CONSTRUCTION OF THE MOTION EQUATIONS BASED ON THE SOLUTION OF THE INVERSE DYNAMICS PROBLEMS

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The approach to construction of program motion based on Erugin's method using the appropriate equations of a motion of considered mechanical system was presented. The constructed system of the equations allows realizing a motion with given properties of trajectory and velocity.

Inverse problems of dynamics are the definition of active forces, applied to the mechanical system, parameters of system and in addition imposed on it constrains, refer to as, at which the motion with given properties is one of possible motions of considered mechanical system. Thus the properties of a motion can be given by a different ways, for example as quantitative and qualitative restrictions for coordinates and speeds of a motion as invariant parities.

The works of the different authors are devoted to the solution of inverse dynamics problems [1, 2, 3]. Here we stop on the basic theoretical preconditions, which are necessary for solution of problems of constructing of program motions.

We assume that properties of a motion of mechanical system, which are defined by a vector $\mathbf{x}(x_1, \ldots, x_n)$ of generalized coordinates and vector $\dot{\mathbf{x}}(\dot{x}_1, \ldots, \dot{x}_n)$ of generalized velocities, are given as variety

$$\omega_\mu(\mathbf{x}, \dot{\mathbf{x}}, t) = c_\mu, \quad \mu = 1, \ldots, m \leq n. \qquad (1)$$

Concerning functions ω_μ we assume, that the equality $\omega_\mu(\mathbf{x}, \dot{\mathbf{x}}, t) = c_\mu$ are joint and are independent in some part of a phase space $G(\mathbf{x}, \dot{\mathbf{x}})$ at $t \geq t_0$.

The given variety of properties of a motion is in essence integrated variety of the appropriate equations of a motion of considered mechanical system. Naturally therefore that for the solution of inverse problems of dynamics it is necessary to construct the equations of a motion of considered mechanical system on given integrated variety so that the expressions $\omega_\mu(\mathbf{x}, \dot{\mathbf{x}}, t) = c_\mu$ are the integrals of these equations. Further, from the constructed equations it is necessary to determine required generalized forces, parameters and connections admitting a motion with given properties (1).

In special cases, when the structure of equations of a motion is known, but required additional forces and parameters of considered mechanical system for realizing of a motion with given properties are unknown, it is necessary to determine the equations of a motion on given integrated variety and to find required unknown equations.

When a part of the equations of a motion of considered mechanical system is a priori known, it is necessary for the solution of inverse problems of dynamics to build the missing equations on given integrated variety and to determine required generalized forces, parameters and interrelations, which allows realizing a motion with given properties. Thus, the solution of inverse problem of dynamics in sufficiently general mathematical interpretation is reduced to construction of the motion equations of mechanical system on a given integrated variety as properties of required motion. And the motion equations are necessary to be defined in the form

$$\ddot{\mathbf{x}} = X(\mathbf{x}, \dot{\mathbf{x}}, t). \qquad (2)$$

The construction of differential equations can be carried out using the Erugin's method [4]. According to this method, at first the necessary and sufficient conditions must be satisfied that the given integrals form the integrated variety of the differential equation system. These conditions can be obtained by comparing of the time derivatives of given integrals in the form of required equations of arbitrary functions, which are equal to zero on given integrated variety.

In our formulation the conditions for feasible motion with given properties (1) can be written as

$$\left(\mathrm{grad}_{\dot{\mathbf{x}}}\omega_\mu \cdot X\right) = R_\mu(\omega, \mathbf{x}, \dot{\mathbf{x}}, \mathbf{t}) - \varphi_\mu, \qquad (3)$$

where

$$\varphi_\mu = \left(\mathrm{grad}_{\mathbf{x}}\omega_\mu \cdot \dot{\mathbf{x}}\right),$$

and $R_\mu(\omega, \mathbf{x}, \dot{\mathbf{x}}, \mathbf{t})$ – functions, which are identically equal to zero at $c_\mu \neq 0$ and any at $c_\mu = 0$, and equal to zero on integrated variety Ω, for example, they can be holomorfic functions of variables $\omega_1, \ldots, \omega_n$ in area Ω_ϵ at $t \geq t_0$, which have members not less than first order in the decomposition on degrees of these variable.

In the case of $m = n$, we can find the required equations directly solving the equations (3):

$$\ddot{x}_v = \sum_{i=1}^{n} \frac{\Delta^{iv}}{\Delta}(R_i - \varphi_i), \qquad (4)$$

where $\Delta = \left|\frac{\partial\omega}{\partial x}\right| \neq 0$; and Δ^{iv} – algebraic adjunct for element i of the determinant Δ.

If $m < n$, it is more convenient to find the vector-function $\mathbf{X}$ of the right parts of the equations as a sum

$$\mathbf{X} = \mathbf{X}^v + \mathbf{X}^\tau, \qquad (5)$$

where vector $\mathbf{X}^v$ is ortogonal to variety $\Omega_{\dot{\mathbf{x}}}\{\omega(\mathbf{x}, \dot{\mathbf{x}}, t)_{x=inv}\} = 0$, and it can be found up to the Lagrange multipliers:

$$\mathbf{X}^v = \sum_{i=1}^{m} \lambda_i \mathrm{grad}_{\dot{\mathbf{x}}}\omega_i,$$

and vector $\mathbf{X}^\tau$ is a component of vector-function along variety $\Omega_{\dot{\mathbf{x}}}$, it is determined by a condition

$$\left(\mathrm{grad}_{\dot{\mathbf{x}}}\omega_\mu \cdot \mathbf{X}^\tau\right) = 0, \quad \mu = 1, \ldots, m. \qquad (6)$$

By substituting vector-function $\mathbf{X}$ in the form (5) into conditions of feasible motion (3), we get

$$\left(\mathrm{grad}_{\dot{\mathbf{x}}}\omega_\mu \cdot \mathbf{X}^v\right) = R_\mu - \varphi_\mu, \qquad (7)$$

and taking $\mathbf{X}^\tau$ into account we get

$$\lambda_i = \frac{1}{\Gamma}\sum_{j=1}^{m} \Gamma_{ij}(R_j - \varphi_j); \quad i = 1, \ldots, m,$$

where

$$\Gamma = |\mathrm{grad}_{\dot{\mathbf{x}}}\omega_i \cdot \mathrm{grad}_{\dot{\mathbf{x}}}\omega_j|_m^m \neq 0,$$

and Γ_{ij} – algebraic adjunct of element i, j of the determinant Γ.

Thus, we finally get

$$\mathbf{X}^v = \frac{1}{\Gamma}\sum_{i,j}^{1,m} \Gamma_{ij}(R_j - \varphi_j)\mathrm{grad}_{\dot{x}}\omega_j.$$

The components of vector-functions $\mathbf{X}^\tau$ are determined solving of a system of the linear equations (6) and can be written as

$$\mathbf{X}_r^\tau = -\sum_{s=m+1}^{n} D^{rs}Q_s, \quad r = 1, \ldots, m,$$

where

$$\mathbf{X}_s^\tau = DQ_s, \quad s = (m+1), \ldots, n;$$

D^{rs} – the determinant, which is received by replacing its column i to column s of a matrix $\left(\frac{\partial\omega}{\partial\dot{\mathbf{x}}}\right)_n^m$;

$Q_s = Q_s(\mathbf{x}, \dot{\mathbf{x}}, t)$ – any functions.

So, the required equation system for construction of the motion equations can be written in the following form:

$$\begin{aligned} &\ddot{x}_r = \frac{1}{\Gamma}\sum_{i,j}^{1,m} \Gamma_{ij}(R_j - \varphi_j)\frac{\partial\omega_i}{\partial\dot{x}_r} - \sum_{s=m+1}^{n} D^{rs}Q_s; \\ &r = 1, \ldots, m; \\ &\ddot{x}_s = \frac{1}{\Gamma}\sum_{i,j}^{1,m} \Gamma_{ij}(R_j - \varphi_j)\frac{\partial\omega_i}{\partial\dot{x}_s} + DQ_s; \\ &s = (m+1), \ldots, n. \end{aligned} \qquad (8)$$

As we see, the solution of a general problem of construction of the motion equations contains unknown functions $R_j(\omega, \mathbf{x}, \dot{\mathbf{x}}, t)$ and $Q_s(\mathbf{x}, \dot{\mathbf{x}}, t)$, which are not considered in this paper. Surely, these functions must be chosen so that conditions of existence and uniqueness of the solution of combined equations (8) in area Ω_ϵ are satisfied.

The constructed system of the equations (8), which allows realizing a motion with given properties (3), can be presented in the form of vector equation

$$\ddot{\mathbf{x}} = \frac{1}{\Gamma}\sum_{i,j}^{1,m} \Gamma_{ij}(R_j - \varphi_j)\mathrm{grad}_{\dot{\mathbf{x}}}\omega_i + \mathbf{X}_s^\tau,$$

where vector $\mathbf{X}_s^\tau$ is determined by conditions (6).

We have got the solution in a wide broad statement of problem using universality of method, and received solution can be used for construction of the equations of a motion in many inverse problems of dynamics.

We notice, when we solve of inverse problems of dynamics in some special cases it is expediently to construct equations of a motion using only some of given integrals at first, and then to build missing equations using the remaining given integrals.

Conclusion

1. In received expressions for control action there are some any functions, equal to zero at the motion program, if the motion occurs without deviations from given. If there is the deviation from the program, control action is necessary to be found in view of stability.

2. A development of the common theory of construction of multilevel multi-coordinate robot systems devices by dynamic criteria based on solution of inverse dynamics problems is described.

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