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BASICS OF DIGITAL PROCESSING OF IMAGES

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Abstract. The article describes replacing the continuous image with a discrete different ways. For example, an image can be replaced with them by choosing a system of any orthogonal functions and calculating the image display coefficients according to this system (based on this). The variety of bases allows you to create different discrete images of a permanent image. However, the most common is periodic sampling, in particular sampling with a rectangular raster, as mentioned above. Such a sampling method can be considered as one of the options for using an orthogonal basis that has been modified as its elements-using functions.

Keywords: distributed system, lossless data processing, ensuring continuity of data flow, various types of information, image analysis, quality image.

Introduction.

Images obtained in information systems are digital. Therefore, transferring them to this form is a mandatory operation if it is necessary to use digital processing, transmission, storage. As with one-dimensional signals, this transformation involves two procedures. The first consists of replacing a continuous frame with a discrete one and is usually called sampling, while the second replaces a set of constant light values with a set of quantized values and is called quantization. In digital form, a binary number is assigned to each of the quantized values of brightness, which allows the image to be inserted into the computer [1].

The two-dimensional nature of the image compared to traditional signals includes additional possibilities for optimizing the digital image to reduce the amount of digital data received. In this regard, the question of the best placement of quantization levels, as well as the use of various rasts [1.1 ... 1.3] and other aspects of this problem were studied. However, it must be said that in most cases, sampling and uniform quantization of brightness are practiced, based on the use of a rectangular raster. This is due to the simplicity of performing the corresponding operations and the relatively small benefit of using optimal transformations. When a rectangular raster is used in the final form, the digital image is usually a matrix, whose rows and columns correspond to the rows and columns of the image [2-4].

Replacing a continuous image with a discrete one can be done in different ways. For example, an image can be replaced with them by choosing a system of any orthogonal functions and calculating the image display coefficients according to this system (based on this). The variety of bases allows you to create different discrete images of a permanent image. However, the most common is periodic sampling, in particular sampling with a rectangular raster, as mentioned above. Such a sampling method can be considered as one of the options for using an orthogonal basis that has been modified as its elements-using functions. Next, mainly after [1.1], we will take a closer look at the main features of rectangular discretization [5].

let $x(t_1, t_2)$ be a continuous image, the corresponding discrite $x(i_1, i_2)$, obtained continuously by rectangular sampling. This means that the relationship between them is determined by the following expression:

$$x(i_1, i_2) = x(i_1 \Delta t_{1,} i_2 \Delta t_2) \quad (1)$$

Where $\Delta t_1, \Delta t_2$ - vertical and horizontal steps or sampling intervals, respectively.

Picture.1 the rectangle shows the location of the samples in the plane with the samples (t_1, t_2).

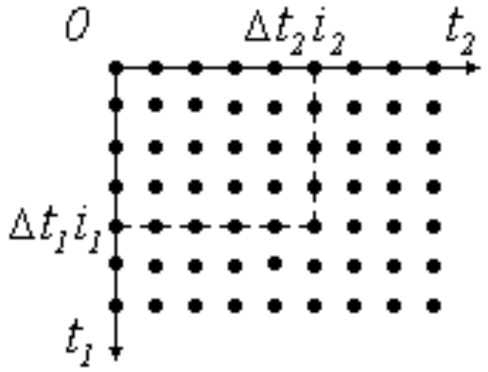

Figure 1. Sample location for rectangular sampling

The main question that arises when replacing a continuous image with a discrete one is to determine the conditions under which such a replacement is complete, i.e. not accompanied by a loss of information in a continuous signal. If it is possible to restore a continuous signal with a discrete signal, there will be no losses. From a mathematical point of view, the question is to restore a continuous signal in two-dimensional intervals between nodes whose values are known, or, in other words, to carry out two-dimensional interpolation. This question can be answered by analyzing the spectral characteristics of continuous and discrete images [6].

2D continuous frequency spectrum $X_H(\Omega_1, \Omega_2)$ continuous signal $x_H(t_1, t_2)$ determined by two-dimensional direct Fourier transform:

$$X_H(\Omega_1, \Omega_2) = \iint_{-\infty -\infty}^{\infty\ \infty} x_H(t_1, t_2) \exp\left(-j\Omega_1 t_1 - j\Omega_2 t_2\right) dt_1 dt_2 \quad (2)$$

This corresponds to a two-dimensional inverse continuous Fourier transform:

$$x_H(\mathrm{t}_1, \mathrm{t}_2) = \frac{1}{4\pi^2} \iint_{-\infty -\infty}^{\infty\ \infty} X_H \cdot (\Omega_1, \Omega_2) \exp\,(j\Omega_1 t_1 + j\Omega_2 t_2) d\Omega_1 d\Omega_2 \ (3)$$

The last relation is true for any values of t_1,t_2, including t_1=i_1 Δt_1 in rectangular grid nodes $t_1 = i_1 \Delta t_1$, $t_2 = i_2 \Delta t_2$ Therefore, taking into account (1), for signal values in nodes (3), the relationship can be written in the following form:

$$x(i_1, i_2) = \frac{1}{4\pi^2} \iint_{-\infty -\infty}^{\infty\ \infty} X_H \cdot (\Omega_1, \Omega_2) \exp\left(j\Omega_1 i_1 t_1 + j\Omega_2 i_2 t_2\right) d\Omega_1 d\Omega_2 \ (4)$$

To shorten

$$\frac{-\pi + 2\pi k_1}{\Delta t_1} \le \Omega_1 \le \frac{\pi + 2\pi k_1}{\Delta t_1}, \qquad \frac{-\pi + 2\pi k_2}{\Delta t_2} \le \Omega_2 \le \frac{\pi + 2\pi k_2}{\Delta t_2},$$

the area of a rectangle in a two-dimensional frequency field we mark with $S(k_1, k_2)$.
Separate the calculation of the integral in (4) by the entire frequency field

$$x(i_1, i_2) = \frac{1}{4\pi^2} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} \iint_{S(k_1,k_2)} X_H(\Omega_1, \Omega_2) \times$$
$$\times \exp(j\Omega_1 i_1 t_1 + j\Omega_2 i_2 t_2)\, d\Omega_1 d\Omega_2.$$

$\Omega_1 \to \Omega_1 - 2\pi k_1 / \Delta t_1$, $\Omega_2 \to \Omega_2 - 2\pi k_2 / \Delta t_2$ by changing the variables according to the rule of the We integration space of k_1 and k_2
we will achieve independence from the numbers:

$$x(i_1, i_2) = \frac{1}{4\pi^2} \int_{-\frac{\pi}{\Delta t_1}}^{\frac{\pi}{\Delta t_1}} \int_{-\frac{\pi}{\Delta t_2}}^{\frac{\pi}{\Delta t_2}} \sum_{k_1} \sum_{k_2} X_H\,(\Omega_1 + \frac{2\pi k_1}{\Delta t_1}, \Omega_2 + \frac{2\pi k_2}{\Delta t_2}) \times$$
$$\times \exp(j\Omega_1 i_1 \Delta t_1 + j\Omega_2 i_2 \Delta t_2)\, d\Omega_1 d\Omega_2.$$

Here for any integer values of k and i

$$\exp(-j2\pi k i) = 1$$

taken into account. This expression is very close in form to the inverse Fourier transform. The only difference is the wrong form of the exponential factor. To give it the desired shape, we use normalized frequencies chastotalarni $\omega_1 = \Omega_1 \Delta t_1, \omega_2 = \Omega_2 \Delta t_2$ we enter and change the variables accordingly. As a result, we get:

$$x(i_1, i_2) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{1}{\Delta t_1 \Delta t_2} \sum_{k_1} \sum_{k_2} X_H(\frac{\omega_1 + 2\pi k_1}{\Delta t_1}, \frac{\omega_2 + 2\pi k_2}{\Delta t_2}) \times$$
$$\times \exp\,(j\omega_1 i_1 + j\omega_2 i_2) d\omega_1 d\omega_2 \qquad (5)$$

Now the expression (5) has the form of reverse Fourier Transform, which means that the function under the integral sign

$$X(\omega_1, \omega_2) = \frac{1}{\Delta t_1 \Delta t_2} \sum_{k_1} \sum_{k_2} X_H(\frac{\omega_1 + 2\pi k_1}{\Delta t_1}, \frac{\omega_2 + 2\pi k_2}{\Delta t_2}) \qquad (6)$$

is a two-dimensional spectrum of a discrete image. In the plane of non-normalized frequencies (6), the expression has the following appearance:

$$X(\Omega_1 \Delta t_1, \Omega_2 \Delta t_2) = \frac{1}{\Delta t_1 \Delta t_2} \sum_{k_1} \sum_{k_2} X_H(\Omega_1 + \frac{2\pi k_1}{\Delta t_1}, \Omega_2 + \frac{2\pi k_2}{\Delta t_2}) \quad (7)$$

(7) from the two-dimensional spectrum of a discrete image Ω_1 and Ω_2 along the frequency axes, respectively $\frac{2\pi k_1}{\Delta t_1}$ and $\frac{2\pi k_1}{\Delta t_2}$ it follows that a rectangle with periods is periodic. Discrete representation

$X(\Omega_1, \Omega_2)$ spectrum apart $\frac{2\pi k_1}{\Delta t_1}$ and $\frac{2\pi k_1}{\Delta t_1}$ infinite number of continuous images that differ in frequency shifts $X_H(\Omega_1, \Omega_2)$ it is formed as a result of the sum of its Spectra. Continuous in Figure 2 (2.a-picture) and discrete (2.B-image) the connection between two-dimensional spectra of images is shown qualitatively [7].

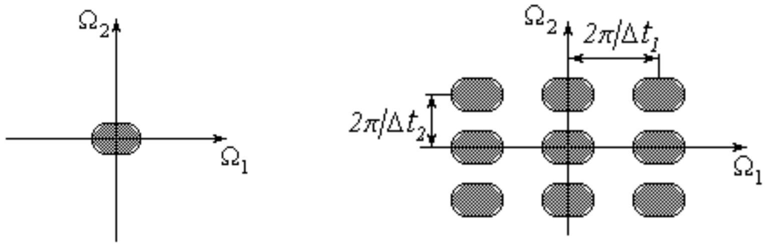

Figure 2. (2.a-picture) and discrete (2.b-image)

The result of the assembly depends mainly on the values of these frequency shifts or in other words, $\Delta t_1, \Delta t_2$ depends on the choice of sampling intervals. Let's assume, $X_H(\Omega_1, \Omega_2)$ the spectrum of a continuous image is not zero in some two-dimensional region close to zero frequency, that is, it is represented by a two-dimensional finite function If in this case the sampling intervals $X_H(\Omega_1, \Omega_2) = 0$ with $|\Omega_1| \geq \pi/\Delta t_1, |\Omega_2| \geq \pi / \Delta t_2$ when choosing with, it does not happen to put separate branches in the formation of the aggregate (7). Consequently, each rectangle has a cross section $S(k_1, k_2)$ in only one term is different from zero. In particular, for $k_1 = 0, k_2 = 0$ we have: $X(\Omega_1, \Omega_2) = \frac{1}{\Delta t_1 \Delta t_2} X_H(\Omega_1, \Omega_2)$ pri $|\Omega_1| \geq \pi/\Delta t_1, |\Omega_2| \geq \pi / \Delta t_2$ (8)

Thus, the frequency field $S(0,0)$ in continuous and discrete spectra of images correspond to a constant factor. In this case, the spectrum of a discrete image in this frequency zone contains complete information about the spectrum of a continuous image. We note that this coincidence occurs only under conditions determined by a good selection of sampling intervals. Note that according to (8) the fulfillment of these conditions must meet the requirements $\Delta t_1, \Delta t_2$, achieved at sufficiently small values of sampling intervals:

$$\Delta t_1 \leq \pi/\Omega_{1gr}, \Delta t_2 \leq \pi / \Omega_{2gr} \qquad (9)$$

Here $\Omega_{1gr}, \Omega_{2gr}$-cutting frequencies of a two-dimensional spectrum.

Attitude (8) $x_H(t_1, t_2)$ discrete continuous image $x(i_1, i_2)$ determines the method of obtaining. To do this, it is enough to carry out two-dimensional filtering of a discrete image with a low-frequency filter with a frequency response.

$$K(j\Omega_1, j\Omega_2) = \begin{cases} \Delta t_1, \Delta t_2 \; pri \; |\Omega_1| \leq \pi/\Delta t_1, |\Omega_2| \leq \pi / \Delta t_2 \\ 0 \; pri \; drugix \; \Omega_1, \Omega_2 \end{cases} \qquad (10)$$

The spectrum of the image at the output is only $S(0,0)$ contains non-zero components in the frequency zone and, according to (8), a constant image $X_H(\Omega_1, \Omega_2)$ is equal to the spectrum. It is an ideal low-frequency filter output image means it is the same with. $x_H(t_1, t_2)$.

Thus, an ideal interpolation reconstruction of a continuous image is carried out using a two-dimensional filter with a rectangular frequency response (10). It is not difficult to accurately write a continuous image recovery algorithm. The two-dimensional pulsed response of the reconstruction filter, which can be easily obtained using the reverse Fourier Transform of (10), has the following form [8]:

$$x(t_1, t_2) = \frac{\sin(\pi t_1 / \Delta t_1)}{\pi t_1 / \Delta t_1} \frac{\sin(\pi t_2 / \Delta t_2)}{\pi t_2 / \Delta t_2}$$

The filter product can be determined using the 2D convolution of the input image and the given impulse response. Input image$x_{vx}(t_1, t_2)$ δ- expression as a two-dimensional sequence of functions.

$$x_{vx}(t_1, t_2) = \sum_{i_1} \sum_{i_2} x(i_1, i_2) \cdot \delta(t_1 - i_1 \Delta t_1) \delta(t_2 - i_2 \Delta t_2)$$

after convolution we find:

$$x(t_1, t_2) = \sum_{i_1} \sum_{i_2} x(i_1, i_2) \cdot \frac{\sin[\pi(t_1 - i_1 \Delta t_1)/\Delta t_1]}{\pi(t_1 - i_1 \Delta t_1)/\Delta t_1} \frac{\sin[\pi(t_2 - i_2 \Delta t_2)/\Delta t_2]}{\pi(t_2 - i_2 \Delta t_2)/\Delta t_2} \qquad (11)$$

The resulting relationship shows a method of accurate interpolation reconstruction of a continuous image from a certain sequence of its two-dimensional samples. According to this expression, for proper reconstruction, two-dimensional functions of the $\sin {}^{x}/_{x}$Type should be used as interpolating functions. Attitude (11) is a two-dimensional variant of the Kotelnikov-Nyquist theorem.

Once again, we note that if the two-dimensional spectrum of the signal is limited and the sampling intervals are small enough, these results are real. If at least one of these conditions is not met, the validity of the conclusions is violated. Real images rarely have Spectra with clear boundary frequencies. One of the reasons that leads to the infinity of the spectrum is the limited size of the image. Therefore,when assembling (7), the movement of terms from adjacent spectral ranges manifests itself in each of the $S(k_1, k_2)$ranges. In this case, it will be completely impossible to accurately reproduce a permanent image. In particular, the use of a rectangular filter does not lead to precise reconstruction [9,10].

Conclusion.

The feature of optimal image reconstruction at intervals between samples (11) is the use of all samples of a discrete image, as shown in the procedure. This is not always convenient, it is often required to reconstruct the signal in the local area, relying on the small number of available discrete values. In such cases, it is recommended to apply a quasi-optimal recovery using various interpolation functions. Such a problem arises, for example, when solving the problem of connecting two images, when, due to the geometric incompatibility of these images, the existing readings of one of them can correspond to certain points located in the intervals between the nodes other.

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ОСНОВЫ ЦИФРОВОЙ ОБРАБОТКИ ИЗОБРАЖЕНИЙ

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Аннотация. В статье описываются методы замены непрерывного изображения дискретным, которые могут осуществляться различными способами. Например, можно выбрать любую систему ортогональных функций и заменить ими изображение, вычислив коэффициенты отображения изображения по этой системе (по этому основанию). Разнообразие основ позволяет создавать различные дискретные изображения непрерывного изображения. Однако наиболее распространенной является периодическая выборка, в частности, как упоминалось выше, выборка с прямоугольным растром. Такой метод выборки можно рассматривать как один из вариантов использования ортогонального базиса, в качестве элементов которого используются модифицированные-функции.

Ключевые слова: распределенная система, обработка данных без потерь, обеспечение непрерывности потока данных, различные виды информации, анализ изображений, качество изображения.