PRIMARY POSTULATES OF THE STANDARD MODEL AS CONSEQUENCES OF THE COMPOSITE NATURE OF THE FUNDAMENTAL FERMIONS

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Abstract

A field model of two-component fermions is described, the consequences of which coincide in the main with primary postulates of the standard model. Such a model can be constructed for 4 generations at the minimum. Peculiarities of the relative coordinate space, determining in general an internal symmetry group, are considered. Analogues of the Higgs fields appear in the model naturally after transition to the Grassmannian extra coordinates.

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1 Introduction

The standard model (see [1]), the result of collective work of many physicists which has not today any non-accordance with phenomenology, contains a number of postulates both very complicated and having weak logical ties among themselves. For example, there are postulates about the internal symmetry group, the existence of a number of generations, transformation of multiplets in different generations by the interwoven representations of the symmetry group and the existence of the Higgs scalar fields. This complication of postulates is conserved also in the Grand Unification theories [2, 3, 4], based on different gauge groups. Spontaneous breaking of a symmetry to the standard model symmetry in such theories is one way to make a theory agree with experimental data. Another way of concordance we see in the theories of the Kaluza-Klein type [5, 6, 7, 8] when by compactification of the additional space dimensions one can introduce manifolds with the symmetry group that occurs in the standard model. These ways are similar while in GUTs one talks about a symmetry group which will be reduced, but in the Kaluza-Klein theories one builds at first a geometrical construction and then only uses its symmetry. In the mechanisms of spontaneous compactification of additional dimensions, the auxiliary fields are used too [9], as in GUTs similar fields are used for breaking of a symmetry. In both the standard model and GUTs one does not raise a question about an internal symmetry nature, and in the Kaluza-Klein theories about the physical sense of the additional dimensions.

In the supersymmetrical models an internal symmetry group arises as result of an equal state of the spinor coordinates [10, 11]. Usually in such models a special stress is laid on the symmetry between fermions and bosons, but it is typical only for the models on the basis of superspaces with torsion when the additional coordinates are transformed by the spinor representations of the Lorentz group of the basic space. For conservation of an internal $SU(N)-$symmetry an availability of superspace torsion is not obligatory.

In the $E_8 \times E_8-$superstring theory, which is the most likely candidate to the role of the realistic Kaluza-Klein theory, a spectrum of the massless four-dimensional fields, appearing as exitations upon the compactified main state of the string, contains fields of four generations of chiral fermions [12].

In this paper a model of the composite fermions - to be more exact, a model of two-component fermions - [13, 14, 15] is described in English; this one is a theory of the Kaluza-Klein type but with the peculiarity that the

internal space in it must be discrete. It turns out that consequences of this model either literally repeat the primary postulates of the standard model - a global symmetry group in a generation is $SU(3)_c \times SU(2)_l \times U(1)$, multiplets in different generations must be transformed by the interwoven representations of the group - or differ from corresponding postulates not essentially - in particular, the composite fermions make up 4 generations in the model, a permissible multiplet of the model contains all states of the standard model multiplet but may contain another $SU(2)$-singlet in the lepton sector [1]. After transition to the Grassmanian variables as additional coordinates, the $SU(2)-$doublets appear in the model naturally, which are the Lorentz-scalar ones and which may be identified with the Higgs fields. Such a coincidence of the composite fermion model consequences with the postulates of the standard model, which is well verified by experience, can be interpreted as an indirect confirmation of the compositeness of the fundamental fermions and this is deserving great attention. On the basis of the simple physical idea about the composite nature of fermions it is possible to logically tie up the main postulates of the standard model among themselves, and it takes away at least one problem - the problem of generations: in the composite fermions model a set of generations appears so naturally as a particle and an antiparticle in the Dirac theory and it is impossible to build a similar model for one generation. In such a context the standard model comes forward as a way for the non-evident description of non-local interactions, the problem whose decision one attempted to find by constructing non- local field theories in the 4-space [16].

The model has a clear physical interpretation of the additional coordinates - these are coordinates of the relative location of the constituents of a composite system.

Usually in the composite particle models [17, 18, 19] it is assumed that its constituents have definite properties on transformations of the 4-space coordinates: there are fermions - charged and neutral rishons in [18], spinor and scalar subquarks in [19]. In contrast to this, in our model constituents of a composite system have not definite properties with respect to transformations of coordinates of its centre of inertia, and the relative coordinates do not form the Minkowski space.

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[1]It gives us a possibility to introduce neutrino masses that is important now - after observation of evidence that these masses are non-zero

Constants of the length dimension appear in the model naturally - four in the lepton sector and one in the quark sector, but only an analysis of the global symmetries of the model does not permit to evaluate their magnitudes and to state any relations among them, and also to compare ones with a fundamental length of other models [20]. It is clear only that all of them cannot be equal to zero - it is necessary to choose the symmetry group $SU(3)_c \times SU(2)_l \times U(1)$.

It is shown in [15] that this model of the composite fermions may considered in the $(N = 2)-$superspace without torsion, with the additional coordinates which are independent of the main ones. A number and a character of ties, choosing a set of supermanifolds, determine an internal symmetry group, while in the supersymmetrical models this one is determined by an extension degree N.

2 Some algebraic remarks

The composite fermion model under consideration is based on one generalization of the Dirac procedure [21] of construction of a linear equation for a multicomponent function, and it is appropriate to consider an algebraic side of things. The Dirac procedure of transition from an equation $E^2 = \bar{p}^2 + m^2$ to a linear equation for a new multicomponent function $\psi_1 = (\psi, ..., \psi_4)^T$ can be devided into two stages: an algebraic stage of linearization of this equation, which is not connected with the introduction of new postulates:

$$E\psi = \beta m\psi + \alpha_k p_k \psi, \tag{1}$$

where β and α_k are the $4 \times 4-$matrices, E, m, p_k do not change, and another stage of transition to the quantum description, connected with the introduction of a postulate about replacement of E and p_k by operators of motion in the space. Here we consider namely the first stage; further it will be used to connect vectorial and spinor additional coordinates.

This Dirac procedure can be generalized to

$$E = \sum_{i=1}^{N}(p_1^r + p_2^r + ... + p_n^r)^{1/k}, \tag{2}$$

with different N, n, k, r; in this paper we deal with $N = k = r = 2, n = 4$, i.e. with an equation for energy of two free particles [13].

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Especially the case of $N = 1, k = r$ must be picked out, i.e. equations

$$E^r = p_1^r + p_2^r + ... + p_n^r. \tag{3}$$

Probably, eqs. (3) with $r > 2$ have not a particular relation with the physical problems though such possibility was considered by Jamaleev [22]; but these are important for the number theory, and I confine myself to formulate a theorem, connected with those whose constructive proof for prime r has been produced in [23]. This theorem is interesting by the circumstance that in it for an algebraic problem "a wave function", well known to physicists, is introduced - it makes one think.

Theorem [23] 1 *With eq. (3), $n \geq 2$, a linear matrix equation*

$$E\psi = p_1 A_1 \psi + p_2 A_2 \psi + ... + p_n A_n \psi, \tag{4}$$

can be juxtaposed, where A_i are the quadratic matrices of order mn, for which the condition holds

$$(p_1 A_1 + p_2 A_2 + ... + p_n A_n)^r = p_1^r + p_2^r + ... + p_n^r, \tag{5}$$

ψ is the column-vector $(\psi_1, ..., \psi_{mn})^T$; if (3) is carried out, a determinant of the system of equations (4) with respect to unknown quantities ψ_i is equal to zero.

The proof of this theorem is based on the study of the properties of algebraic objects, called in [23] generalized anticommutators:

$$(ABC...)_+ \equiv \sum_P ABC...,$$

where summing up has been made on all different permutations of matrices $A, B, C, ...$.

Let us consider the simplest example of eq. (4) to show that "a wave function" in number theory problems - only without such a name - is well known for mathematicians. For the Diophantine equation $z^2 = x^2 + y^2$ the parametrization of its solutions: $x = 2\psi_1 \psi_2$, $y = \psi_1^2 - \psi_2^2$, $z = \psi_1^2 + \psi_2^2$ is known; but the solutions of then equation

$$z\psi = \sigma_1 x\psi + \sigma_3 y\psi,$$

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where σ_i are the Pauli matrices, are just of this kind, if $\psi = (\psi_1, \psi_2)^T$.

By linearization of an equation

$$E = (m_1^2 + \bar{p}_1^2)^{1/2} + (m_2^2 + \bar{p}_2^2)^{1/2} \tag{6}$$

considered below, defining relations of an algebra of introduced matrices contain both commutators and anticommutators.

3   Description of the two-component composite system

To make the physical idea clear, lying on the basis of the composite fermion model, before formulation of its postulates we consider the description of a composite system with the help of a wave function in the $8-$space [13]. With eq. (6) we juxtapose a linear equation ($c = h = 1$)

$$E\psi = (\beta^1 m_1 + \alpha^{1k} p_{1k} + \beta^2 m_2 + \alpha^{2k} p_{2k})\psi \tag{7}$$

and substitute E, p_{ik} by operators of system energy and constituent momenta in it. To obey the conformity principle, matrices β^i, α^{ik} should have the dimension 16×16 and should satisfy the conditions:

$$\{\alpha^{ik}, \alpha^{il}\}_+ = 2\delta^{kl}, \{\alpha^{il}, \beta^i\} = 0,$$
$$\left[\beta^i, \beta^j\right]_- = \left[\alpha^{ik}, \beta^j\right] = \left[\alpha^{ik}, \alpha^{jl}\right] = 0,$$
$$i \neq j, \ \beta^{i2} = I^{16} \tag{8}$$

(there is not summation on i in the last line). The form of eq. (7) coincides with that of the Bethe-Salpeter equation [24, 25], which is used for the description of such composite systems as mesons.

Further interpretation of eq. (7) depends essentially on geometrical properties, which are provided by the eight-dimensional coordinate space of a model: if one suggests that coordinates of every constituent x_i^μ belong to the Minkowski space $R_{1,3}^4$ we get a description of a system of two fermions. Another possibility, considered in [13, 14], leads to the composite fermions model: it is assumed in this one that only coordinates of a centre of inertia x^μ belong to $R_{1,3}^4$ while about internal coordinates (the ones of the relative

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location) it is assumed only, that these are transformed independently x^μ. Besides that, additional restrictions are taken on a wave function which do not contain evidently derivatives with respect to time, to be like the condition on a wave function for particles with spin 3/2, when field equations are written in the Rarita-Schwinger form [26].

Let us define four-momentum operators

$$p_\mu = i\partial/\partial x^\mu,\ \pi_\mu = i\partial/\partial y^\mu,\ p_{i\mu} = i\partial/\partial x_i^\mu,\ i = 1, 2,\ \mu = 0, 1, 2, 3,$$

and designate corresponding coordinate spaces by X, Y, X_1, X_2. We deal with two 8−spaces, connected among themselves by a linear transformation. If $z = (x, y)$, $z_{12} = (x_1, x_2)$, then these are $Z = \{z\}$ and $Z_{12} = \{z_{12}\}$. In eq. (7) operators of motion in both Z and Z_{12} enter; as was shown in [13], if $p_\mu\psi = p_{1\mu}\psi + p_{2\mu}\psi$, $\pi_\mu\psi = p_{1\mu}\psi - p_{2\mu}\psi$, the additional condition on ψ ($k = 1, 2, 3$)

$$\alpha^{1k}p_{1k}\psi + \alpha^{2k}p_{2k}\psi = A^k p_k\psi, \tag{9}$$

with some matrices A^k permits to split (7) into four Dirac equations for some four components of ψ. If one considers that every constituent of a composite system moves according to the equation

$$p_{i0}\psi = \alpha^{ik}p_{ik}\psi + \beta^i m_i\psi, \tag{10}$$

one gets that a system motion is described by equations

$$E\psi = (\beta^1 m_1 + \beta^2 m_2)\psi + \bar{A} \cdot \bar{p}\psi, \tag{11}$$

$$\pi_0\psi = (\beta^1 m_1 - \beta^2 m_2)\psi + \bar{A} \cdot \bar{\pi}\psi. \tag{12}$$

We have equivalent descriptions of the system, using only eqs. $(10) - (12)$ or $(7) - (10)$. Later on eqs. $(10) - (12)$ with $m_1 = m_2 = 0$ are taken for the basis. In such a manner, at present we make certain that the description of a composite system can be done such that it will correspond for $m_1 = m_2 = 0$ to four "generations" of massless fermions. What symmetries has this model? But before we should formulate clearly primary postulates of the model and write its equations without picking out operators of energy [14].

4   Primary postulates

Let us introduce the following postulates:

a) a fermion is a composite two-component system;

b) the coordinate space is an eight-dimensional and real one, the coordinates of a centre of inertia x^μ set up the Minkowski space $R^4_{1,3}$;

c) internal coordinates y^μ are transformed independently of x^μ (compare with [27]).

Namely the latter of them leads in the model to the consequence: the internal coordinate space is not the Minkowski one. To speak about description with the help of this model of four generations of fermions in a space X, it is necessary to have a possibility to average on coordinates y the function $\psi(x, y)$, introducing in (11) or its analogue, with conservation of the equation form; but the independence of y on x is necessary for it.

It is assumed that the physical space-time coincides with X.

5   Field equations of the two-component fermions

Taking into account our designations, the main equations for the $16-$component wave function of massless, non-charged, free fermions can be written as four Dirac-Kahler equations (see [28]):

$$\Gamma_1^\mu p_\mu \psi(x, y) = 0, \tag{13}$$

$$\Gamma_1^\mu \pi_\mu \psi(x, y) = 0, \tag{14}$$

$$\Gamma_2^\mu p_{1\mu} \psi(x_1, x_2) = 0, \tag{15}$$

$$\Gamma_3^\mu p_{2\mu} \psi(x_1, x_2) = 0, \tag{16}$$

which are equivalent to $(10)-(12)$, if matrices Γ_a^μ, $a = 1, 2, 3$, obey conditions (compare with (8)):

$$\{\Gamma_1^\mu, \Gamma_1^\nu\} = \{\Gamma_2^\mu, \Gamma_2^\nu\} = \{\Gamma_3^{'\mu}, \Gamma_3^{'\nu}\} = 2g^{\mu\nu}, \ \{\Gamma_2^\mu, \Gamma_3^{'\nu}\} = 0, \tag{17}$$

where $g^{\mu\nu} = diag(1, -1, -1, -1)$, $\Gamma_3^{'\mu} \equiv K\Gamma_3^{'\mu}$; $K = \sigma^3 \times I^8$ for the representation of Γ_a^μ from [14].

As in the Dirac theory, eqs. $(13) - (16)$ must be considered also as postulates of the model. These contain non-evidently condition (9), connected not only with coordinate spaces, but also with a wave function.

After averaging of $\psi(x, y)$ with respect to y in (13): $\tilde{\psi}(x) = <\psi(x, y)>_y$ by taking into account the postulate c), we see that the model really corresponds to four generations of fermions, being described by $\tilde{\psi}(x)$. Though (15),(16) have the same structure, one cannot say that the two-component system is composed by two fermions - below it will be shown that, for taken axioms, permissible transformations of coordinates x_1, x_2 do not belong to the Lorentz group or the Poincare one.

6 A kind of permissible transformations of coordinates

Coordinates of two eight-dimensional spaces Z and Z_{12} have been connected by the linear transformation with a matrix λ (compare with [17]):

$$z^\alpha = \lambda^\alpha_\beta z^\beta_{12}, \ \alpha, \beta = 1, ..., 8, \tag{18}$$

where $\lambda = (\sigma^1 + \sigma^3) \times I^4/2$, $\lambda^{-1} = 2\lambda$, $|\lambda| = -2^{-4}$. Accordingly operators $\partial_\alpha = i\partial/\partial z^\alpha$ and $D_\alpha = i\partial/\partial z^\alpha_{12}$ (i.e. $\partial = (p, \pi)$, $D = (p_1, p_2)$) have been connected by the formula

$$D_\alpha = \lambda^\alpha_\beta \partial_\beta.$$

In accordance with c), matrices of permissible transformations of coordinates of Z should have the block-diagonal kind:

$$\Lambda = diag(\Lambda_x, \Lambda_y), \tag{19}$$

where Λ_x and Λ_y are matrices of transformations of x and y. The transformation $z_{12} \to L z_{12}$ corresponds to the one $z \to \Lambda z$ with

$$L = \lambda^{-1}\Lambda\lambda = \left[I^2 \times (\Lambda_x + \Lambda_y) + \sigma^1 \times (\Lambda_x - \Lambda_y) \right]/2. \tag{20}$$

It is natural to interpret transformations $\Lambda' = diag(\Lambda_x, I^4)$, where Λ_x is the Lorentz transformation, as the "coordinate" ones of the physical space-time, and these of the kind $\Lambda'' = diag(I^4, \Lambda_y)$ - as the "internal symmetry" ones. There exists every $\Lambda = \Lambda'\Lambda''$.

A matrix L is not diagonal for $\Lambda_x \neq \Lambda_y$.

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7 A global symmetry group of the model

An internal space turns out to be discrete for extra coordinates y of the vectorial kind in the model, in contrast to the situation in others Kaluza-Klein theories. In such a case a continuous group may be a symmetry group of the model, if an additional postulate about conservation of a norm for a set of model solutions is introduced [14]. Probably, the model may be supplemented - for example, by introduction of some "observation space", differing from the "existence space" of fermions - in such a manner, that one will manage to find a proof of this postulate. It is probable also that the restriction $\pi_\mu \psi(z) = 0$, taken below from the model covariance condition, may be replaced by some softer one.

A group $SU(n)$ is a symmetry one for a field model if

1) a model permits n different solutions $\psi_A, \psi_B, ...$;

2) a direct transition from any ψ_A to any ψ_B is possible;

3) any linear composition of these solutions is a model solution too;

4) for $\varphi^T \equiv (\psi_A, \psi_B, ...)$ the norm $\varphi^\dagger \varphi$ is conserved.

Three first signs of the presence of a group $SU(n)$ can be ascertained in this model and the fourth one may be introduced as an additional postulate. Such a postulate is much simpler than the choice of the definite group in the standard model.

It was shown in [13] that model $(13) - (16)$ permits eight different solutions, which are $G_A \psi(x, y)$, if $\psi(x, y)$ is some solution. These solutions are broken up by the sign 2) on sets of $1, 2, 3$ elements, thus taking condition 4) of conservation of the norm $\varphi^\dagger \varphi$ for the sets as postulate, one has that $SU(3)_c \times SU(2)_l \times U(1)$ appears as the global symmetry group of the model. Matrices G_A, $A = 1, ..., 8$, set up a representation of the discrete group D_4 [29]. The discrete symmetry groups are used in field theories - for example, see [30]. Indeed, as one may check, eight matrices G_A exist for eq. (13), such that $[G_A, \Gamma_1^\mu] = 0$. The representation has been given for them in [14]. The multiplication law of these corresponds to the one of D_4. One of the ideas of the composite fermions model consists in the fact that eight solutions of eq. (13),(14) of the kind $\psi_A(x, y) = G_A \psi(x, y)$ have been assumed to be induced by transformations Λ_{yA} of a space Y, also setting up a representation of D_4. It leads to the fact that a given algebraic structure of field equations puts hard restrictions on the geometrical properties of a space of internal coordinates. An additional demand of covariance of these equations leads to a set of ma-

trices Λ_{yA} with imaginary elements, and in this connection the postulate b) can be conserved only at the price of fixing of a definite topology for every A. If $\Lambda_A = diag(I^4, \Lambda_{yA})$, a transformation $Z \to \Lambda_A Z$ would be accompanied by $\psi(z) \to \psi_A(z)$, and a condition of covariance of (14) will be

$$\Gamma^\mu_{1\pm} = M_A \Gamma^\nu_{1\pm} \Lambda^\mu_{yA\nu} G_A, \qquad (21)$$

where $\Gamma^\mu_{a\pm} \equiv \Gamma^\mu_a(1 \pm \Gamma^5)$, $\Gamma^5 = i\Gamma^0_1\Gamma^1_1\Gamma^2_1\Gamma^3_1$, M_A is some matrix. It appears that, if Λ_{yA+} transforms coordinates in an equation for right components $\psi_R \equiv (1 + \Gamma^5)\psi/2$, and Λ_{yA-} for left ones $\psi_L \equiv (1 - \Gamma^5)\psi/2$, then condition $\Lambda_{yA} = \Lambda_{yA+} = \Lambda_{yA-}$ makes only the trivial solution of (21) possible: $\Lambda_{yA} = aI^4$ [14].

Removing this restriction, i.e. considering $\Lambda_{yA+} \neq \Lambda_{yA-}$, we obtain from (21) (see [14]) that any matrix $\Lambda_{yA\pm}$ must have the structure

$$\Lambda_{yA+} = aI^4 + bB_1 + cB_2 + dB_3, \Lambda_{yA-} = a'I^4 + b'B_1^* + c'B_2^* + d'B_3^*, \quad (22)$$

where a, b, c, d and a', b', c', d' are some numbers, $*$ is complex conjugation, B_1, B_2, B_3 are the matrices

$$B_1 \equiv r^1 \times \sigma^1 - r^3 \times \sigma^2, B_2 \equiv \sigma^1 \times r^1 + \sigma^2 \times r^3, B_3 \equiv r^2 \times r^2 + r^4 \times r^4 + ir^4 \times r^4 - ir^4 \times r^2,$$

and matrices r^a have the form

$$r^1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, r^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, r^3 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, r^4 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

The following matrices, given upon the field C, set up representations of D_4 $(B_k^2 = I^4)$:

$$\{\Lambda_{yA+}\} = \{I^4, -iB_1, -B_2, B_3; -I^4, iB_1, B_2, -B_3\}, \Lambda_{yA-} = \Lambda_{yA+}^*. \qquad (23)$$

This fact can be made to agree with postulate b) about the real type of Y by introduction of restrictions on values of y^μ for every A. If real y^μ corresponds to the case $A = 1$, then the representation (23) does not contradict postulate b) for the following restrictions on y^μ: $y^0 = y^1 = 0$, $A = 2, 6$; $y^1 = y^3 = 0$, $A = 3, 7$; $y^1 = y^2 = 0$, $A = 4, 8$, i.e. the internal space for all three solutions with $A = 2, 3, 4$ and $A = 6, 7, 8$ must consist of separate two-dimensional planes. For all these 6 solutions (states of field) $y^1 = 0$, i.e. there is reduction to the $3-$dimensional pseudo-Euclidean space with

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the topology: $y^0 = 0 \bigvee y^2 = 0 \bigvee y^3 = 0$, in which its own plane corresponds to every pair of solutions.

The demand of covariance and eqs. (15),(16) leads, as we shall see, to further restrictions on internal coordinates - it turns out that y^μ must be constant, though these may differ for various A. Let us consider transformations of (15),(16), which accompany those of Λ in Z. The transformation $\Lambda' \Lambda_A$ induces $z_{12} \to L_A z_{12}$. We designate $\Gamma_2^\alpha \equiv \{\Gamma_2^\mu; 0\}^\alpha$, $\Gamma_3^\alpha \equiv \{0; \Gamma_3^\mu\}^\alpha$, $\alpha = 1, ..., 8$; then eqs. (15),(16) after transformation are

$$\tilde{\Gamma}_a^\alpha D_\alpha' \psi'(z_{12}') = 0, \ \alpha = 2, 3, \tag{24}$$

where $D_\alpha' = L_{A\alpha}^\beta D_\beta$, $\psi'(z_{12}') = V_A \psi(z_{12})$, V_A are matrices, also setting up a representation of D_4 (this is given in [14]), and the covariance condition is

$$M_A \tilde{\Gamma}_a^\alpha L_{A\alpha}^\beta V_A = \Gamma_a^\beta, \ \alpha = 2, 3. \tag{25}$$

It results from (20) that, if $\Lambda_x \neq \pm \Lambda_{yA}$, $\tilde{\Gamma}_a^\alpha$ may contain more then 4 nonzero matrices. A conservation of postulate c) and a covariance of eqs. (15),(16) can be secured for some M_A, V_A, if the condition is carried out

$$p_{1\mu} \psi(z_{12}) = p_{2\mu} \psi(z_{12}). \tag{26}$$

In such a case $\pi_\mu \psi(z) = 0$, i.e. $\psi(x, y) = \psi(x, y_A^\mu)$, where y_A^μ are constants, which may differ for various A. If one assumes that transitions are possible between all states $\psi_A(z)$, then from restrictions on y^μ for $A = 2, 3, 4; \ 6, 7, 8$ made above, it should follow $y_A^\mu = 0 \forall A$, i.e. the theory should be local with the symmetry group $SU(8)$.

A conservation of a non-local type of the theory is provided by isolation of field states with $A = 1, 5$ from the rest; as a result one has the symmetry group $SU(3)_c \times SU(2)_l$. One gets up such isolation by means of the choice of values of y_A^μ. Then states with $A \neq 1, 5$ may be interpreted as the colour states of field since their symmetry group will be $SU(3)$. Coordinates of the colour space y_A^μ, $A = 2, 3, 4; \ 6, 7, 8$, can be chosen in such a manner, that matrices of their transformations T_b set up a discrete group, which is isomorphic to $D_3 : \ \{T_b \mid b = 1, ..., 6\}$, with $T_1 = I^4$, $T_2 = T_3^{-1}$, $T_2^3 = T_4^2 = T_5^2 = T_6^2 = I^4$. A set of y_A^μ for such A turns reserved, i.e. transformations T_b do not lead to appearance of new values of y_A^μ, only for the condition

$$y_A^\mu = 0 \bigvee y_A^\mu = \pm l, \ A = 2, 3, 4; \ 6, 7, 8; \tag{27}$$

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where l is a constant with dimension of length. Thus all quark sector of the model has been characterized by one length l. This condition does not affect the values of y_A^μ for $A = 1, 5$, so as there are five constants with such dimension (it is $y_1^\mu = -y_5^\mu$) in the model, and four of them concern the lepton sector. There are not any restrictions, that all these may be reduced to zero or $\pm l$, but there are not any grounds for such conclusion at the global symmetry level.

An action of transformations T_b induces transitions $\psi_A(z_{12}) \to \psi_A(z_{12})$, and it turns out to be impossible to construct a representation of D_3, the matrices of which transform states $\psi(z_{12})$ separately. But a representation of D_3 exists, the matrices of which permute components of triplets $(\psi_A, \ \psi_{A'}, \ \psi_{A''})$, where $(A, A', A'') = (2, 3, 4)$ or $(6, 7, 8)$. The kind of matrices S_b, $b = 1, ..., 6$, of this representation may be easy determined by action of T_b on the set of y_A^μ for $A = 2, 3, 4$. With an additional demand of conservation of a norm $\varphi^\dagger \varphi$, $SU(3)_c$ will be the symmetry group of these states.

The $SU(2)$ symmetry is connected with the transformation $(x, y) \to (x, -y)$, and the chirality of it has been caused by the algebraic structure of eqs. (15),(16).

The replacement $y \to -y$ leads to the permutation $(x_1, x_2) \to (x_2, x_1)$, so that eqs. (15) and (16) must turn into one another. But the construction of matrices Γ_2^μ and Γ_3^μ permits them to make a transition (15) $\leftrightarrow$ (16) by means of transformation $\psi(z_{12}) \to V\psi(z_{12})$, with some matrix V only for ψ_L or ψ_R. So, a matrix V exists with the property

$$\Gamma_{3-}^\mu V = V\Gamma_{2-}^\mu, \tag{28}$$

but in this case

$$\Gamma_{3+}^\mu V \neq V\Gamma_{2+}^\mu, \tag{29}$$

because components ψ_L and ψ_R should be transformed differently by $y \to -y$. Eqs. (15) and (16) will be covariant, if

$$\psi_R(z_{12}) \to \psi_R(z_{12}), \ \psi_L(z_{12}) \to V\psi_L(z_{12}), \tag{30}$$

and for covariance of (13),(14), by taking into account the correspondence of matrices $V_5 = V$ and G_5 in representations of D_4, it is enough

$$\psi_R(z) \to \psi_R(z), \ \psi_L(z) \to G_5\psi_L(z). \tag{31}$$

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Thus, for $(x, y) \to (x, -y)$ solutions $\psi_R(z)$ and $G_5\psi_R(z)$ are singlets, and $\psi_L(z)$ and $G_5\psi_L$ are components of a doublet of $SU(2)$, and this is true also for the colour states with $A = 2, 3, 4; 6, 7, 8$. Equations (13)-(16) are invariant also with respect to a global $U(1)$. Because of this, $SU(3)_c \times SU(2)_l \times U(1)$ will be the global symmetry group of the model. The structure of a permissible multiplet of the group differs from this one, taken in the standard model, only on a possibility of appearance of another $SU(2)_r-$singlet for states with $A = 1, 5$, i.e. in the lepton sector, that may provide a non-zero mass of neutrino.

8   Covariance of the model under the global Lorentz transformations

If $\Lambda = (\Lambda_x, I^4)$, $\theta_{\mu\nu}$ are parameters of the Lorentz transformation, $G_x = \exp(-i\sigma^{\mu\nu}\theta_{\mu\nu}/4)$, where $\sigma^{\mu\nu} = [\Gamma_1^\mu, \Gamma_1^\nu]$, then a transformation

$$\psi(z) \to G_x\psi(z) \tag{32}$$

keeps (13) covariant, being $[G_x, G_A] = 0$. Equations (15),(16) by condition (26) will be covariant, if

$$\psi(z_{12}) \to S_x\psi(z_{12}), \tag{33}$$

where $S_x = \exp(-i(\sigma_2^{\mu\nu} + \sigma_3^{\mu\nu})\theta_{\mu\nu}/4)$, $\sigma_a^{\mu\nu} = [\Gamma_a'^\mu, \Gamma_a'^\nu]$, $a = 2, 3$, $\Gamma_2'^\mu = \Gamma_2^\mu$.

In the 4−dimensional case transformation (33) would correspond to the field states of spin $0, 1$ or $1/2$ [28], but in the 8−dimensional case such an interpretation has not any sense. In the 4−space-time, states of field $\tilde\psi(x)$, describing four generations of fermions, will be observable.

9   Introduction of the Grassmannian extra co-ordinates

It is shown in [15] that transition from internal coordinates of the vectorial type y^μ to the Grassmannian ones $\chi_a, \bar\chi_a$, $a = 1, 2, 3, 4$, leads to the appearance of the effective four-dimensional fields in the model's Lagrangian, which

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are $SU(2)-$doublets and analogs of the Higgs fields and may play their part in the mass splitting mechanism.

We show here, following [15], that one may consider the model described above as a model of field in the $(N = 2)-$superspace without torsion, with independent additional coordinates, in which with the help of ties a set of supermanifolds has been picked out, and one interprets mixing of solutions, to be set on these supermanifolds, as an internal symmetry. For this, with lack of torsion there is not any symmetry between bosons and fermions. Another difference from the supersymmetrical models is that an internal symmetry group is not defined now by an extension degree N, but it is given by a character of ties and the supermanifolds number, which are picked out. An extension with $N = 2$ is necessary only to provide an arbitrary "norm" $y^\mu y_\mu$ of relative coordinates - as it is shown in [15], the $(N = 1)-$super-space without torsion may correspond to a space, for which $y^\mu y_\mu = 0$ only.

For the substitution of the spinor coordinates χ_a for the vectorial ones y^μ we use an equation of the type (2)

$$y^\mu y_\mu = s^2 \tag{34}$$

and its linearized form:

$$\gamma^\mu y_\mu \chi = s\chi, \tag{35}$$

where γ^μ are the Dirac matrices, χ is a bispinor, y_μ and s are the same as in (34) - they are not operators, but usual numbers. From (35) one obtains the relation of connection of vectorial y_μ and spinor $\chi_a, \bar\chi_a$ coordinates, $a = 1, 2, 3, 4$:

$$y^\mu = \bar\chi\gamma^\mu\chi, \tag{36}$$

and (34) is carried out by some additional restrictions on $\chi_a, \bar\chi_a$. If all $\chi_a, \bar\chi_a$ commute, it is necessary (compare with [31]) that

$$R^2 = 0, \tag{37}$$

where $R = \bar\chi\gamma^5\chi$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and, if these anticommute, the condition must be carried out

$$R^2 + 8M = 0, \tag{38}$$

where $M = \bar\chi_1\chi_1\bar\chi_4\chi_4 + (\bar\chi_2\chi_2 + \bar\chi_1\chi_1)\bar\chi_3\chi_3 + \bar\chi_1\chi_4\bar\chi_2\chi_3 + \bar\chi_3\chi_2\bar\chi_4\chi_1$. For this the dependence $\bar\chi = \chi\dagger\gamma^0$, $\chi_a^* = \chi_a$, leads to identity in case of (37) and to a local variant of the composite fermions model in the case of (38).

One must consider $\chi_a, \bar{\chi}_a$, to be independent of x^μ, taking into account the postulate c), and for this $\chi_a, \bar{\chi}_a$ may commute or anticommute between themselves [15]. In last case, $(x, \bar{\chi}, \chi)$ is the $(N = 2)$−superspace without torsion. The substitution $\gamma^\mu \to \gamma_A^\mu$ in (36) permits to get the set of y_A^μ, if $\gamma_A^\mu = \gamma^\mu \Gamma_A B_A^\mu$, where $B_A^\mu \in \{\pm I^4, \pm \gamma^5\}$, $B_1^\mu = I^4\ \forall \mu$, $\Gamma_1 = I^4$, $\Gamma_2 = iI^2 \times \sigma^1$, $\Gamma_3 = \sigma^1 \times \sigma^2$, $\Gamma_4 = \sigma^1 \times \sigma^3$, $\gamma_{A+4}^\mu = -\gamma_A^\mu$. Matrices $B_A^\mu = \pm \gamma^5$ only for $y_A^\mu = 0$, so that in the quark sector of the model

$$y_A^\mu = 0 \ \Rightarrow \bar{\chi}\gamma^\mu\gamma^5\chi_A = 0, \ y_A^\mu = \pm l \ \Rightarrow \bar{\chi}\gamma^\mu\chi_A = \pm l, \qquad (39)$$

where $\chi_A = \Gamma_A \chi$ and in the last line the inverse correspondence of signs is also possible.

Thus, the $(N = 2)$−superspace without torsion $(x, \bar{\chi}, \chi)$ with the set of ties

$$y_A^\mu = \bar{\chi}\gamma_A^\mu\chi = Re\bar{\chi}\gamma_A^\mu\chi, \qquad (40)$$

picking out 8 supermanifolds, may be represented on the space $Z = (x, y_A^\mu)$ of the model: $x \to x$, $(\bar{\chi}, \chi) \to y_A^\mu$ by formulae (40). Then one may consider an internal symmetry as a consequence of mixing of solutions, which are defined on different manifolds. The chirality of $SU(2)$ is caused by the field equations as before, but it is not connected with any peculiarities of a basic space $(x, \bar{\chi}, \chi)$.

Rejection of the axioms of the supersymmetrical models about the presence of torsion and about the dependent transformation of x and $\bar{\chi}, \chi$ changes essentially an interpretation of component fields, which are contained by any scalar superfield in a space $(x, \bar{\chi}, \chi)$ - now all of them must be scalar with respect to the Lorentz group of a basic space X. The Lagrangian of the model, being considered as such superfield, must contain terms of expansion on degrees of $\bar{\chi}, \chi$; factors by the first degree of χ_A must be the sets of scalars with respect to the Lorentz group and $SU(2)$−doublets, i.e. these are analogs of the Higgs fields of the standard model.

10   Conclusion

The interpretation of the composite fermion model, given here, is based essentially on investigations of local symmetries and gauge interactions, which have been made in the frame of the standard model. One may say that an

assumption about the composite nature of the fundamental fermions permits not only to describe the physics of particles, as the standard model does with success, but and to explain this. In the composite fermions model, the basic propositions of the standard model go into the common logically harmonious picture, their ties and an interconditionality are visible. In such a context the standard model may be interpreted as a way to describe nonlocal interactions of composite objects, and its experimental corroboration may be interpreted as an indirect confirmation of the composite nature of the fundamental fermions.

In the text, the postulate of the standard model about mixing of generations in the quark sector was not mentioned. A similar phenomenon takes place in the composite fermions model, but to evaluate this effect, its description in the model, based on introduction of 4 generations at once, will be be made to agree with the conception of mixing in the standard model language. The representation for matrices G_A [14] is such that matrices G_6 and G_7 mix generations in pairs in the quark sector - in the sense that if, for instance, in the lepton sector some wave function components set up the first generation, then in the quark one a mixture of components of the first generation and of the third one corresponds to this (here the numbering of the generations corresponds only to the indices of ψ) [2].

PS to this reprint

It is a reprint of author's paper of 1992. I am grateful to the editors of Nuovo Cimento for a permission to re-print it. I think that after the observation of evidence of non-zero neutrino mass difference by the Super-Kamiokande collaboration [32], my study may be more actual than ten years ago. Another reason to enter this paper into the LANL archive is my recent study of the hypothetical background of super-strong interacting gravitons [33, 34, 35]. Perhaps, such gravitons would be very appropriate candidates to a role of constituents of the composite fundamental fermions.

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