**Affine connections on three-dimensional pseudo-Riemannian homogeneous spaces. I**

* **N. P. Mozhei**

[Email author](mailto:mozheynatalya@mail.ru)

* + *Kazan (Volga Region) Federal University*

Article

**First Online:**

28 November 2013

**Received:**

21 August 2012

DOI: 10.3103/S1066369X13120050

**Cite this article as:**

Mozhei, N.P. Russ Math. (2013) 57: 44. doi:10.3103/S1066369X13120050

**Abstract**

The aim of this paper is to describe all invariant affine connections on pseudo-Riemannian homogeneous spaces of dimensions 2 and 3. We present a complete local classification of Riemannian homogeneous spaces which is equivalent to the description of effective pairs of Lie algebras supplied with an invariant nondegenerate symmetric bilinear form on the isotropy module. The classification of pseudo-Riemannian homogeneous spaces is given in a separate paper (Part 2). We describe all invariant affine connections together with their curvature and torsion tensors and single out affine connections on Riemannian homogeneous spaces and Riemannian connections.

**Keywords and phrases**

invariant affine connectionpseudo-Riemannian homogeneous space

Original Russian Text © N.P. Mozhei, 2013, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2013, No. 12, pp. 51–68.

References

1.

T. Levi-Civita, “Nozione di Parallelismo in una Varietà Qualunque,” Rend. Circ. Mat. Palermo **42**, 173–205 (1917).[CrossRef](http://dx.doi.org/10.1007/BF03014898" \t "_blank)

2.

H. Weyl, *Raum, Zeit, Materie* (6e Aufl., 1923; Springer, 1970).[CrossRef](http://dx.doi.org/10.1007/978-3-642-98950-6)

3.

É. Cartan “Les Espacesà Connexion Conforme,” Ann. Soc. Polon. Math. **2**, 171–211 (1923).

4.

Yu. G. Lumiste, “The Theory of Connections in Fiber Bundles,” Itogi Nauki. Algebra. Topologiya. Geometriya (VINITI, Moscow, 1971), p. 123–168.

5.

G. I. Kruchkovich, “Classification of Three-Dimensional Riemannian Spaces by Groups of Motions,” Usp. Mat. Nauk **9**(1), 3–40 (1954).[MATH](http://www.emis.de/MATH-item?0055.40201)

6.

W. Thurston, “Three-DimensionalManifolds, KleinianGroups and Hyperbolic Geometry,” Bull. Amer. Math. Soc. **6**, 357–381 (1982).[MathSciNet](http://www.ams.org/mathscinet-getitem?mr=648524" \t "_blank)[CrossRef](http://dx.doi.org/10.1090/S0273-0979-1982-15003-0)[MATH](http://www.emis.de/MATH-item?0496.57005)

7.

P. Scott, “The Geometries of 3-Manifolds,” Bull. LondonMath. Soc. **15**, part 5 (56), 401–487 (1983).[CrossRef](http://dx.doi.org/10.1112/blms/15.5.401" \t "_blank)[MATH](http://www.emis.de/MATH-item?0561.57001)

8.

P. A. Shirokov, *Selected Papers in Geometry* (Kazan Univ. Press, Kazan, 1966).

9.

A. Z. Petrov, “On Geodesic Mappings of Riemannian Spaces with Indefinite Metric,” Uchen. Zap. Kazan. Univ. **109**(3), 7–36 (1949).

10.

A. L. Onishchik, *Topology of Transitive Lie Groups of Transformations* (Fizmatlit, Moscow, 1995).

11.

S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry* (New York-London, 1963), Vol. 1.[MATH](http://www.emis.de/MATH-item?0119.37502)

12.

S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry* (New York-London, 1969), Vol. 2.[MATH](http://www.emis.de/MATH-item?0175.48504)

13.

G. D. Mostow, “The Extensibility of Local Lie Groups of Transformations and Groups on Surfaces,” Ann. Math. **52**(3), 606–636 (1950).[MathSciNet](http://www.ams.org/mathscinet-getitem?mr=48464" \t "_blank)[CrossRef](http://dx.doi.org/10.2307/1969437)[MATH](http://www.emis.de/MATH-item?0040.15204)

14.

S. Lie, *Theorie der Transformationsgruppen*. Abschn. III (Teubner, Leipzig, 1893).[MATH](http://www.emis.de/MATH-item?JFM%2025.0623.01" \t "_blank)

15.

B. Komrakov, A. Churyumov, and B. Doubrov, “Two-Dimensional Homogeneous Spaces,” Preprint No. 17 (Univ. Oslo, 1993).

16.

K. Nomizu, “Invariant Affine Connections on Homogeneous Spaces,” Amer. J. Math. **76**(1), 33–65 (1954).[MathSciNet](http://www.ams.org/mathscinet-getitem?mr=59050)[CrossRef](http://dx.doi.org/10.2307/2372398)[MATH](http://www.emis.de/MATH-item?0059.15805)

17.

O. Kowalski, “Riemannian Manifolds with Homogeneous Geodesics,” Boll. Unione Math. Ital. VII. Ser. B **5**(1), 189–246 (1991).[MATH](http://www.emis.de/MATH-item?0339.62018)

18.

B. Komrakov, A. Tchourioumov, N. Mozhey et al. “Three-Dimensional Isotropically-Faithful Homogeneous Spaces,” Preprints Nos. 35–37 (Univ. Oslo, 1993).