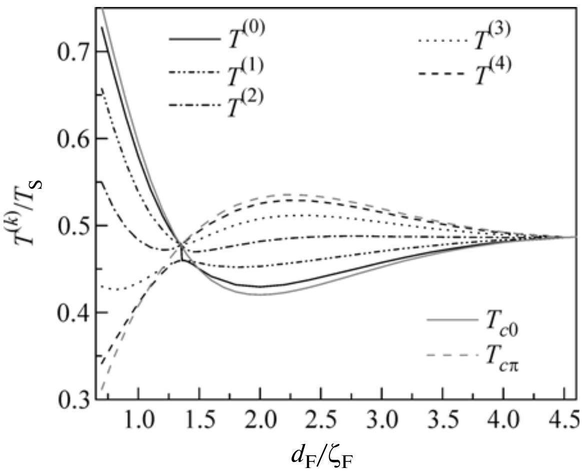

Fig. 1. Eigenvalues of the temperature of the F/5(S/F) structure versus the thickness of the F layer calculated with the thickness $d_S = 4.67\xi_S$.

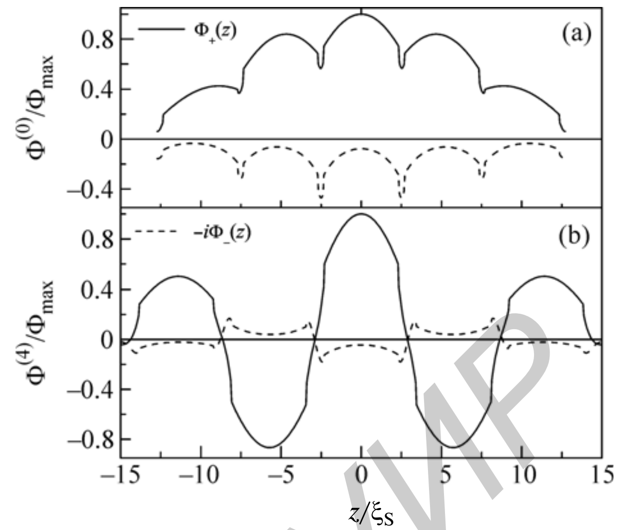

Fig. 2. Real, $\Phi_+(z)$, and imaginary, $-i\Phi_-(z)$, parts of the functions of the critical state calculated with the thickness $d_F =$ (a) $0.7\zeta_F \approx 0.52\,d_F^*$ and (b) $2.2\zeta_F \approx 1.63\,d_F^*$.

the inverse inequality at $d_F > d_F^*$. The 0 state described by the vector function $\Phi^{(0)}(z)$ and the π state described by the vector function $\Phi^{(4)}(z)$ are implemented in the former and latter cases, respectively (see Fig. 2).

Let us consider the immediate vicinity of the 0–π crossover (see Fig. 3). As is seen in Fig. 3, $T^{(0)}$ decreases when the crossover point is passed along the curve C_{02} with an increase in d_F. Simultaneously, the spatial distribution of eigenfunctions $\Phi^{(0)}(z)$ corresponding to this state is gradually transformed from that shown in Fig. 2a to those shown in Fig. 4a. A further increase in d_F leads to the transformation of $\Phi^{(0)}(z)$ to the spatial distribution corresponding to the eigenfunctions $\Phi^{(2)}(z)$, i.e., to the solution corresponding to the eigenvalue with the critical temperature $T^{(2)}$. Simultaneously with this process, an increase in d_F is accompanied by the transformation of the state $\Phi^{(4)}(z)$ corresponding to the temperature $T^{(4)}$ to $\Phi^{(0)}(z)$. Thus, the critical temperature $T^{(0)}$ corresponding to the zeroth state (see Fig. 2a) has a jump in the crossover region as a function of the parameter d_F. It is clearly seen that the absence of continuity is a common property of all $T^{(k)}(d_F)$ dependences. In particular, when moving along the curve C_{24}, the eigenstate $\Phi^{(2)}(z)$ corresponding to the temperature $T^{(2)}$ is transformed to the π-state $\Phi^{(4)}(z)$ (see Fig. 2b) with the temperature $T^{(4)}$, which is critical for large d_F values.

According to Fig. 3, the temperatures $T^{(1)}$ and $T^{(3)}$ corresponding to antisymmetric functions $\Phi^{(1)}(z)$ and $\Phi^{(3)}(z)$, respectively, are critical in a very narrow range near the point d_F^*.

Figures 4a–4c show the functions of the critical states, which successively change each other with an increase in d_F from $1.33\zeta_F$ to $1.37\zeta_F$. Figure 4d shows the symmetric state function that corresponds to the intersection of curves C_{02} and C_{24} and has the eigenvalue of the temperature slightly lower than the critical temperature. An interesting feature in the distribution of the superconducting condensate at $d_F \sim d_F^*$ is remarkable. It is seen in Fig. 4 that superconductivity in outer S layers is suppressed almost completely. Superconductivity at the crossover point also disappears in the central S layer. The crossover of states is also manifested in the response to the measuring transport current $J_x(z)$. Namely, the transport current induces weak countercurrents flowing along the central ferromagnetic layers (see Fig. 4a).

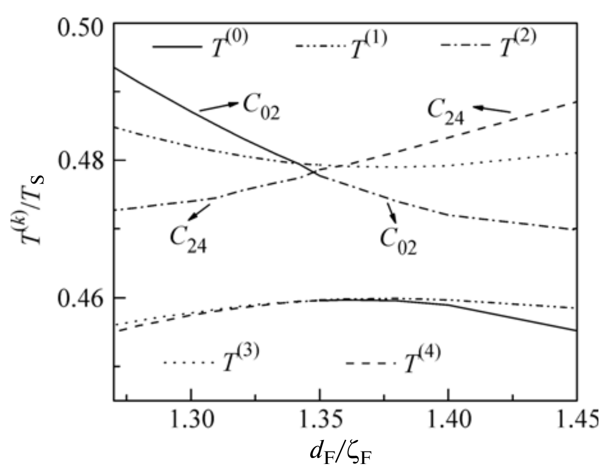

Fig. 3. Eigenvalues of the critical temperature $T^{(k)}$ of the F/5(S/F) structure versus the thickness of the F layer near crossover (see Fig. 2).

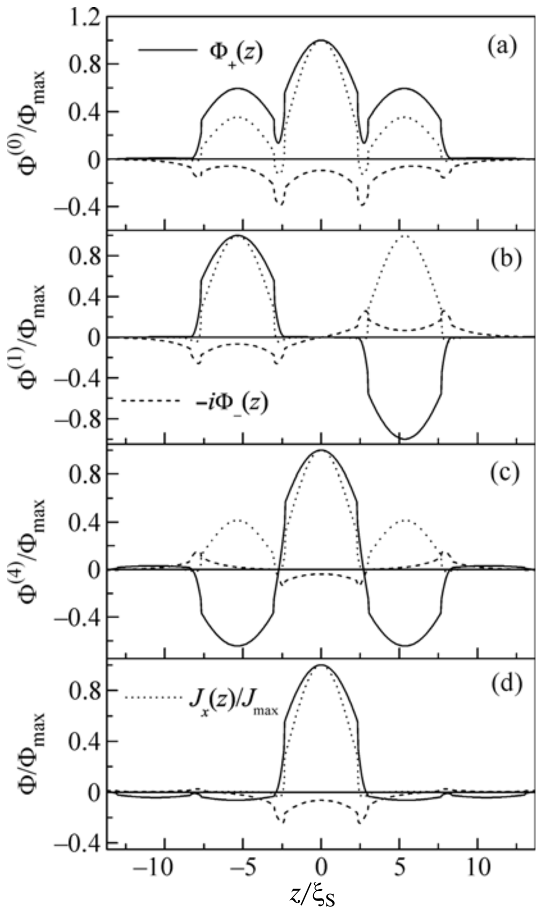

Fig. 4. Real, $\Phi_+(z)$, and imaginary, $-i\Phi_-(z)$, parts of the state functions and the transport current density $J_x(z)$ calculated for the F/5(S/F) with the thickness d_F = (a) $1.33\zeta_F$, (b, d) $1.35\zeta_F$, and (c) $1.37\zeta_F$.

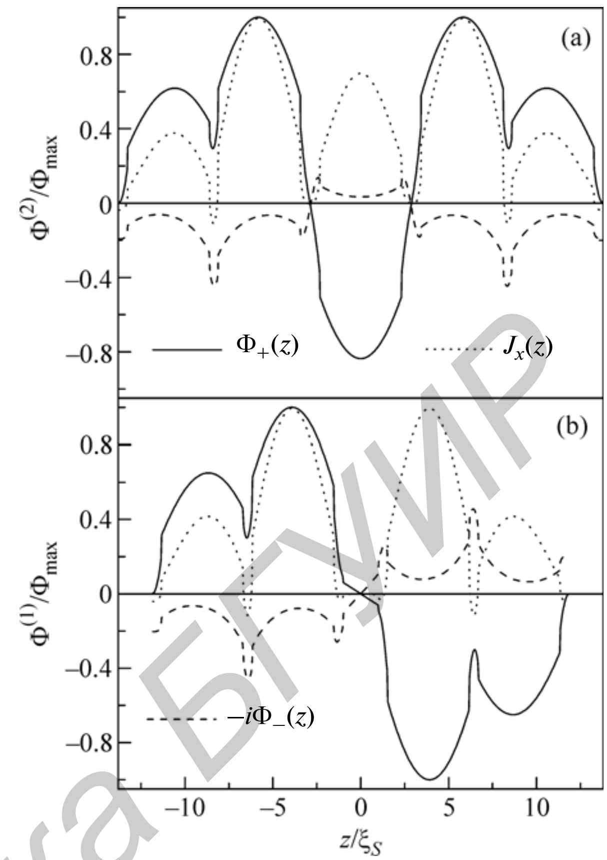

Fig. 5. Functions of the critical state and distribution of the transport current density calculated for the F/5(S/F) structure with (a) the increased thicknesses of the central F layers and (b) the decreased thickness of the central S layer (see comments in the main text).

Thus, states different from 0 and π are specifically manifested in the regular F/N_{bl}(S/F) structure at the thicknesses $d_F \sim d_F^*$; for other d_F values, they remain "hidden" (see Fig. 1). At the same time, there is an obvious method for implementing a given eigenstate [7]. Namely, if the thickness of F layers containing the zeros of a given symmetric state function is increased or the thickness of S layers containing the zeros of the antisymmetric function is decreased, the given state becomes critical; i.e., its eigenvalue becomes maximal. Indeed, this conclusion is confirmed by calculations. Figure 5 shows the functions of the critical states calculated for the five-layer structure under consideration where (a) thicknesses of the F layers neighboring the central S layer are increased to $d_F = 2.2\zeta_F$ and (b) the central S layer is narrowed to the thickness $d_S = 2\xi_S$. The states described by the functions $\Phi^{(2)}(z)$ and $\Phi^{(1)}(z)$ have the highest eigenvalue of the temperature in the former and latter cases, respectively. It is worth noting that (as is seen in Fig. 5) the process of the measurement of the resistive characteristics of structures with thicknesses changed in this way is accompanied by countercurrents.

DISCUSSION AND CONCLUSIONS

According to the calculations, the results obtained in the numerical example, namely, the suppression of superconductivity in outer S layers, the induction of weak countercurrents along the central F layers of the regular S/F structure with the thicknesses $d_F \sim d_F^*$ remain at any reasonable parameters (at least, if F layers are prepared from a weak ferromagnet). Moreover, the configurations of the state functions shown in Fig. 4 are quite universal. The existence of induced countercurrents is indirectly confirmed by the existence of spontaneous closed currents [17, 18], which have the same nature; in addition, this conclusion is easily tested in experiments. It is also worth noting that countercurrents generated due to the measuring transport current assignment in the process of the measurement of the resistive curve of the phase transition $R(T)$ tend to closure through shortest trajectories using natural inhomogeneities of the spatial distribution of nuclei of the superconducting phase, i.e., promote the clustering of the structure into individual conglomerates interacting with each other, as, for example, was proposed in [9].

The simplicity of the calculations by the presented formula makes it possible to experimentally form structures with given critical states of superconductivity (two examples of their configurations are given in Fig. 5). All results obtained in the single-mode approximation should obviously be confirmed by exact calculations and experiments. This will be an aim of the next work.

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