Algebraic Model for Automated Detection of Human's Fundus Morphometrical Characteristics Abnormal Changes

Igor Gurevich
Department of Mathematical Problems of Information Recognition and Analysis
Federal Research Center
"Computer Science and Control"
of the Russian Academy of Sciences
Moscow, Russian Federation
igourevi@ccas.ru,
ORCID *0000-0002-9171-1005*

Vera Yashina
Department of Mathematical Problems of Information Recognition and Analysis
Federal Research Center
"Computer Science and Control"
of the Russian Academy of Sciences
Moscow, Russian Federation
werayashina@gmail.com,
ORCID *0000-0002-5120-3081*

***Abstract*— We give the algebraic model of solving the problem of optical coherence tomography angiogram image analysis for automatization of detection of abnormal changes in morphometrical characteristics of human's fundus. This model deals with a new class of specialized algebras, viz. descriptive image algebras, which is an original mathematical language used to formalize and standardize the processing procedures for image models and transformations over them. Descriptive image algebras are used to describe problems, objects, and transformations involved in extracting information from images.**

Keywords*— *image analysis, descriptive image analysis, mathematical theory of image analysis, descriptive algorithmic schemes, descriptive image models, image representations, representations of image analysis procedures, descriptive approach to image analysis, image algebras, descriptive image algebra with one ring, optical coherence tomography angiogram, human fundus

I. Introduction

The paper presents the algebraic model of solving the problem of atomization of ophthalmological diagnostics described by a new class of specialized algebras, viz. descriptive image algebras (DIA) [6, 8]. DIA are used to describe problems, objects, and transformations considered in the course of information extraction from images. When we describe algorithmic schemes of image formal representation, processing, analysis and recognition using DIA, each element of the scheme and any transformation used in the scheme are given by the structures constructed by applying DIA operations to the set of DIA operands. With such approach, we can vary methods of solving the subproblem by using image analysis operations as DIA elements while preserving the overall scheme of the technique of information extraction from images.

When speaking of DIA in this publication, we consider DIA with one ring (DIA1R) in more detail. This class of algebras falls into the class of universal linear algebras with sigma-associative ring with identity.

DIA is the principal branch of the mathematical apparatus of descriptive analysis (DA) of images [1-10], which is a logically arranged totality of descriptive methods and models designed for image analysis and estimation.

Modern state and trends of DA development are specified by methods, models and results of the descriptive approach (DAp) to image analysis and understanding proposed by I. B. Gurevich and developed by his fellow scholars and students [1-10]. Since 2002, V. V. Yashina has made a significant contribution to the theory. As its methods and apparatus were being developed and refined, the interpretation of the descriptive approach was proposed and defined as DA. Mathematical basis of image analysis and recognition is now studied [1-10] in Federal Research Center "Computer Science and Control" of the Russian Academy of Sciences.

Dap[1-3], general algebraic methods, methods of mathematical theories of image processing, image analysis and pattern recognition form the theoretical basis for the studies.

As we noted above, within DA, we propose to perform 'algebraization' of image analysis and recognition using DIA [6, 8]. This new class of image algebras was developed based on researches in the domain of 'algebraization' of pattern recognition and image analysis performed since the 1970s. Creation of the new algebra was directly influenced by works of S. Sternberg [13-15] and G. Ritter [11-12], who defined the classical variants of image algebras.

This paper consists of Introduction, three principal sections, Conclusions, and References.

Section II, Descriptive Image Models (DIM), deals with constructing image models (representations, formalized descriptions) for algebraic model construction.

Section III, Descriptive Image Algebras, describes several classes of specialized DIA1R for algebraic model construction.

Section IV, Algebraic Model, gives the algebraic model of solving the problem of automatization of ophthalmological diagnostics, viz. analysis of angiogram images obtained by the method of optical coherence tomography (OCT) used to automate detection of abnormal changes in morphometrical characteristics of human's fundus.

II. Descriptive Image Models

This section presents, respectively, the DIM [7, 9] necessary for constructing an algebraic model for the analysis of OCT-angiograms (OCT-A) to automate the detection of pathological changes in the morphometric characteristics of the fundus [4]: M_{T1}, M_{T2}, M_{T3}, M_{T4}, M_{T5}, M_{P1}, M_{P2} for the image I_1 given in Fig. 1.

Images are obtained by the OCT-A method for layer-by-layer analysis of vascular plexus of the retina [4]. The image is the courtesy of the Research Institute of Eye Diseases and is a color picture of the retina tissues decomposed into several layers.

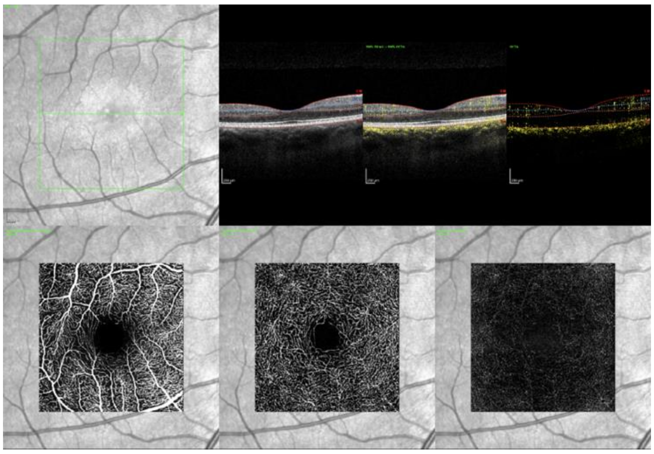

Fig. 1.Image of the OCT angiogram.

DIM presented below are described as follows: the number and type of the model class; image representations/information generated when using DIM; the purpose of the introduction:

- procedural DIM 1 M_{T1}; gray-scale realizations of the initial images; representation of fragments of OCT-A images corresponding to 3 layers of the retina;
- procedural DIM 2 M_{T2}; binary realizations of initial images; representation of vessel images;
- procedural DIM 3 M_{T3}; binary realizations of initial images; representation of vessel skeleton images;
- procedural DIM 4 M_{T4}; binary realizations of initial images; representation of images of vessel boundaries;
- procedural DIM5 M_{T5}; binary realizations of initial images; representation of foveolar avascular zone and ischemia zone obtained by representation of vessel images;
- parametric DIM1 M_{P1}; context/semantic information about images; preliminary image classification on the base of pixel brightness;
- parametric DIM2 M_{P2}; context/semantic information about images; quantitative assessment of vascular tortuosity;
- parametric DIM3 M_{P3}; context/semantic information about images; quantitative assessment of foveolar avascular zone and ischemia zone.

III. Descriptive image algebras

This section presents, respectively, the DIA [6, 8] necessary for constructing an algebraic model for the analysis of OCT-angiograms (OCT-A) to automate the detection of pathological changes in the morphometric characteristics of the fundus.

DIA presented below are described as follows: the name of DIA1R, algebra operands, algebra operations, the purpose of the introduction:

- $DIA1R1.3$; color images in the RGB model; algebraic point operations by module of maximum brightness; description of color image representations;
- $DIA1R1.4$; grey-scale images represented by function of brightness in each point; algebraic point operations by module of maximum brightness; description of grey-scale image representations;
- $DIA1R1.5$; grey-scale images represented as matrix of brightness values; matrix algebraic operations; description of grey-scale image representations;
- $DIA1.6$; binary images; logical operations; description of binary image representations;
- $DIA1R2.2$; procedural transformations of color images to grey-scale ones; implementation of transformations of color images to grey-scale ones and implementation of algebraic operations to the obtained grey-scale images; transformation of color image representation to the grey-scale ones;
- $DIA1R2.2.1$; procedural transformation of grey-scale images to binary ones; implementation of grey-scale images to binary ones and implementation of algebraic operations to the obtained binary images; transformation of grey-scale images to the binary ones;
- $DIA1R2.3$; procedural transformations of getting fragments of grey-scale images represented by matrix; operations of union, intersection and scale of image fragments; obtaining of fragments of grey-scale image representations;
- $DIA1R2.4$; procedural transformations of filtrations of grey-scale images; implementation of transformations of filtrations to grey-scale images and implementation of algebraic operations over obtained grey-scale images; filtration of grey-scale images;
- $DIA2.5$; procedural transformations of binary images; implementation of transformations and implementation of algebraic operations over obtained binary images; transformation of binary image representations;
- $DIA1R2.6$; parametric transformations; implementation of parametric transformations and implementation of algebraic operations over parametric representations of images; construction of parametric image models;
- $DIA1R3.1$; different parametric image representations; union, intersection and scale of parametric image representations; choice of parametric image representations;
- $DIA1R3.2$; parametric image representations; algebraic operations over vectors of feature values; construction of parametric image representations.

IV. Algebraic Model

We give the algebraic model of solving the problem of OCT-A image analysis for automatization of detection of abnormal changes in morphometrical characteristics of human's fundus (for the detailed description of the problem, see [4]). This algebraic model generates a set of specialized descriptive algorithmic schemes (DAS)[10] when one refines the particular transformations and representations from the classes of specialized DIA and DIM.

The sign → in (1)-(13) designates that the transformation of the respective algebra on its left is applied to the operand on its right. I_2 is the original image.

We can write the algebraic model as follows.

Step 1. Within the algebra of color images, the image is manipulated with.

$$DIA1R1.3 \to I_2 = DIA1R1.3(I_2) \quad (1)$$

Step 2. Within the algebra of procedural transformations, the color image is transformed to a gray-scale one.

$$DIA1R2.2 \to DIA1R1.3(I_2) = = DIA1R2.2(DIA1R1.3(I_2)) \quad (2)$$

Step 3. Within the algebra of gray-scale images, the image fragments are manipulated with.

$$DIA1R1.4 \to DIA1R2.2(DIA1R1.3(I_2)) = = DIA1R1.4(DIA1R2.2(DIA1R1.3(I_2))) \quad (3)$$

Step 4. Within the algebra over matrices, the gray-scale image is transformed into the matrix form for the following transformations to be further applied.

$$DIA1R1.5 \to DIA1R1.4(DIA1R2.2(DIA1R1.3(I_2))) = = DIA1R1.5(DIA1R1.4(DIA1R2.2(DIA1R1.3(I_2)))) \quad (4)$$

Step 5. Within the algebra of image fragments, necessary fragments are singled out on the image; for instance, the image model $M_{T1}(I_2)$ is constructed.

$$DIA1R2.3 \to DIA1R1.5(DIA1R1.4(DIA1R2.2(DIA1R1.3(I_2)))) = = DIA1R2.3(DIA1R1.5(DIA1R1.4(DIA1R2.2(DIA1R1.3(I_2))))) = = M_{T1}(I_2) \quad (5)$$

Step 6. Within the algebra of gray-scale images, the image fragments represented in the matrix form are manipulated with.

$$DIA1R1.5 \to M_{T1}(I_2) = DIA1R1.5(M_{T1}(I_2)) \quad (6)$$

Step 7. Within the algebra of filtering transformations, the filtering operations are chosen and applied.

$$DIA1R2.4 \to DIA1R1.5(M_{T1}(I_2)) = = DIA1R2.4(DIA1R1.5(M_{T1}(I_2))) \quad (7)$$

Step 8. Within the algebra of transformations of gray-scale images into binary images, the gray-scale image is transformed into the binary image.

$$DIA2.2.1 \to DIA1R2.4(DIA1R1.5(M_{T1}(I_2))) = = DIA2.2.1(DIA1R2.4(DIA1R1.5(M_{T1}(I_2)))) = M_{T2}(I_2) \quad (8)$$

Step 9. Within the algebra of binary images, the binary representation of an image is manipulated with.

$$DIA1.6 \to M_{T2}(I_2) = DIA1.6(M_{T2}(I_2)) \quad (9)$$

Step 10. Within the algebra of transformations of binary images, binary transformations of binary image representations are chosen and applied; for instance, the following procedural models can be obtained $M_{T3}(I_2)$), $M_{T4}(I_2)$, $M_{T5}(I_2)$).

$$DIA2.5 \to DIA1.6(M_{T2}(I_2)) = = DIA2.5(DIA1.6(M_{T2}(I_2))) = = \{M_{T3}(I_2), M_{T4}(I_2), M_{T5}(I_2)\} \quad (10)$$

Step 11. Within the algebra of parametric transformations, parametric transformations are chosen and applied to calculate features by the constructed image models; for instance, the following parametric image models can be obtained $M_{P1}(I_2), M_{P2}(I_2), M_{P3}(I_2)$).

$$DIA1R2.6 \to \{M_{T1}(I_2), M_{T2}(I_2), M_{T3}(I_2), M_{T4}(I_2), M_{T5}(I_2)\} = = DIA1R2.6(\{M_{T1}(I_2), M_{T2}(I_2), M_{T3}(I_2), M_{T4}(I_2), M_{T5}(I_2)\}) = = \{M_{P1}(I_2), M_{P2}(I_2), M_{P3}(I_2)\} \quad (11)$$

Step 12. Within the algebra over various parametric representations, the essential features for all classes of the recognition problem are chosen.

$$DIA1R3.1 \to \{M_{P1}(I_2), M_{P2}(I_2), M_{P3}(I_2)\} = = DIA1R3.1(\{M_{P1}(I_2), M_{P2}(I_2), M_{P3}(I_2)\}) \quad (12)$$

Step 13. Within the algebra over parametric representations, the chosen features are corrected.

$$DIA1R3.2 \to DIA1R3.1(\{M_{P1}(I_2), M_{P2}(I_2), M_{P3}(I_2)\}) = = DIA1R3.2(DIA1R3.1(\{M_{P1}(I_2), M_{P2}(I_2), M_{P3}(I_2)\})) \quad (12)$$

One can apply various transformations of image representations within each class of algebras. In the previous sections of the chapter, we introduced the principal theoretical aspects needed to understand the proposed ways and tools of DAS description. We will give a more detailed description of the model in our future works. Algebraic model (1)-(13) of solving the problem generates a set of image analysis DAS.

In [4], an example of DAS for solving the problem of analyzing OCT-angiograms to automate the detection of abnormal changes in morphometrical fundus characteristics is given.

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