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## EVOLUTIONARY CHAOTIC DYNAMICS IN A GUNN-INSTABILITY SYSTEM

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A drift-diffusion Gunn effect model is used to analyse the complex behaviour of the natural and driven Gunn oscillations. The results of the numerical simulation of the spatio-temporal chaotic dynamics are presented. It was shown that Gunn device might exhibit a low-dimensional chaotic behavior having the slow evolution throughout a long transient from one to another clumped state.

*Keywords:* nitrides, millimetric waves, Monte Carlo method, scattering frequency.

### Introduction

The most common microwave diode sources are the Gunn diode and the IMPATT diode, both of which directly convert a dc bias to RF power in the frequency range up to about 300 GHz (usually in the second harmonic mode). The Gunn diode is a transferred electron device that uses a bulk semiconductor, usually GaAs or InP, where the negative differential resistance effect leads to the high-field domain dynamics and the well-known Gunn microwave oscillations. The experimental discovery of oscillating microwave current in GaAs samples with ohmic contacts subjected to the high electric fields was made by Gunn still in 1963 [1, 2]. But even nowadays the Gunn effect is a subject of the experimental and theoretical investigations [3, 4]. Having found a system with a natural oscillation due to the travelling-wave motion, it is natural to ask whether harmonic forcing would lead to chaos with spatial structure. Understanding transport behavior of the driven oscillatory semiconductors is important both fundamentally and technologically. From a fundamental perspective, applied signal adds several degrees of freedom to the dynamics of charge transport and makes possible the observation of various interesting phenomena such as dynamical non-linearities, instabilities, and chaotic phenomenon. From a technological point of view, the presence of a forcing signal in semiconductor microwave oscillators can lead to a variety of parasitic effects such as multifrequency regime, complicated high-frequency oscillations in biasing circuit and so on. During the last fifteen years the nonlinear dynamics of periodically forced Gunn devices has been investigated [5–11]. One of the frequently observed phenomena is phase locking of the transit Gunn oscillation to the periodical forcing. Among observed phenomena also were period doubling, chaotic response, and phase singularity.

This work is closely connected with [12, 13]. Here is also described the nonlinear dynamical properties obtained numerically for 3  $\mu\text{m}$  GaAs Gunn device driven by harmonic microwave signal. We present here the new results concerning the natural Gunn oscillations and evolutionary dynamics of the natural and driven Gunn current oscillation.

### Description of the Gunn device model

In this chapter, we give the only some remarks about our mathematical model. One can find its detailed description in Ref. [13]. We used the drift-diffusion model for nonlinear electronic trans-

port in in  $n^+ - n - n^+$  GaAs structures that yields complex and chaotic dynamic behavior under fixed time-independent external dc biasing voltage and time-dependent external dc voltage (together with microwave signal). The application of an external dc bias voltage  $V_{dc} > V_{th}$  ( $V_{th}$  — the threshold voltage for Gunn device used) leads to the periodic formation near one contact  $n^+$  and propagation to another contact  $n^+$  of bunches of electrons (high-field domains) and causes microwave self-sustained oscillation in an external circuit. Periodic external excitation, i.e. forced oscillation, is known to be an important means in the investigation of a system, which has an inherent oscillatory character. Usually the applied signal  $V(t)$  essentially changes and complicates the oscillation dynamics.  $V(t)$  is the total voltage applied to a GaAs structure, consisting of the dc voltage  $V_{dc}$  and the external microwave signal with amplitude  $V_{ac}$  and frequency  $f_d$  :

$$V(t) = V_{dc} + V_{ac} \sin(2\pi f_d \cdot t). \quad (1)$$

The drift-diffusion model, consisting of the current continuity and particle current relationships, and Poisson's equation is a system of coupled nonlinear partial differential equations. They were integrated numerically using a well-known Runge-Kutta scheme.

### Numerical simulation

In all simulations to be discussed, we have fixed the following parameters of the  $n$ -GaAs sample: length of the transit region  $L = 3 \mu\text{m}$ , cross-sectional area  $S = 10^{-5} \text{cm}^2$ , static diode capacitance  $C_{diode} = 0.036 \text{pF}$ , threshold voltage  $V_{th} = 0.77 \text{V}$ , homogeneous doping density profile with  $n_o(x) = 5 \cdot 10^{15} \text{cm}^{-3}$  and with doping notch  $0.25 \mu\text{m}$  length and  $3.5 \cdot 10^{15} \text{cm}^{-3}$  doping density, located  $0.3 \mu\text{m}$  from cathode. For dc bias below the threshold we find steady-state, non-oscillatory behaviour. Above threshold, self-sustained periodic oscillations occur. The typical shape of the natural oscillation with frequency  $f_0 \approx 27.43 \text{GHz}$  is shown in fig. 1 ( $V_{ac} = 0$ , it means, that  $A = V_{ac} / (V_{dc} - V_{th}) = 0$ ). The space-time electric field and electron concentration characteristics for this case are given in [9]. Fig. 2, 3 show the attractor and the Poincare section of this oscillation. To our surprising we can see that they have non-trivial strange geometrical structure. Taking into consideration that the calculated correlation dimension is approximately  $\nu = 1.08$  and the computed first Lyapunov exponent value is nearly 0.5 one can conclude that we deal with the oscillation, which formally can be defined as chaotic [12–15]. Here chaotic refers to exponential divergence of nearby trajectories and strange means that the dimension of the attractor is not an integer. We can also see that, on the whole, the oscillation shape is not changed. It can be supposed that a strong convergence during fairly short time intervals is outweighed by the divergence of nearby trajectories that occurs within other time intervals. This example shows that the evolutionary stability and chaotic dynamics are perfectly compatible.

Now, we turn to the study of the responses of this self-oscillatory system to periodically varying applied voltage. This problem is related to the operation of Gunn diodes inserted in a microwave resonant circuit [16, 17]. In fig. 4 one can see that the influence of the external forcing leads to the specific modulation of the natural Gunn oscillation. The resultant shape is a sequence of complicated asymmetrical oscillations, which seem almost periodic and similar to polar-modulated ones [9, 15]. To understand physical processes in the Gunn-effect structure leading to current oscillations, we modelled earlier the travelling of a charge layer in more detail [9]. Modelling shows that the oscillations rather complicated shape is the result of a specific modulation of the effective length of the Gunn device transit region. Fig. 5 shows that the increase of amplitude leads to the successive multiplication of the applied oscillation period by  $N$ , where  $N = 1, 2, 3, \dots, 35$ . Denoting the period of the resultant oscillations as  $T$ , we have that  $T \approx N \cdot T_d$ , where  $T_d = 1/f_d$ . The shape of the curve is similar to a staircase, the steps length of which (stable regions) is gradually decreasing with the growth of the microwave amplitude  $A$ .

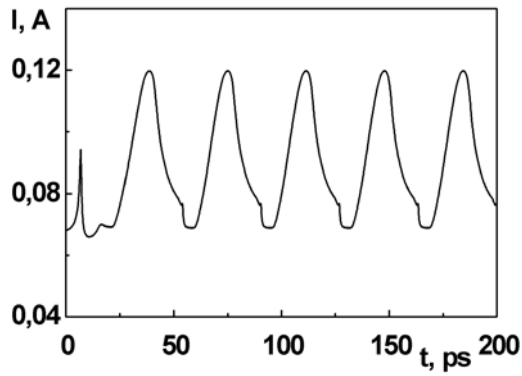


Fig. 1. Current time dependence of the transit Gunn oscillation at  $V_{dc}/V_{th}=3,9$ ;  $V_{ac}=0$ ;  $T_0=36,43$  ps

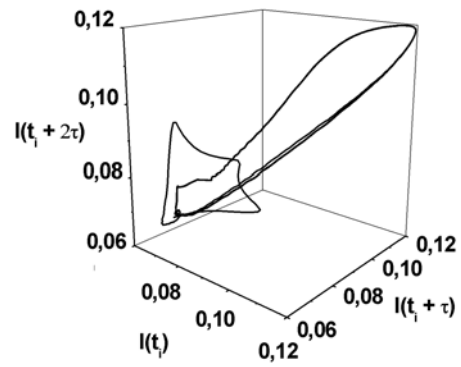
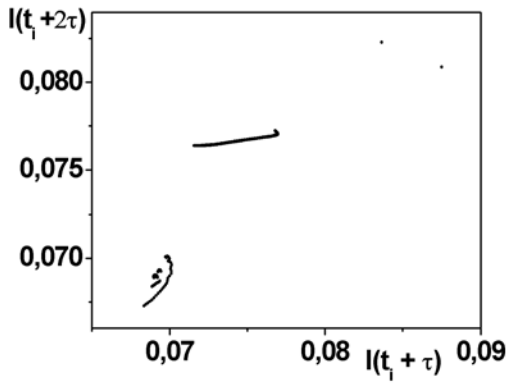
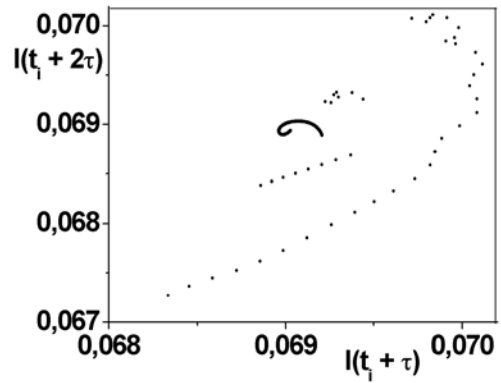


Fig. 2. Attractor of the transit Gunn oscillation for  $V_{dc}/V_{th}=3,9$ ;  $V_{ac}=0$ ;  $\tau=1$  ps;  $t_i=0,1 \cdot i$  ps;  $i=0 \dots 306 \cdot 10^3$ ; ( $t_i \approx 0 \dots 840 \cdot T_0$ )



a



b

Fig. 3. a — Poincaré section of attractor for  $A=0$ ; b — enlarged part of the Poincaré section:  $V_{dc}/V_{th}=3,9$ ;  $T_0=36,43$  ps;  $\tau=1$  ps;  $I(t_i)=0,07$  A=const;  $t_i=0,1 \cdot i$  ps;  $i=0 \dots 306 \cdot 10^3$ ; ( $t_i \approx 0 \dots 840 \cdot T_0$ )

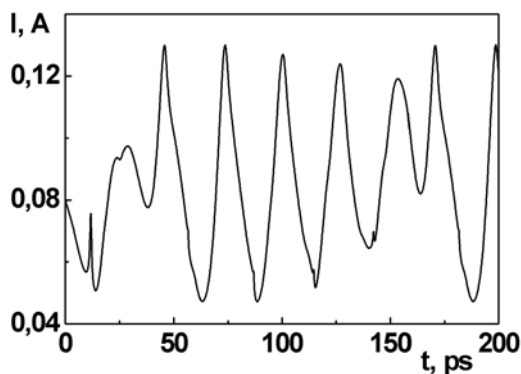


Fig. 4. Current time dependence of the forced Gunn oscillation at  $V_{dc}/V_{th}=3,9$ ,  $V_{ac}=1,35$  V,  $A=0,605$ ,  $f_d=40$  GHz,  $f_d/f_0=1,456$

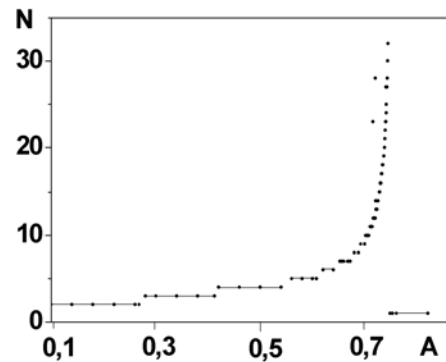


Fig. 5. Period multiplication of the driven Gunn oscillation as a function of the applied microwave voltage for  $V_{dc}/V_{th}=3,9$ ;  $f_d=40$  GHz;  $f_d/f_0=1,456$

The transition between the nearby stable regions gives a rise to the narrow windows of non-periodic responses like shown in fig. 5 and more complicated oscillations. The correlation dimension values of the resultant oscillations less than two. As we can see the competition between the natural oscillation due to the space-charge domain dynamics and the periodic forcing can result in a low-dimensional fractal oscillation. It is important that the current forms and bifurcation processes obtained numerically fairly well agree with the experimental data obtained earlier using a millimetre-wave Gunn oscillator [10]. It means that the model used reflects the real mechanism of the nonlinear interaction in Gunn devices. Fig. 6, 7 show the attractor and the Poincaré section of the current oscillation at  $A=0.605$  (see also fig. 4). The calculations show that the first Lyapunov exponent is positive and the correlation dimension is about 1.95. It means that we also deal with the chaotic behaviour.

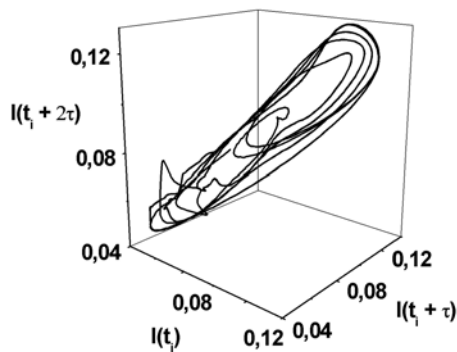


Fig. 6. Attractor of the driven Gunn oscillation for  $A=0,605$ ,  $T_d=25,0$  ps,  $\tau=1$  ps;  $t_i=0,1 \cdot i$  ps;  $i=0 \dots 306 \cdot 10^3$ ; ( $t_i \approx 0 \dots 1224 \cdot T_d$ )

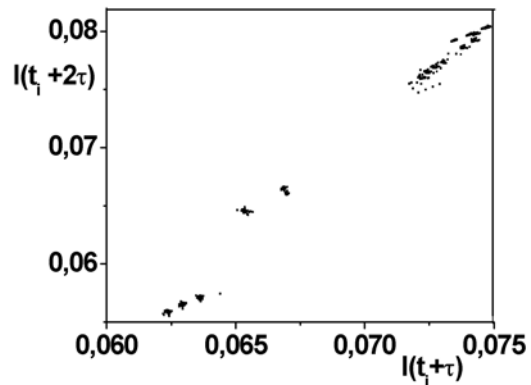
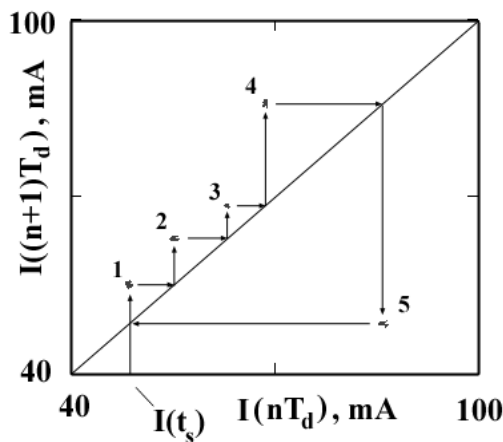
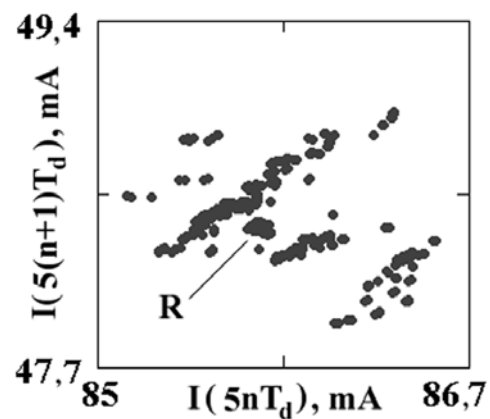


Fig. 7. Poincaré section of the attractor in fig. 6 for  $A=0,605$ ;  $V_{dc}/V_{th}=3,9$ ;  $\tau=1$  ps;  $I(t_i)=0,07$  A=const;  $t_i=0,1 \cdot i$  ps;  $i=0 \dots 306 \cdot 10^3$ ; ( $t_i \approx 0 \dots 1224 \cdot T_d$ )

Finally, in fig. 8 we present the return maps of this oscillation. To our surprise one can see that after fairly long period of comparative stability the oscillation becomes more irregular. The comparison of figures 8,*b,c* shows that the part of the return map in figure 8,*b*, marked R, is practically the full return map shown in fig. 8,*c*. It means that the used model demonstrates the evolutionary dynamics by slow changing from one to another clumped state. Nevertheless, even by the end of the calculation, the oscillation had a high predictability.



a



b

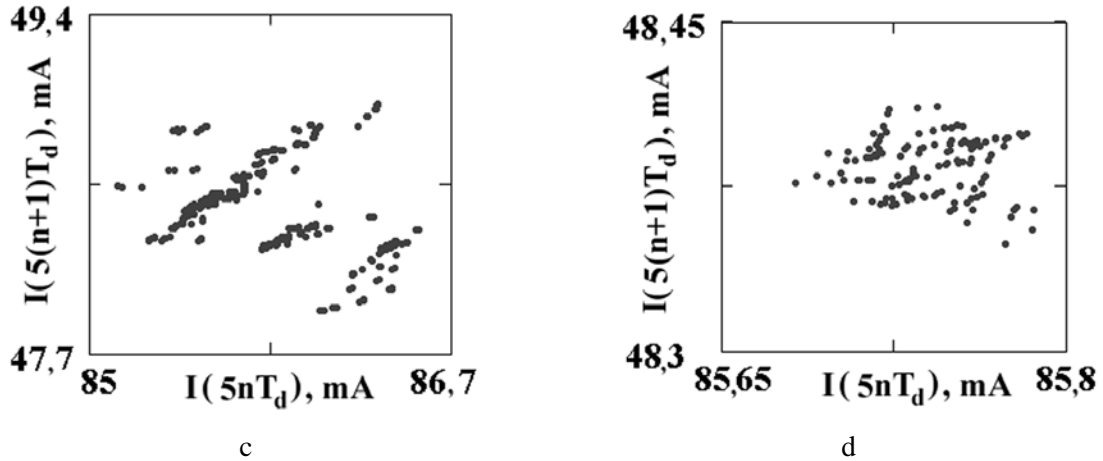


Fig. 8. Return maps for  $A=0,605$ ,  $T_d=25,0$  ps,  $f_d/f_0=1,456$ ,  $t_s=12$  ps,  $a$  —  $n=0\dots3900$ ;  $b$  —  $n=1\dots3900$ ;  $c$  —  $n=1\dots570$ ;  $d$  —  $n=600\dots3900$

### Conclusions

It was shown that the drift-diffusion transport model of the Gunn-effect structure could reproduce strange chaotic behaviour. Nevertheless, the time-dependence of the current keeps a very high level of predictability. It fails to detect the chaotic nature of the system. The examples given show that the evolutionary stability and chaotic dynamics are fairly well compatible. This phenomenon connected not only with the applying of the external sinusoidal signal, that leads to the suppression or maintenance of the travelling charge layers [9]. It turned out that even the transit Gunn oscillations also reproduce the low-dimensional strange chaotic behavior. It is not an ordinary phenomenon and we connect it with the complex form of the drift velocity and diffusivity characteristics depending on electric field [9, 13]. However, we want to emphasize that the obtained results are only preliminary and do not give detailed information about reasons of such behaviour. The obtained results can be used to design chaos free Gunn oscillators. But nowadays, the more attractive stimulus to study self-oscillating system of this type is associated with their prevalence in the majority of branches of science and engineering. For example, a similar nonlinear dynamics one can see in Nature evolution. The fundamental aim of modeling of real ecosystems is to better understand the processes and mechanisms underlying the evolution of patterns in ecosystems. There is evidence of the evolutionary chaotic dynamics and sudden changes in ecosystems, which have led to mass extinctions or large-scale alterations in species diversity [18,19]. It may be useful to make an attempt to apply our model when studying these processes.

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### Abstract

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