A model of gravitation with global U(1)-symmetry

By M.A.Ivanov
Chair of Physics,
Belarus State University of Informatics and Radioelectronics,
6 Pyatrus Brovka Street, 220027, Minsk, Republic of Belarus.
E-mail: ivanovma@gw.bsuir.unibel.by

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Abstract

It is shown that an embedding of the general relativity $4-$space into a flat $12-$space gives a model of gravitation with the global $U(1)-$symmetry and the discrete D_1-one. The last one may be transformed into the $SU(2)-$symmetry of the unified model, and the demand of independence of $U(1)-$ and $SU(2)-$transformations leads to the estimate $\sin^2\theta_{min} = 0,20$ where θ_{min} is an analog of the Weinberg angle of the standard model.

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1 Introduction

In Einstein's theory of gravitation, the flat Minkowski space is not used as a background one that differs the theory from other physical theories. From a geometrical point of view, a curved space can be embedded into a flat one of an enlarged dimension. A similar embedding, but into a nonflat space with the additional spinor coordinates, is factically used in the Ashtekar approach [1], that makes possible to introduce the new variables for a description of gravitational field.

Many-dimensional spaces are widely used in physics, for example, in theories of supersymmetry, supergravitation, and superstrings [2]. In theories of the Kaluza-Klein type in the 8-space, a description of a system of generations of the fundamental fermions is possible, by which primary postulates of the standard model are the consequences of the hypothesis about compositness of the fermions [3]. It is shown in this paper, how one can realize such an embedding of the general relativity 4-space into a flat 12-space. The additional coordinates are choosen to have a clear interpretation. If in the four-dimensional case, three points — a point of the Minkowski space, a trial material point (its position), and some point of observation — can be described by one set of the coordinates x, then in the model it is postulated, that these three points are described by three independent sets of coordinates x, A, B accordingly.

At an initial stage, the connection is introduced with a trivial curvature in the model, which is caused by local rotations under transitions from one four-dimensional subspace to another. A linear deformation of such the connection lets to introduce the 4-manifold with a non-trivial curvature. Then one obliges the Ricci tensor of the deformed connection to Einstein's equation. The equations, which picks out the curved manifold, will be the algebraic ones for components of the connection of flat space. All three flat subspaces are locally isomorphic to each other, i.e. the additional coordinates have the "vector" kind.

The model lets a natural unification with the composite fermions model by the author [3], for which one needs to enlarge the space dimension up to 16. It is an important fact, that the model has the $U(1)-$symmetry, therewith it is the symmetry of field equations. Another interesting fact is an existence of the discrete D_1-symmetry. By an introduction of spinor fields of the composite fermions with a demand of conservation its norm [3], the last symmetry will lead to the global $SU(2)-$symmetry of the model. But the parameters of the $U(1)-$transformations and the $SU(2)-$ones will depend from each other in a general case. To provide its independence, both conditions are necessary: 1) the $SU(2)-$doublet states are massless, and 2) a rotation must be executed on some angle in the parameter space. It is similar to a situation with the Weinberg angle rotation in the standard model, still a logical sequence of actions differs from the one that leads to an introduction of the Weinberg angle.

2 Connections with a trivial curvature, introduced by mappings in a flat 12-space

Let us consider the flat 12-space $(x^a, A^{\tilde{\mu}}, B^{\mu})$, where (x) is the Minkowski space, (A) is a trial body (a material point) coordinate space, and (B) is an observation point coordinate space. Let us use simbols $a, b, \dots$ for indices in $(x)-$subspace, $\tilde{\alpha}, \tilde{\beta}, \dots$ in $(A)-$one, and $\alpha, \beta \dots$ in $(B)-$one. I.e. we shall suggest that the diffeomorphisms exist: $x^a \to A^{\tilde{\mu}}(x), x^a \to B^{\mu}(x)$, which describe motions of a trial body and of an observation point in the flat space (x) in some systems of reference. These mappings are characterized by the functions $h_a^{\tilde{\mu}} \equiv \partial A^{\tilde{\mu}}/\partial x^a$ and $h_a^{\mu} \equiv \partial B^{\mu}/\partial x^a$, for which the reverse functions exist: $h_a^{\mu} h_{\mu}^{b} = \delta_a^b$, e.c. Let us consider that the functions $h_a^{\tilde{\mu}}$ and h_a^{μ} describe the local Lorentz transformations $dx^a \to dA^{\tilde{\mu}}(x), dx^a \to dB^{\mu}(x)$, if one projects both flat 4-spaces (A) and (B) onto (x).

By the sequential mappings

$$x^a \to A^{\tilde{\mu}} \to B^\mu(x),$$

one has for the metric tensors of the subspaces $(x), (A), (B):$

$$\eta^{ab} \to \eta^{\tilde{\mu}\tilde{\nu}} \to \eta^{\mu\nu}$$

and for the corresponding metric connections:

$$0 = \Gamma^a_{bc} \to \tilde{\Gamma}^{\tilde{\mu}}_{\tilde{\nu}\tilde{\epsilon}} \to \Gamma^\mu_{\nu\epsilon},$$

where [4] $(h^\mu_{\tilde{\mu}} \equiv \partial B^\mu / \partial A^{\tilde{\mu}}):$

$$\Gamma^\mu_{\nu\epsilon} = h^\mu_{\tilde{\mu}} h^{\tilde{\nu}}_\nu h^{\tilde{\epsilon}}_\epsilon \tilde{\Gamma}^{\tilde{\mu}}_{\tilde{\nu}\tilde{\epsilon}} + h^\mu_{\tilde{s}} \partial_\nu h^{\tilde{s}}_\epsilon, \tag{1}$$

$$\tilde{\Gamma}^{\tilde{\mu}}_{\tilde{\nu}\tilde{\epsilon}} = h^{\tilde{\mu}}_a \partial_{\tilde{\nu}} h^a_{\tilde{\epsilon}}. \tag{2}$$

Let us denote the first part of (1) as $\tilde{\Gamma}^\mu_{\nu\epsilon}$ and the second one as $\bar{\Gamma}^\mu_{\nu\epsilon}$ and rewrite (1) as

$$\Gamma^\mu_{\nu\epsilon} = \tilde{\Gamma}^\mu_{\nu\epsilon} + \bar{\Gamma}^\mu_{\nu\epsilon}. \tag{3}$$

Relatively to the local coordinates dB^μ, i.e. for $\partial_\mu = \partial/\partial B^\mu$ in the definition of the curvature tensor:

$$R^\alpha_{\beta\gamma\delta} \equiv 2(\partial_{[\gamma}\Gamma^\alpha_{\delta]\beta} + \Gamma^\alpha_{\epsilon[\gamma}\Gamma^\epsilon_{\delta]\beta}), \tag{4}$$

the curvature of the connections $\Gamma^\mu_{\nu\epsilon}$ and $\bar{\Gamma}^\mu_{\nu\epsilon}$ is equal to zero. The incomplete connection $\tilde{\Gamma}^\mu_{\nu\epsilon}$ (the tensor part of Γ) has a nontrivial curvature, but it is equal to zero by $\tilde{\Gamma}^\mu_{\nu\epsilon} = 0$, therefore the one cannot be used as Einstein's connection.

3 A linear deformation of the connection

The connection $\gamma^\mu_{\nu\epsilon}$, which can be Einstein's one on some 4-manifold, should have the following properties: 1) it must sutisfy the transformation law (1) under transition to a new system of reference of an observer $B^\mu \to C^\mu$; 2) its curvature tensor must be not trivial: $r^\alpha_{\beta\gamma\delta} \not\equiv 0$; 3) $r^\alpha_{\beta\gamma\delta} \not\equiv 0$, if $\gamma^\mu_{\nu\epsilon} = 0$.

The linear form γ of the connections $\tilde{\Gamma}$ and $\bar{\Gamma}$, with one parameter f, satisfies these demands:

$$\gamma^\mu_{\nu\epsilon} = f\tilde{\Gamma}^\mu_{\nu\epsilon} + \bar{\Gamma}^\mu_{\nu\epsilon}. \tag{5}$$

The transformation $\Gamma = \tilde{\Gamma} + \bar{\Gamma} \to \gamma = f\tilde{\Gamma} + \bar{\Gamma}$ is called here a linear deformation of the connection. In the paper, the parameter f is global that provides the global $U(1)-$symmetry of the model (with the peculiarity which is discussed below).

The curvature tensor of this connection relatively to the local coordinates dB^μ is nontrivial by $f \neq 0; 1$:

$$r^\alpha_{\beta\gamma\delta} = 2(f^2 - f)\tilde{\Gamma}^\alpha_{\epsilon[\gamma}\tilde{\Gamma}^\epsilon_{\delta]\beta}, \tag{6}$$

if $\tilde{\Gamma}^\mu_{\nu\epsilon} \not\equiv 0$. Under the condition $\gamma^\mu_{\nu\epsilon} = 0$, we have

$$r^\alpha_{\beta\gamma\delta} = ((f-1)/2)\tilde{R}^\alpha_{\beta\gamma\delta},$$

where $\tilde{R}^\alpha_{\beta\gamma\delta}$ is the curvature tensor of the incomplete connection $\tilde{\Gamma}$.

4 Picking out of the four-dimensional curved manifold

Let us denote as $g_{\mu\nu}$ the metric tensor which corresponds to the metric connection $\gamma^\mu_{\nu\epsilon}$. In the flat 12-space, let us pick out the four-dimensional curved manifold Σ^4 with the metric tensor $g_{\mu\nu}$ and the metric connection $\gamma^\mu_{\nu\epsilon}$, on which Einstein's field equations are satisfied:

$$r_{\mu\nu} = k(T_{\mu\nu} - g_{\mu\nu}T/2), \tag{7}$$

where $r_{\mu\nu}$ is the Ricci tensor, k is Einstein's constant, and $T_{\mu\nu}$ is the matter energy-momentum tensor.

We have

$$r_{\mu\nu} = 2(f^2 - f)\tilde{\Gamma}^\alpha_{\epsilon[\alpha}\tilde{\Gamma}^\epsilon_{\nu]\mu}. \tag{8}$$

Then by $T_{\mu\nu} = 0$, the manifold Σ^4 are picked out by the algebraic equations for $\tilde{\Gamma}^\mu_{\nu\epsilon}$, which do not depend on the parameter f (under the condition $f^2 - f \neq 0$):

$$\tilde{\Gamma}^\alpha_{\epsilon[\alpha}\tilde{\Gamma}^\epsilon_{\nu]\mu} = 0. \tag{9}$$

One can find the metric tensor $g_{\mu\nu}$ from the definition of the metric connection:

$$g^{\mu\alpha}(g_{\alpha\nu,\epsilon} + g_{\alpha\epsilon,\nu} + g_{\nu\epsilon,\alpha})/2 = \gamma^\mu_{\nu\epsilon}. \tag{10}$$

In a general case, a situation is more complex. For $T_{\mu\nu} \neq 0$, instead of (9) we have the algebraic equations for the manifold Σ^4 :

$$2(f^2 - f)\tilde{\Gamma}^\alpha_{\epsilon[\alpha}\tilde{\Gamma}^\epsilon_{\nu]\mu} = k(T_{\mu\nu} - g_{\mu\nu}T/2), \tag{11}$$

therewith $T_{\mu\nu}$ must be computed on Σ^4, and $g_{\mu\nu}$ should satisfy the equation (10).

The equation of a trial body motion on Σ^4 on the coordinates B^μ :

$$d^2B^\mu/ds^2 + \gamma^\mu_{\nu\epsilon}u^\nu u^\epsilon = 0, \tag{12}$$

where $u^{\nu} = dB^{\nu}/ds$, can be rewritten on the coordinates $A^{\tilde{\mu}}$ as

$$d^2 A^{\tilde{\mu}}/ds^2 + f\tilde{\Gamma}^{\tilde{\mu}}_{\tilde{\nu}\tilde{\epsilon}} u^{\tilde{\nu}} u^{\tilde{\epsilon}} = 0, \tag{13}$$

where $u^{\tilde{\nu}} = dA^{\tilde{\nu}}/ds$, therewith it has such the view independently from a choice of the coordinates B^{μ}. A motion of a picked out observation point on Σ^4 is described by the equation:

$$d^2 B^{\mu}/ds^2 + f\Gamma^{\mu}_{\nu\epsilon} u^{\nu} u^{\epsilon} = 0. \tag{14}$$

5 The additional coordinates as fields in the Minkowski space

On the first view, the equations (9) are seemed to depend on coordinates B^{μ}. But these equations are easy transformed to the following form:

$$h^{s}_{\tilde{\nu}} h^{m}_{\tilde{\mu}} h^{\tilde{\mu}}_{s[m} h^{\tilde{\nu}}_{c]d} = 0, \tag{15}$$

for which its independence from B^{μ} is obvious. The equations (15), rewritten in details, can be interpreted as the nonlinear differential equations of the second order for the "field" $A^{\tilde{\mu}}$ in the Minkowski space (x) :

$$\frac{\partial x^s}{\partial A^{\tilde{\nu}}} \frac{\partial x^m}{\partial A^{\tilde{\mu}}} \frac{\partial^2 A^{\tilde{\mu}}}{\partial x^s \partial x^{[m}} \frac{\partial^2 A^{\tilde{\nu}}}{\partial x^{l]} \partial x^d} = 0. \tag{16}$$

Its coefficients are independent from B^{μ}, therefore Σ^4 is a "cylindrical" hypersurface in the 12-space (x, A, B).

6 The global symmetries of the model

Let us denote $F = f^2 - f$, and let $F_1 = F_2$ be the function values for two values f_1 and f_2 of the parameter f. It follows from Eqs. (7), (8), and (11), that the same connection $\gamma^{\mu}_{\nu\epsilon}$ and two different connections $\tilde{\Gamma}^{\mu}_{\nu\epsilon}$ (so as $\gamma = f\tilde{\Gamma} + \bar{\Gamma}$) correspond to these two values f_1 and f_2. The discrete D_1-symmetry will take place on Σ^4. By an introduction of spinor fields of the composite fermions [3], the last symmetry would be transformed into the global $SU(2)$-symmetry of the unified model.

The parameter f can have any value, excluding $f = 0; 1$. On the manifold Σ^4, the global variations of f are not observable. An existence of two peculiar points will be not essential by localization of variations of f. So one can consider $U(1)$ to be the global symmetry group of the model, with the made note.

In a general case, transformations of the groups $SU(2)$ and $U(1)$ will be connected between themselves. $SU(2)$-transformations correspond to "rotations" around the axis F, where $F = f^2 - f$, in a transformation parameters space. It means that a variation of the parameter f can lead to a permutation of a pair of solutions which will be transformed by the group $SU(2)$. One needs of additional restrictions to have the $SU(2) \times U(1)-$symmetry of the model.

The transformations can be independent if: 1) the $SU(2)$-doublets are massless, and 2) a region of permissible variations of the parameter f is such a one that for any pair $f_1, f_2 : F(f_1) \neq F(f_2)$. To satisfy the second condition, one can perform a rotation in the plane (f, F) on some angle θ. Under an additional condition that one component of the $SU(2)$-doublet should be massless after breaking of the $SU(2)$-symmetry (that is equivalent to the demand $f \to 1$ for the component), θ has the minimum value θ_{min}, for which $\sin^2 \theta_{min} = 0,20$ (θ_{min} is the angle between the axis f and the straight line, which goes through the points $(1/2, -1/4)$ and $(1, 0)$ on the plane (f, F)). It is approximate enough to the value of the same function for the Weinberg angle of the standard model. There is a very close analogy with the situation in the standard model, when gauge fields of the groups $U(1)$ and $SU(2)$ are linear transformed to get a gauge field of the observable $U(1)-$symmetry [5].

Out of the massless limit, the $SU(2)-$symmetry will be broken automatically, and the $U(1)-$one will be preserved.

7 Conclusion

The considered linear deformation of the connection, with the global parameter f, will be a universal method to embed the general relativity 4-space into a flat 12-space, if any its Ricci tensor would be presented as the quadratic form (8). This question needs an additional research.

A local action of the group $U(1)$ should be accompanied by a statistical description of trajectories of a trial particle, because non-observable variations of f from one point of the space (x) to another will provide fluctuations of particle's trajectory in the space.

To unify the described model of gravitation in the flat 12-space with the composite fermions model with the $SU(3)_c \times SU(2)_L-$symmetry in the 8-space (x, y) [3], we would use the flat 16-space $(x, A = (x_1 + x_2)/2, B, y)$, where x_1, x_2 are the coordinates in the flat 8-space with torsion, for which we had in [3]: $(x_1 + x_2)/2 = x,\ (x_1 - x_2)/2 = y$. The structure of the discrete space (y) is caused by symmetry properties of the equations of motion of the fundamental fermions [3]. Namely the structure leads to an appearance of the exact $SU(3)_c-$symmetry of the composite fermions.

Equations (9) and (11) are uniform relatively to $\tilde{\Gamma}$. It is an additional advantage, which gives us a possibility to linearize these equations relatively to $\tilde{\Gamma}$, introducing new variables [6, 7].

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