**Affine connections on three-dimensional pseudo-Riemannian homogeneous spaces. I**

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**Abstract**

The aim of this paper is to describe all invariant affine connections on pseudo-Riemannian homogeneous spaces of dimensions 2 and 3. We present a complete local classification of Riemannian homogeneous spaces which is equivalent to the description of effective pairs of Lie algebras supplied with an invariant nondegenerate symmetric bilinear form on the isotropy module. The classification of pseudo-Riemannian homogeneous spaces is given in a separate paper (Part 2). We describe all invariant affine connections together with their curvature and torsion tensors and single out affine connections on Riemannian homogeneous spaces and Riemannian connections.

**Keywords and phrases**

invariant affine connectionpseudo-Riemannian homogeneous space

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