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МАТЕМАТИКА

В ПРИМЕРАХ И ЗАДАЧАХ

Допущено
Министерством образования Республики Беларусь
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для иностранных студентов
учреждений высшего образования
по техническим и экономическим специальностям

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Р е ц е н з е н т ы: кафедра высшей математики Белорусского государственного аграрного технического университета (профессор кафедры доктор физико-математических наук, профессор *И.В. Белько*); профессор кафедры теоретической и прикладной механики Белорусского государственного университета доктор педагогических наук, кандидат физико-математических наук, доцент *Д.Г. Медведев*; доцент кафедры теории и практики перевода № 1 Минского государственного лингвистического университета кандидат филологических наук, доцент *В.Г. Минина*

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Содержатся теоретические сведения по математике, включающие определения, утверждения, теоремы и формулы. Теория сопровождается решенными типовыми примерами, содержащими пояснения. Задания для решения систематизированы по трем уровням сложности, для осуществления контроля правильности их решений даны ответы.

Для иностранных студентов учреждений высшего образования по техническим и экономическим специальностям.

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CONTENTS

PREFACE

While creating this training manual, the authors set several goals: firstly, to give a significant number of problems (sample and original) that would sufficiently reflect the essence of basic mathematical concepts; secondly, to provide the necessary theoretical information for their solving; thirdly, to give the solution of the main types of problems on each topic; fourth, to distribute the set of tasks proposed for the solution into three levels of complexity. All these goals determined the structure of the book.

The training manual "Mathematics in problems and tasks" consists of 14 chapters, each of which is subdivided into sections. Each section contains theoretical information on mathematics, including definitions, statements, theorems and formulas. The theory is accompanied by solved typical examples with explanations. The tasks to be solved by students are categorised into three levels of difficulty, and answers are given to control the correctness of their solutions.

The training manual allows to implement a differentiated approach in teaching: each student can solve tasks of an accessible level of complexity. This book can be used in studying in various specialities of the higher vocational system with curricula of the discipline "Mathematics", "Linear algebra and analytical geometry", "Mathematical analysis", "Higher mathematics".

The proposed structure of this training manual makes it possible to study mathematics independently.

The book is addressed to foreign students of higher education institutions in technical and economic specialities.

The authors hope that the proposed book will contribute to the activation of students mental activity and increase the effectiveness of the mathematics teaching process.

Please send all feedback and suggestions to the following address: "Вышэйшая школа" publishing house, Pobediteley Ave., 11, 220004, Minsk; e-mail: info@vshph.com

The authors

Chapter 1. GATEWAYS TO MATHEMATICS

1.1. Sets and operations on them. Number sets

A set is an indefinable primary concept. Sets are indicated by capital letters A, B, C, X, ...of the Latin alpabet. Set can be characterised as a group (multitude, family, e.t.c.) of elements, which have the same properties. Sets are represented by *Euler – Venn's diagrams* (*circles*) (Fig. 1.1).

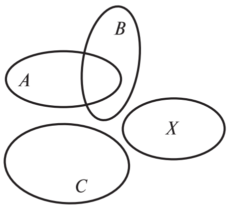
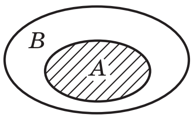


Fig. 1.1

If element a belongs to set A, then write $a \in A$; if element a does not belong to set A, then write $a \notin A$.

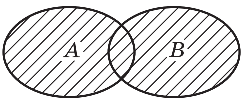
A set can be the one with the indication of its elements (for example, $A = \{1, 3, 8\}$) or specifying a characteristic property (for example, if B consists of elements x for which the property $P(x)$ is executed, then write $B = \{x \mid P(x)\}$).



$A \subset B$

Fig. 1.2

If each element of set A can also be found in set B, then set A is called *subset of set B* (or *set A included* in B), writing $A \subset B$ (or $B \supset A$) (Fig. 1.2). Two sets A, B are *equal* ($A = B$), if both of them consist of equal elements: $A = B$ then and only then, when $A \subset B$ and $B \subset A$. If $A \subset B$ or $A = B$ then write $A \subseteq B$. A set without any element is called *empty* and is indicated by the symbol $\varnothing$.

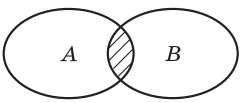


$A \cup B$

Fig. 1.3

The main operations on sets include union, intersection, difference, complement.

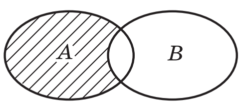
A set $A \cup B$, consisting of all the elements that belong to set A, or to set B (at least to one of the sets A, B) is called ***the union of sets*** A, B (Fig. 1.3).



$A \cap B$

Fig. 1.4

A set $A \cap B$, consisting of all the elements that belong to both set A and set B is called ***the intersection of sets*** A, B (Fig. 1.4).



$A \setminus B$

Fig. 1.5

A set, consisting of all the elements that belong to set A and do not belong to set B is called ***the difference of sets*** $A \setminus B$ (Fig. 1.5).

The complement $\overline{A}$ of set A up to the (*universal*) set U is defined by equality $\overline{A} = U \setminus A$ (Fig. 1.6).

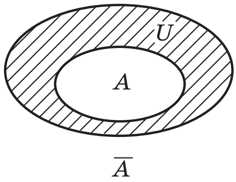


Fig. 1.6

Properties of operations on sets

The following properties are true for arbitrary sets A, B, C:

1) $A \cup B = B \cup A -$ commutativity of the union;
2) $A \cap B = B \cap A -$ commutativity of the intersection;
3) $A \cup (B \cup C) = (A \cup B) \cup C -$ associativity of the union;
4) $A \cap (B \cap C) = (A \cap B) \cap C -$ associativity of the intersection;
5) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$,
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ – distributivity;
6) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$;
7) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$;
8) $\overline{A \cap B} = \overline{A} \cup \overline{B}$;
9) $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

May $m(A)$, $m(B)$ be the number of elements of sets A and B respectively, then the formula is valid:

$$m(A \cup B) = m(A) + m(B) - m(A \cap B). \tag{1.1}$$

Number sets

$\mathbf{N} = \{1, 2, 3, \ldots\}$ – ***Set of natural numbers***.

$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ – ***Set of integers.***

$\mathbf{Q}$ – ***Set of rational numbers***: this is the set of all ordinary fractions $\frac{m}{n}$, where $m \in \mathbf{Z}$, $n \in \mathbf{N}$;

set $\mathbf{Q}$ is also defined as the set of all infinite periodic decimals.

$\mathbf{I}$ – ***Set of irrational numbers***: this is the set of all infinite non-periodic decimals.

$\mathbf{R}$ – ***Set of real numbers***: $\mathbf{R} = \mathbf{Q} \cup \mathbf{I}$.

The following ratios are correct

$$\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R} \subset \mathbf{C}, \quad \mathbf{I} \subset \mathbf{R}, \quad \mathbf{Q} \cap \mathbf{I} = \varnothing.$$

The product of the first n natural numbers is called ***factorial***, a special character has been entered for it:

$$n! = 1 \cdot 2 \cdots n.$$

By definition they accept $0! = 1$.

The following concepts are defined for every $x \in \mathbf{R}$:

$[x]$ – ***the entire of number*** x (or *whole part*), is defined as an integer such that

$$[x] \leqslant x < [x]+1;$$

$\{x\}$ – ***the mantissa of number*** x (or *fractional part*), is defined by equality

$$\{x\} = x - [x].$$

The ***signum*** (or *sing of a number*), is defined as follows:

$$\operatorname{sign} x = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x = 0, \\ -1, & \text{if } x < 0. \end{cases}$$

If $a_1, a_2, ..., a_n$ are some real numbers, then the sum of these quantities is denoted by using ***the sum sign***:

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + ... + a_n,$$

where k – *summation index*.

Sum properties

1. $\sum_{k=1}^{n} a_k = \sum_{p=1}^{n} a_p$ – the sum does not depend on which letter indicates the summation index.
2. $\sum_{k=1}^{n} (c \cdot a_k) = c \sum_{k=1}^{n} a_k$, where $c = \text{const}$.
3. $\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$.
4. $\sum_{k=1}^{n} a_k = \sum_{k=2}^{n+1} a_{k-1} = \sum_{k=0}^{n-1} a_{k+1}$ – the “shift” property of the summation index.

To prove the truth of some statement $A(n)$ for all values of the natural variable n, on which it depends (starting from n_0, $n_0 \in \mathbf{N}$), the ***mathematical induction method*** is often used. To do this, you need to undertake the following three steps:

1) check the truth $A(n_0)$ by direct verification;
2) assume that $A(k)$ is true for any $k \geqslant n_0$;
3) prove that $A(k+1)$ is true for all $k \in \mathbf{N}$, $k \geqslant n_0$.

Sample problems

Problem 1. Prove equality

$$A \setminus \left(A \setminus B\right) = A \cap B. \qquad (1.2)$$

P r o o f. Let $x \in A \setminus \left(A \setminus B\right)$. According to the definition of the difference, we obtain $x \in A$ and $x \notin \left(A \setminus B\right)$. Since both these conditions are satisfied, this is possible only in the case $x \in B$. We obtain that $x \in A$ and $x \in B$, i.e. $x \in A \cap B$. By this we have proved that

$$\left(A \setminus (A \setminus B)\right) \subset (A \cap B). \qquad (1.3)$$

Let us assume that $x \in (A \cap B)$. This means that $x \notin (A \setminus B)$. The two conditions $x \in A$ and $x \in B$ which take place mean that $x \in \left(A \setminus (A \setminus B)\right)$, i.e.

$$(A \cap B) \subset (A \setminus (A \setminus B)). \qquad (1.4)$$

The equality (1.2) is proved, since the inclusions (1.3) and (1.4) are established.

Problem 2. There are 200 first course students: 175 of them passed the maths test in a timely manner, 185 people passed the physics test, 10 people did not pass the test either in mathematics or physics. Determine how many students have passed both tests.

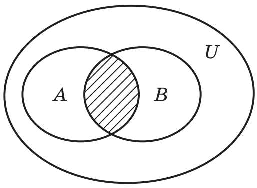


Fig. 1.7

S o l u t i o n. Let U be the set of all students of the course; A is the set of students who passed the test in mathematics, B – in physics (Fig. 1.7).

According to the problem condition $m(U) = 200$, $m(A) = 175$, $m(B) = 185$, $m(U \setminus (A \cup B)) = 10$ and we need to find $m(A \cap B)$.

We find how many people have passed at least one test:

$$m(A \cup B) = m(U) - m(U \setminus (A \cup B)) = 200 - 10 = 190.$$

Next we use the formula (1.1) from which we express

$$m(A \cap B) = m(A) + m(B) - m(A \cup B).$$

We get

$$m(A \cap B) = 175 + 185 - 190 = 170.$$

Problem 3. Shorten the fraction $\dfrac{(2n-1)!+(2n)!}{(2n+1)!}$.

S o l u t i o n. Let's highlight the common factor in the numerator and denominator. Obviously,

$$(2n)! = 1\cdot 2\cdots(2n-1)\cdot 2n = (2n-1)!2n,$$

$$(2n+1)! = 1\cdot 2\cdots(2n-1)\cdot 2n(2n+1) = (2n-1)!(2n)(2n+1).$$

Therefore

$$\frac{(2n-1)!+(2n)!}{(2n+1)!} = \frac{(2n-1)!(1+2n)}{(2n-1)!(2n)(2n+1)} = \frac{1}{2n}.$$

Problem 4. Calculate the sum $\sum_{n=1}^{7} \dfrac{(-1)^{\left[\frac{n}{2}\right]}}{(n-1)!}$.

S o l u t i o n. Let's write down the summands sequentially, giving n values 1, 2, ..., 7 to the variable:

$$\sum_{n=1}^{7} \frac{(-1)^{\left[\frac{n}{2}\right]}}{(n-1)!} = \frac{(-1)^{\left[\frac{1}{2}\right]}}{0!} + \frac{(-1)^{[1]}}{1!} + \frac{(-1)^{\left[\frac{3}{2}\right]}}{2!} + \frac{(-1)^{[2]}}{3!} + \frac{(-1)^{\left[\frac{5}{2}\right]}}{4!} + \frac{(-1)^{[3]}}{5!} +$$

$$+\frac{(-1)^{\left[\frac{7}{2}\right]}}{6!} = \frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^1}{2!} + \frac{(-1)^2}{3!} + \frac{(-1)^2}{4!} + \frac{(-1)^3}{5!} + \frac{(-1)^3}{6!} =$$

$$= 1 - 1 - \frac{1}{2} + \frac{1}{6} + \frac{1}{24} - \frac{1}{120} - \frac{1}{720}.$$

After calculating we come to the answer:

$$\sum_{n=1}^{7} \frac{(-1)^{\left[\frac{n}{2}\right]}}{(n-1)!} = -\frac{217}{720}.$$

Problem 5. Prove the validity of the formula

$$1+3+5+\ldots+(2n-1) = n^2 \tag{1.5}$$

for any $n \in \mathbf{N}$.

S o l u t i o n. To solve this problem we will use mathematical induction method.

1. We check the validity of equality (1.5) when $n=1$. To do this we assume that $n=1$ in equality (1.5). The left part of the equality consists of one term:

$$(2\cdot 1-1)=1^2.$$

That means that $1=1$

2. Let's assume that the statement (1.5) is true for $n=k$, $k\in\mathbf{N}$, $k\geqslant 1$, i.e.

$$1+3+5+\ldots+(2k-1)=k^2. \tag{1.6}$$

3. To prove the validity of formula (1.5) for $n=k+1$. We consider the left side of equality (1.5) when $n=k+1$ and transform it:

$$1+3+5+\ldots+(2k-1)+(2(k+1)-1)=(1+3+5+\ldots+2k-1)+2k+1.$$

We further use the fact that the expression in the last brackets, according to the relation (1.6), is equal to k^2. As a result we get

$$k^2+2k+1=(k+1)^2.$$

The right side of equality (1.5) for $n=k+1$ has the form $(k+1)^2$. It is obvious that the left and right sides of equality (1.5) are equal when $n=k+1$.

Since all three steps of mathematical induction method are implemented, formula (1.5) is true for any $n\in\mathbf{N}$.

Problem 6. Find all natural numbers n for which the inequality

$$2^n>n^2. \tag{1.7}$$

is true.

S o l u t i o n. The statement (1.7) that could be proved by the mathematical induction method is not explicitly formulated.

For this reason let us find out the pattern of interdependence of the values 2^n and n^2. Assign to the number n the values 1, 2, 3, 4, 5, 6 sequentially. We obtain $2^1>1$, $2^2=2^2$, $2^3<3^2$, $2^4=4^2$, $2^5>5^2$, $2^6>6^2$.

Thus, we can make a hypothesis: the original inequality is true for $n=1$ and every natural $n\geqslant 5$. Let us prove this statement.

1. The truth of the inequality (1.7) for $n=5$ has already been proved.

2. Suppose that inequality (1.7) is true for any $n=k$, $k\geq 5$, $k\in\mathbf{N}$, i.e.

$$2^k>k^2. \tag{1.8}$$

Using inequality (1.8), we prove the inequality

$$2^{k+1} > (k+1)^2. \qquad (1.9)$$

Based on the inequality (1.8), we have:

$$2^{k+1} = 2 \cdot 2^k > 2 \cdot k^2. \qquad (1.10)$$

Note that $(k-1)^2 > 2$ for all natural $k \geq 3$, which can be seen, for example, graphically by examining the function $y = (x-1)^2$ for $x = 3$, 4, 5, ... (Fig. 1.8).

Then $k^2 > 2 + 2k - 1$, or $k^2 > 2k + 1$. Add k^2 to both parts of the last inequality. We get $2k^2 > k^2 + 2k + 1$.

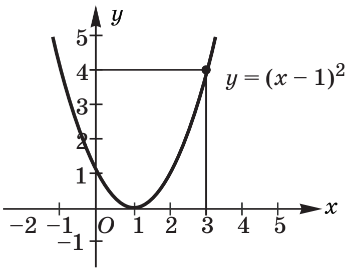


Fig. 1.8

The obtained inequality can be written in the form $2k^2 > (k+1)^2$. Together with the inequality (1.10) it proves the validity of the inequality (1.9).

By the mathematical induction method we conclude that the original inequality is true for every $n \geq 5$, $n \in \mathbf{N}$, and also true for $n = 1$.

Tasks for solving

Level I

1.1. Let $A = (-\infty, 2]$, $B = [-3, 5)$ are subsets of universal set $U = \mathbf{R}$. Find the set:

1) $A \cup \overline{B}$; 2) $\overline{A} \cap B$; 3) $\overline{A \cup B}$; 4) $\overline{A} \cap \overline{B}$.

1.2. There are 28 students in the group, each of them can ski or skate. At the same time, 20 people can ski, 15 people can skate. Determine how many students can both ski and skate.

1.3. Calculate:

1) $3! + 2!$; 2) $\dfrac{5!}{3!}$; 3) $\dfrac{(2 \cdot 3)!}{2 \cdot 3!}$; 4) $\dfrac{(5-2)!}{5! - 2!}$.

1.4. Shorten the fraction:

1) $\dfrac{(n+1)!}{2 \cdot n!}$; 2) $\dfrac{(2n)!}{(2n+1)!}$.

1.5. Determine the entire of the number:

1) $\dfrac{23}{2}$; 2) $\dfrac{3}{28}$; 3) $-5{,}2$; 4) $3{,}25$.

1.6. Calculate:

1) $[2{,}8]+3[-2{,}8]-2\{2{,}25\}$; 2) $\dfrac{\{6{,}25\}}{[5{,}25]}+[-7{,}08]$.

1.7. Write down the sum, specifying each term, and calculate it:

1) $\sum_{n=1}^{5}\dfrac{1}{n}$; 2) $\sum_{n=2}^{6}\dfrac{(-1)^n}{n-1}$; 3) $\sum_{n=0}^{4}\dfrac{(-1)^{n+1}}{(n+1)!}$.

Level II

2.1. The sets

$$A=\{a_n \mid a_n=2n,\ n\in\mathbf{N}\},\quad B=\{b_n \mid b_n=4n-2,\ n\in\mathbf{N}\},$$

$$C=\{c_n \mid c_n=4n+2,\ n\in\mathbf{N}\}$$

are considered. Find set:

1) $A\cup B$; 2) $A\cap B$; 3) $B\setminus C$;
4) $A\setminus B$; 5) $A\cap B\cap C$; 6) $A\cup B\cup C$.

2.2. Write down by which operations on sets A, B, C the shaded set is obtained (Fig. 1.9):

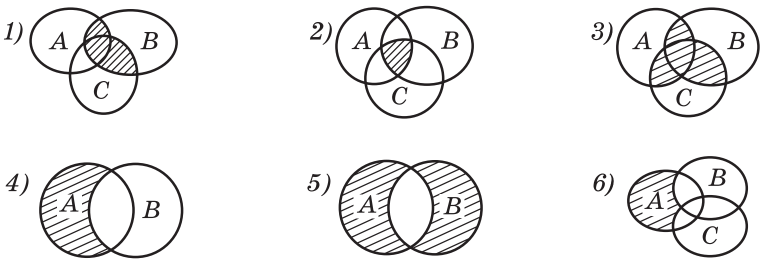


Fig. 1.9

2.3. 100 students participated in the first round of the competition, 70 of them received the right to participate in the second round of the competition in physics, 45 in mathematics. It is known that 23 people can participate both in the second round of the physics competition and the mathematics competition. Determine how many students are not admitted to the second round either in physics or mathematics.

2.4. Shorten the fraction and simplify the resulting expression:

1) $\dfrac{(n-1)!+3n!}{(n+1)(n-1)!-(n-2)!}$; 2) $\dfrac{(n-1)!+(n-3)!}{2n^2(n-3)!+(n-2)!}$.

2.5. Prove the equality is true for all n ($n \in \mathbf{N}$):

1) $1^2 + 3^2 + \ldots + (2n-1)^2 = \dfrac{n(4n^2-1)}{3}$;

2) $1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \ldots + (n-1)n = \dfrac{(n-1)n(n+1)}{3}$;

3) $\dfrac{1}{1\cdot 3} + \dfrac{1}{3\cdot 5} + \ldots + \dfrac{1}{(2n-1)(2n+1)} = \dfrac{n}{2n+1}$.

Level III

3.1. For universal set $\mathbf{R}$ subsets $A = \{x \mid x^2 - 4 \leqslant 0, x \in \mathbf{R}\}$, $B = \{x \mid x^2 - 6x + 5 > 0, x \in \mathbf{R}\}$ are considered. Find the set:

1) $A \cap \bar{B}$; 2) $\overline{A \cup B}$; 3) $(\overline{A \cap \bar{B}}) \setminus B$.

3.2. Among the applicants who successfully passed the university entrance exams, 48 people received an excellent grade in mathematics, 37 in physics, 42 in belarusian, 75 in mathematics or physics, 76 in mathematics or belarusian, 66 in physics or belarusian, and 4 people in all three disciplines. Find out:

1) how many applicants received at least one «excellent» gratitude;
2) how many applicants received only one «excellent» gratitude.

3.3. Compare the fractions

$$a = \frac{(2n)! - (2n-2)!}{(2n-1)!}, \quad b = \frac{2n! - 2(n-1)!}{(n+1)!}.$$

3.4. Prove the inequality by mathematical induction method starting from some natural n:

1) $2^n < n!$; 2) $n^n > 2^n \cdot n!$; 3) $\dfrac{4n}{n+1}(n!)^2 > 2n!$

3.5. Plot the graph of the function $y = f(x)$, if $x \in R$:

1) $f(x) = \operatorname{sgn} x$; 2) $f(x) = [x]$; 3) $f(x) = \{x\}$.

1.2. The concept of a complex number, an rectangular form of its notation

The number of the type

$$z = a + bi, \tag{1.11}$$

where $a, b \in \mathbf{R}$, i – *imaginary unit*, defined by equality $i^2 = -1$, is called ***a complex number.*** The number a is called *the real part of the complex* number z and is denoted by $a = \mathrm{Re}\, z$; the number b is called *the imaginary part of the complex number* z and is denoted by $b = \mathrm{Im}\, z$.

Writing a complex number in the form (1.11) is called ***the rectangular form of a complex number.***

If $a = 0$, $b \neq 0$, then complex number (1.11) is called *purely imaginary*. By $b = 0$ it turns out a real number.

The set of all complex numbers is denoted by **C**. There is an inclusion $\mathbf{R} \subset \mathbf{C}$. The following ratios $\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R} \subset \mathbf{C}$ are correct, as a result of expanding the meaning of the set of number.

In a rectangular Cartesian's coordinate system, a complex number $z = a + bi$ is represented by a point with coordinates (a, b) (Fig. 1.10). There is a one-to-one correspondence between the set of all points in the coordinate plane and the set of all complex numbers. The coordinate plane is called *the complex plane*. The *Ox* axis is called *the real axis*, the *Oy* axis is called *the imaginary axis*.

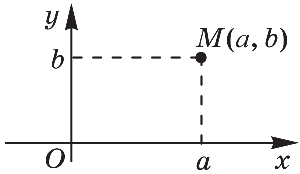


Fig. 1.10

Two complex numbers $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$ are *equal* if and only if their real and imaginary parts are equal, i.e. $a_1 = a_2$, $b_1 = b_2$.

It's impossible to arrange complex numbers by using the inequality relation (it can be done to real numbers). Therefore, we can say that complex numbers can not be compared.

If $z = a + bi$, then $\bar{z} = a - bi$ is called *conjugate number to the number* z.

Conjugate numbers z, $\bar{z}$ are represented by symmetrical points in the coordinate system with respect to the *Ox* axis.

Operations on complex numbers in rectangular form

Let $z_1 = a_1 + b_1 i$, $z_2 = a_2 + b_2 i$, then

$$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i, \tag{1.12}$$

$$z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)i, \tag{1.13}$$

$$z_1 \cdot z_2 = (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1)i. \tag{1.14}$$

Formulas (1.12)–(1.14) show that the operations of addition, subtraction and multiplication are performed similarly to the same operations on polynomials (taking into account $i^2 = -1$ in multiplying). To find the quotient of complex numbers z_1 and z_2 first the numerator and

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