Analytic properties of solutions belonging to a family of third-order nonlinear dynamical systems with no chaotic behavior

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The Painlevé analysis of solutions is made to a family of dissipative third-order dynamical systems showing no chaotic behavior. It is found, that none of the systems under study is of the Painlevé type. Moreover, two systems from the above family are characterized by the fact that one of their solution components has no movable critical singular points at all.

One of the imporatant events in classical nonlinear physics of mid-twentieth century was the realization that nonlinear determenistic equations could have chaotic solutions that exhibited both a sensitive dependence on initial conditions and long-term unpredictability. An interesting and remaining to be solved is the problem of identifying minimum necessary conditions for chaos.

In [1], computer simulation was used to obtain 19 third-order dynamic systems with complex chaotic behavior, which are algebraically simpler than the well-known Lorenz and Rössler ones. The distinctive difference of the above Sprott systems is that their right-hand parts contain either 6 components and one quadratic nonlinearity or 5 components and two quadratic nonlinearities.

1. The authors of [2] proved that none of the systems listed below had chaotic behavior.

$$\dot{x} = y^2 - x, \quad \dot{y} = z, \quad \dot{z} = x. \tag{1.1}$$

$$\dot{x} = y^2 + z, \quad \dot{y} = x, \quad \dot{z} = -z. \tag{1.2}$$

$$\dot{x} = yz - x, \quad \dot{y} = x, \quad \dot{z} = y. \tag{1.3}$$

$$\dot{x} = y^2, \quad \dot{y} = x + z, \quad \dot{z} = -z. \tag{1.4}$$

$$\dot{x} = y^2, \quad \dot{y} = z - y, \quad \dot{z} = x. \tag{1.5}$$

$$\dot{x} = y^2, \quad \dot{y} = z, \quad \dot{z} = x - z. \tag{1.6}$$

$$\dot{x} = yz, \quad \dot{y} = x, \quad \dot{z} = x - z. \tag{1.7}$$

$$\dot{x} = yz, \quad \dot{y} = x, \quad \dot{z} = y - z. \tag{1.8}$$

Note, that the right-hand parts of each of dissipative systems (1.1)—(1.8) contain one quadratic nonlinearity. Assuming the independent variable t to be complex, let us determine whether the general solution of the systems (1.1)—(1.8) has no moving critical singular points. that is, whether the so called Painlevé property is fulfilled for them.

The following statements are true

Theorem 1. None of the systems (1.2), (1.4) passes the Painlevé test and they do not possess the Painlevé property. At the same time, a component (z) of these systems has no movable critical singular points at all.

Theorem 2. None of the systems (1.1), (1.3), (1.5)—(1.8) passes the Painlevé test and they do not have the Painlevé property.

Theorem 3. Equation

$$y\dddot{y} + y\ddot{y} - \dot{y}\ddot{y} - y^2\dot{y} = 0 \tag{1}$$

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is not a Painlevé-type equation.

The validity of this statement follows from the fact that the system (1.7) is equivalent to equation (1).

2. In [2], it is also proved the absence of chaos in systems of differential equations

$$\dot{x} = y^2 + yz, \quad \dot{y} = x, \quad \dot{z} = -z. \tag{2.1}$$

$$\dot{x} = y^2 + z^2, \quad \dot{y} = x, \quad \dot{z} = -z. \tag{2.2}$$

$$\dot{x} = y^2 - x, \quad \dot{y} = xz, \quad \dot{z} = \varepsilon y. \tag{2.3}$$

$$\dot{x} = y^2 - x, \quad \dot{y} = xz, \quad \dot{z} = kz. \tag{2.4}$$

$$\dot{x} = y^2 - x, \quad \dot{y} = z^2, \quad \dot{z} = x. \tag{2.5}$$

$$\dot{x} = y^2 + y, \quad \dot{y} = xz, \quad \dot{z} = -z. \tag{2.6}$$

$$\dot{x} = y^2 + z, \quad \dot{y} = x^2, \quad \dot{z} = -z. \tag{2.7}$$

$$\dot{x} = y^2 + z, \quad \dot{y} = xz, \quad \dot{z} = -z. \tag{2.8}$$

$$\dot{x} = yz - x, \quad \dot{y} = x^2, \quad \dot{z} = \varepsilon x. \tag{2.9}$$

$$\dot{x} = yz - x, \quad \dot{y} = x^2, \quad \dot{z} = y. \tag{2.10}$$

$$\dot{x} = yz - x, \quad \dot{y} = x^2, \quad \dot{z} = kz. \tag{2.11}$$

$$\dot{x} = yz - x, \quad \dot{y} = \varepsilon xz, \quad \dot{z} = y. \tag{2.12}$$

$$\dot{x} = yz - x, \quad \dot{y} = z^2, \quad \dot{z} = \varepsilon x. \tag{2.13}$$

$$\dot{x} = \varepsilon y - x, \quad \dot{y} = xz, \quad \dot{z} = x^2. \tag{2.14}$$

$$\dot{x} = \varepsilon y - x, \quad \dot{y} = xz, \quad \dot{z} = y^2. \tag{2.15}$$

$$\dot{x} = y - x, \quad \dot{y} = z^2, \quad \dot{z} = x^2. \tag{2.16}$$

$$\dot{x} = y - x, \quad \dot{y} = z^2, \quad \dot{z} = xy \tag{2.17}$$

where k is a parameter ($k < 1$) and $\varepsilon^2 = 1$.

Systems (2.1)—(2.17) are dissipative systems, and their right-hand parts contain two quadratic nonlinearities.

The following statements are true

Theorem 4. None of the systems (2.1), (2.2), (2.4), (2.6)—(2.8), (2.11) is of the Painlevé type. At the same time, a component (z) of these systems has no movable critical singular points at all.

Theorem 5. None of the systems (2.3), (2.5), (2.9), (2.10), (2.12)—(2.17) is of the Painlevé type.

Theorem 6. The system (2.3) is equivalent to equation

$$z\dddot{z} = \dot{z}\ddot{z} - z\ddot{z} + \varepsilon z^2\dot{z}^2. \tag{2}$$

Theorem 7. The system (2.9) is equivalent to equation

$$z\dddot{z} = \dot{z}\ddot{z} - z\ddot{z} + \varepsilon z^2\dot{z}^2 + \dot{z}^2. \tag{3}$$

Theorem 8. The system (2.12) is equivalent to equation

$$z\dddot{z} = \dot{z}\ddot{z} - z\ddot{z} + \varepsilon\dot{z}z^3. \tag{4}$$

Theorem 9. The systems (2.13) and (2.14) are equivalent to equation

$$w\dddot{w} = \dot{w}\ddot{w} - w\ddot{w} + \dot{w}^2 + \varepsilon w^4 \tag{5}$$

by z and x respectively.

Theorem 10. The system (2.15) is equivalent to equation

$$x\dddot{x} = \dot{x}\ddot{x} - x\ddot{x} + \dot{x}^2 + \varepsilon x^2(x + \dot{x}^2). \tag{6}$$

Theorem 11. None of the equations (2)—(6) is of the Painlevé type.

References

[1] Sprott J C 1994 *Some simple chaotic flows Phys. Rev E* **50**. pp R647-R650.

[2] Heidel J and Zhang Fu 1999 *Nonchaotic behaviour in three-dimensional quadratic systems Nonlinearity* **10**. pp 1289 -1303

Vlasov-Maxwell-Einstein equation and analysis of Λ-term with the help of kinetic theory and post-Newtonian approximation

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The possibility to obtain an analog of Milne–McCrea model with the help of Vlasov–Poisson equation is considered. The simplest nonrelativistic analog of Maxwell–Einstein action is introduced, from which we deduce Vlasov-Poisson-Poisson equation for an electrostatics with gravitation. The nonrelativistic limit of Einstein–Gilbert action is studied and Vlasov–Poisson–Poisson equation is also obtained with cosmological Λ–term then the accounting of electromagnetism is added. An equation of Vlasov type is derived which can be proposed for dark matter and perhaps for dark energy.

The nonrelativistic analog of Friedmann equation is a self–gravitating sphere or Milne–McCrea model [1]. At the same time Friedmann model can be it is obtained as the exact solution of Vlasov–Poisson equations for system of massive particles:

$$\frac{\partial f}{\partial t} + \left(\frac{\mathbf{p}}{m}, \frac{\partial f}{\partial \mathbf{x}}\right) - \left(m\frac{\partial U}{\partial \mathbf{x}}, \frac{\partial f}{\partial \mathbf{p}}\right) = 0, \quad \Delta U = -4\pi m\gamma \int f(\mathbf{x}, \mathbf{p}, t)\, d\mathbf{p}. \tag{1}$$

Solutions of Milne–McCrea type can be found by substitution $f(\mathbf{x}, \mathbf{p}, t) = \int \delta\big(\mathbf{x} - \mathbf{X}(\mathbf{q}, t)\big)\delta\big(\mathbf{p} - \mathbf{P}(\mathbf{q}, t)\big)\rho(\mathbf{q})d\mathbf{q}$, using Lagrangian coordinates $\mathbf{q}$ [2]. The task of obtaining of the relevant solution

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