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New Approach to the Generalized Poincare Conjecture

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ABSTRACT

Using our proof of the Poincare conjecture in dimension three and the method of mathematical induction a short and transparent proof of the generalized Poincare conjecture (the main theorem below) has been obtained. **Main Theorem.** *Let M^n be a n-dimensional, connected, simply connected, compact, closed, smooth manifold and there exists a smooth finite triangulation on M^n which is coordinated with the smoothness structure of M^n. If S^n is the n-dimensional sphere then the manifolds M^n and S^n are homemorphic.*

Keywords: Compact Smooth Manifolds; Riemannian Metric; Smooth Triangulation; Homotopy-Equivalence; Algorithms

1. Introduction

We can fix some Riemannian metric g on a manifold M^n of dimension n which defines the length of arc of a piecewise smooth curve and the continuous function $\rho(x;y)$ of the distance between two points $x,y\in M^n$. The topology defined by the function of distance (metric) ρ is the same as the topology of the manifold M^n [1].

In Section 1, using a smooth triangulation considered in the main theorem and a Riemannian metric we construct an algorithm of extension of coordinate neighborhood. With the help of this algorithm we get that every compact, connected, closed manifold M^n of dimension n having the triangulation above can be represented as a union of a n-dimensional cell C^n and a connected union K^{n-1} of some finite number of simplexes of the triangulation having dimension less or equal $(n-1)$. A sufficiently small closed neighborhood of K^{n-1} is called a *geometric black hole* [2]. Simplexes with boundaries can be retracted *i.e.* a decomposition $M^n=\tilde{C}^n\cup\tilde{K}^{n-1}$ can be obtained where $\tilde{K}^{n-1}$ contains less simplexes than $\tilde{K}^{n-1}$ does.

In Section 2, we consider the proof of the main theorem consisting of the realization of several algorithms. Using the method of mathematical induction and the algorithms we retract all the simplexes from $\tilde{K}^{n-1}$ to a point x_0, therefore a decomposition $M^n=C^n\cup\{x_0\}$ is obtained and M^n is homeomorphic to the sphere S^n.

2. On Algorithm of Extension of Coordinate Neighborhood

1) Let M^n be a connected, compact, closed and smooth manifold of dimension n and C^n be a cell (coordinate neighborhood) on M^n. A standard simplex Δ^n of dimension n is the set of points $x=(x_1,x_2,\cdots,x_n)\in\boldsymbol{R}^n$ defined by conditions

$$0\le x_i\le 1,\ i=1,\cdots n,\ x_1+x_2+\cdots+x_n\le 1.$$

We consider the interval of a straight line connected the center of some face of Δ^n and the vertex which is opposite to this face. It is clear that the center of Δ^n belongs to the interval. We can decompose Δ^n as a set of intervals which are parallel to that mentioned above. If the center of Δ^n is connected by intervals with points of some face of Δ^n then a subsimplex of Δ^n is obtained. All the faces of Δ^n considered, Δ^n is seen as a set of all such subsimplexes. Let $U(D^n)$ be some open neighborhood of Δ^n in $\boldsymbol{R}^n$. A diffeomorphism $\varphi:U(\Delta^n)\to M^n\left(\delta^n=\varphi(\Delta^n)\right)$ is called a singular n-simplex on the manifold M^n. *Faces*, *edges*, *the center*, *vertexes* of the simplex δ^n are defined as the images of those of Δ^n with respect to φ.

The manifold M^n is triangulable [3]. It means that for any l, $0\le l\le n$ such a finite set Φ^l of diffeomorphisms $\varphi:\Delta^l\to M^n$ is defined that

a) M^n is a disjunct union of images $\varphi(\mathrm{Int}\Delta^l)$, $\varphi\in\Phi^l$;

b) if $\varphi \in \Phi^l$ then $\varphi \circ \varepsilon_i \in \Phi^{l-1}$ for every i where $\varepsilon_i : \Delta^{k-1} \to \Delta^k$ is the linear mapping transferring the vertexes $v_0, \cdots, v_{k-1}$ of the simplex Δ^{k-1} in the vertexes $v_0, \cdots, \hat{v}_i, \cdots, v_k$ of the simplex Δ^k.

We suppose that there exists a smooth finite triangulation on M^n which is coordinated with the smoothness structure of M^n and fix the triangulation. Such triangulations exist for manifolds of dimension 2 or 3.

2) Let δ_0^n be some simplex of the fixed triangulation of the manifold M^n. We paint the inner part $\operatorname{Int}\delta_0^n$ of the simplex δ_0^n white and the boundary $\partial\delta_0^n$ of δ_0^n black. There exist coordinates on $\operatorname{Int}\delta_0^n$ given by diffeomorphism φ_0. A subsimplex $\delta_{01}^{n-1} \subset \delta_0^n$ is defined by a black face $\delta_{01}^{n-1} \subset \delta_0^n$ and the center c_0 of δ_0^n. We connect c_0 with the center d_0 of the face δ_{01}^{n-1} and decompose the subsimplex δ_{01}^n as a set of intervals which are parallel to the interval $c_0 d_0$. The face δ_{01}^{n-1} is a face of some simplex δ_1^n that has not been painted. We draw an interval between d_0 and the vertex v_1 of the subsimplex δ_1^n which is opposite to the face δ_{01}^{n-1} then we decompose δ_1^n as a set of intervals which are parallel to the interval $d_0 v_1$. The set $\delta_{01}^n \cup \delta_1^n$ is a union of such broken lines every one from which consists of two intervals where the endpoint of the first interval coincides with the beginning of the second interval (in the face δ_{01}^{n-1}) the first interval belongs to δ_{01}^n and the second interval belongs to δ_1^n. We construct a homeomorphism (extension) $\varphi_{01}^1 : \operatorname{Int}\delta_{01}^n \to \operatorname{Int}\left(\delta_{01}^n \cup \delta_1^n\right)$. Let us consider a point $x \in \operatorname{Int}\delta_{01}^n$ and let x belong to a broken line consisting of two intervals the first interval is of a length of s_1 and the second interval is of a length of s_2 and let x be at a distance of s from the beginning of the first interval. Then we suppose that $\varphi_{01}^1(x)$ belongs to the same broken line at a distance of $\dfrac{s_1 + s_2}{s_1} \cdot s$ from the beginning of the first interval. It is clear that φ_{01}^1 is a homeomorphism giving coordinates on $\operatorname{Int}\left(\delta_{01}^n \cup \delta_1^n\right)$. We paint points of $\operatorname{Int}\left(\delta_{01}^n \cup \delta_1^n\right)$ white. Assuming the coordinates of points of white initial faces of subsimplex δ_{01}^n to be fixed we obtain correctly introduced coordinates on $\operatorname{Int}\left(\delta_0^n \cup \delta_1^n\right)$. The set $\sigma_1 = \delta_0^n \cup \delta_1^n$ is called a *canonical polyhedron*. We paint faces of the boundary $\partial\sigma_1$ black.

We describe the contents of the successive step of the algorithm of extension of coordinate neighborhood. Let us have a canonical polyhedron σ_{k-1} with white inner points (they have introduced *white coordinates*) and the black boundary $\partial\sigma_{k-1}$. We look for such an n-simplex in σ_{k-1}, let it be δ_0^n that has such a black face, let it be δ_{01}^{n-1} that is simultaneously a face of some n-simplex, let it be δ_1^n, inner points of which are not painted. Then we apply the procedure described above to the pair δ_0^n, δ_1^n. As a result we have a polyhedron σ_k with one simplex more than σ_{k-1} has. Points of $\operatorname{Int}\sigma_k$ are painted white and the boundary $\partial\sigma_k$ is painted black. The process is finished in the case when all the black faces of the last polyhedron border on the set of white points (the cell) from two sides.

After that all the points of the manifold M^n are painted in black or white, otherwise we would have that $M^n = M_0^n \cup M_1^n$ (the points of M_0^n would be painted and those of M_1^n would be not) with M_0^n and M_1^n being unconnected, which would contradict of connectivity of M^n.

Thus, we have proved the following.

Theorem 1. *Let M^n be a connected, compact, closed, smooth manifold of dimension n. Then $M^n = C^n \cup K^{n-1}$, $C^n \cap K^{n-1} = \varnothing$, where C^n is an n-dimensional cell and K^{n-1} is a union of some finite number of $(n-1)$-simplexes of the triangulation.*

3) We consider the initial simplex δ_0^n of the triangulation and its center c_0. Drawing intervals between the point c_0 and points of all the faces of δ_0^n we obtain a decomposition of δ_0^n as a set of the intervals. In 2) the homeomorphism $\psi : \operatorname{Int}\delta_0^n \to C^n$ was constructed and ψ evidently maps every interval above on a piecewise smooth broken line γ in C^n. We denote $\tilde{M}^n = M^n \setminus \{c_0\}$. $\tilde{M}^n$ is a connected and simply connected manifold if M^n is that. Let $I = [0;1]$, we define a homotopy $F : \tilde{M}^n \times I \to \tilde{M}^n : (x;t) \mapsto y = F(x;t)$ in the following way

a) $F(z;t) = z$ for every point $z \in K^{n-1}$;

b) if a point x belongs to the broken line γ in C^n and the distance between x and its limit point $z \in K^{n-1}$ is $s(x)$ then $y = F(x;t)$ is on the same broken line γ at a distance of $(1-t)s(x)$ from the point z.

It is clear that $F(x;0) = x$, $F(x;1) = z$ and we have obtained the following.

Theorem 2. *The spaces $\tilde{M}^n$ and K^{n-1} are homotopy-equivalent, in particular, the groups of singular homologies $H_k\left(\tilde{M}^n\right)$ and $H_k\left(K^{n-1}\right)$ are isomorphic for every k.*

Corollary 2.1. *The space K^{n-1} is connected and if M^n is simply connected then K^{n-1} is simply connected too.*

Remark 1. *The white coordinates are extended from the simplex δ_0^n in the simplex δ_1^n through the face δ_{01}^{n-1} hence $\operatorname{Int}\delta_{01}^{n-1}$ has also the white coordinates. On the other hand there exist two linear structures (intervals, the center etc.) on δ_{01}^n induced from δ_0^n and δ_1^n respectively. Further, we set that the linear structure of δ_{01}^{n-1} is the structure induced from δ_0^n.*

Remark 2. *In the process of getting of C^n in 2) we can construct a maximal tree L connecting by intervals all the centers of the n-simplexes of the triangulation via the centers of some white faces.*

Conversely, if we have such a maximal tree L connecting by intervals all the centers of the n-simplexes of *the triangulation via the centers of some faces* (*any from two possible centers of a face can be chosen*) *then we can extend white coordinates from any simplex* δ_0^n *on the maximal cell* C^n *as it was shown in* 2). *Thus*, *the maximal tree L defines the maximal cell* C^3 *and white faces.*

4) Definition 1. a) *A simplex* $\delta^k \in K^{n-1}\left(k=\overline{1,n-1}\right)$ *is called free if it has at least one free face* δ^{k-1} *i.e. such a face that it is not a face of any other k-simplex from* K^{n-1}.

b) *An edge* $\delta^1 = x_0x_1$ *is called semi-isolated if it is not an edge of any simplex from* K^{n-1}. *A semi-isolated edge* δ^1 *is called isolated if it is free.*

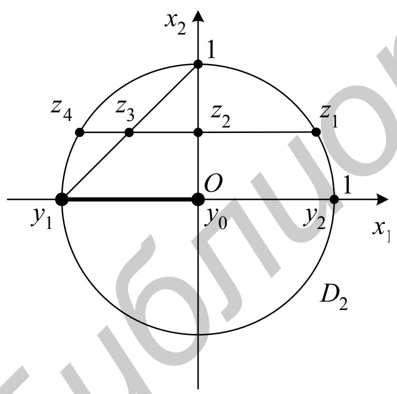
Let us have a free simplex $\delta^k \in K^{n-1}$ *with some free face* δ^{k-1}. *We consider such a polyhedron* σ *that* σ *is the set of all n-simplexes having common point with* δ^{k-1}.

Proposition 3. *We can redistribute coordinates of white points of the polyhedron* δ (*retract the free simplex* δ^k) *i.e. construct the corresponding mapping* φ_σ *in such a way that the following conditions are fulfilled*:

a) *all the points of* $\mathrm{Int}\sigma$ *are painted white i.e. have new white coordinates*,

b) *white coordinates of points of boundary faces of the polyhedron* σ *are not changed.*

Proof. a) We consider the unit disk D^2 having the center in the origin $O = y_0$ of the coordinate system Ox_1x_2 of $\mathbf{R}^2$ and the radius y_0y_1.



We define a mapping $\psi : D^2 \to D^2$ by the following way:

- $\psi\left(y_0y_2\right) = y_1y_2$, $\psi\left(y_0\right) = y_1$, $\psi\left(y_2\right) = y_2$;
- for any chord z_1z_4 which is parallel to y_0y_1 $\psi\left(z_1z_2\right) = z_1z_3$, $\psi\left(z_2\right) = z_3$, $\psi\left(z_1\right) = z_1$, $\psi\left(z_2z_4\right) = z_3z_4$, $\psi\left(z_4\right) = z_4$.

It is clear that ψ maps $\mathrm{Int}D^2 \setminus y_0y_1$ onto $\mathrm{Int}D^2$ and $\psi = id$ on the boundary circle of D^2.

b) We consider the unit disk $D^{k-1} : x_1^2 + x_2^2 + \cdots + x_{k-1}^2 \le 1$ having the center in the origin O of the coordinate system $Ox_1x_2\cdots x_{k-1}$ and the semidisk $SD^{k-2} : x_{k-1} = 0$, $x_{k-2} \le 0$, $x_1^2 + x_2^2 + \cdots + x_{k-2}^2 \le 1$. By inductive hypothesis we assume that such a mapping $\psi : D^{k-1} \to D^{k-1}$ has been constructed that ψ maps $\mathrm{Int}D^{k-1} \setminus SD^{k-2}$ onto $\mathrm{Int}D^{k-1}$ and $\psi = id$ on the boundary of D^{k-1}.

Further, we consider the unit disk $D^k : x_1^2 + x_2^2 + \cdots + x_k^2 \le 1$ in the coordinate system $Ox_1x_2\cdots x_k$, the semidisk $SD^{k-1} : x_k = 0,\ x_{k-1} \le 0$, $x_1^2 + x_2^2 + \cdots + x_{k-1}^2 \le 1$ and the family of disks $D_t^{k-1} : x_1^2 + x_2^2 + \cdots + x_k^2 \le 1$, $x_{k-2} = t$, $t \in [-1;1]$. We denote $SD_t^{k-2} = D_t^{k-1} \bigcap SD^{k-1}$. By inductive hypothesis there exists such the family of mappings $\psi_t : D_t^{k-1} \to D_t^{k-1}\left(t \in [-1;1]\right)$ that every ψ_t maps $\mathrm{Int}D_t^{k-1} \setminus SD_t^{k-2}$ onto $\mathrm{Int}D_t^{k-1}$ and $\psi_t = id$ on the boundary of D_t^{k-1}. Union of all ψ_t gives the mapping $\psi : D^k \to D^k$, ψ maps $\mathrm{Int}D^k \setminus SD^{k-1}$ onto $\mathrm{Int}D^k$ and $\psi = id$ on the boundary of D^k.

Thus, the mapping ψ is constructed for any $n \in \mathbf{N}$ by the method of mathematical induction.

c) It is clear that there exists such a homeomorphism $\varphi : \sigma \to D^n$ that $\varphi\left(\partial\sigma\right) = \partial D^n$ and $\varphi\left(\delta^k\right) \subset SD^{n-1}$. We define the mapping $\varphi_\sigma = \varphi^{-1} \circ \psi \circ \varphi$ then $\varphi_\sigma : \mathrm{Int}\sigma \setminus \sigma^k \to \mathrm{Int}\sigma$ is a required homeomorphism introducing new white coordinates in $\mathrm{Int}\sigma$.

QED.

Remark 3. *In is clear that the rebuilt complex* K^{n-1} *is connected and simply connected because of a homotopy-equivalence.*

5) We assume that in the process of painting free simplexes white by the Proposition 3 we get a representation $M^n = C^n \bigcup K^1$, $C^n \bigcap K^1 = \varnothing$, where K^1 is the connected union of black edges of the triangulation. Since the process of painting free simplexes white does not influence simply connectivity of a space that has been obtained every step then K^1 is a tree if the complex K^{n-1} is simply connected. Painting isolated edges of K^1 white by the Proposition 3 we have got unique black point x_0 as result. Thus, we obtain a representation $M^n = C^n \bigcup B^n\left(x_0;\varepsilon\right)$, where $B^n\left(x_0;\varepsilon\right)$ is an open geodesic ball with the center in x_0 and of a radius ε. The manifold M^n is homeomorfic to the sphere S^n by the following lemma 4.

Lemma 4 [1]. *If a topological manifold* M^n *is a union of two n-dimensional cells then* M^n *is homeomorfic to the sphere* S^n.

3. Proof of the Main Theorem

The proof has a combinatorial nature and assumes the realization of a number of algorithms. We consider that step by step. The initial complex K^{n-1} is assumed to be connected, simply connected and without free simplexes.

1) Proposition 5 (opening an input). *Let* δ_1^n *be some n–simplex of the triangulation having a black face* $\delta_{01}^{n-1} \in K^{n-1}$. *Then* $\mathrm{Int}\delta_{01}^{n-1}$ *can be repainted white to get a new decomposition* $M^n = C^n \bigcup K^{n-1}$, *where* K^{n-1} *is a new connected and simply connected complex.*

Proof. The face δ_{01}^{n-1} is the common face of *n*-simplexes δ_0^n and δ_1^n. We cansel the white painting of points of δ_1^n and paint the *n*-simplexe δ_1^n black. Repainting of δ_1^n black brings to a gap of the maximal tree *L* (see the Remark 2) on *n* subtrees $L_1, L_2, \cdots, L_n$ or less where the center of δ_0^n belongs to L_1. Further, we extend white coordinates from δ_0^n into δ_1^n through the face δ_{01}^{n-1} as it was shown in 2), 1 and connect the centers of δ_0^n, δ_{01}^{n-1}, δ_1^n by intervals. Those centers belong to the subtree L_1. Other faces of δ_1^n are black and they are simultaneously some faces of other *n*-simplexes.

We consider the following cases.

a) $L_1 = L$ or we have no a gap. The black faces of δ_1^n remain black.

b) We have got *k* subtrees $L_1, L_2, \cdots, L_k \left(k = \overline{2,n}\right)$ where the subtrees $L_2, \cdots, L_k$ define cells called *dead ends*. We repaint the closures of the dead ends black. Further, we are looking for a black face of δ_1^n which is simultaneously a face of other *n*-simplex with the center from L_1. This face remains black. For every subtree $L_i \left(i = \overline{2,k}\right)$ we consider a *n*-simplex with the center from L_i that has a common black face δ_{1i}^{n-1} with δ_1^n. We extend white coordinates from δ_1^n through δ_{1i}^{n-1} along the subtree L_i as it was shown in 2), 1 and repaint inner points of this face and points of the corresponding dead end white. Further, we connect by intervals the centers of δ_{1i}^{n-1} with the centers of δ_1^n and the other simplex connecting L_1 and L_i.

After repainting all the dead ends white we obtain a new maximal tree *L* defining a new maximal cell C^3. Retracting all the free simplexes by the Proposition 3 a new rebuilt complex K^{n-1} is obtained which is connected and simply connected because of homotopy-equivalence.

QED.

Remark 4. *A broken line has been obtained in the proof above which connects by intervals the centers of* δ_0^n, δ_{01}^{n-1}, δ_1^n. *This broken line is a part of the subtree* L_1 *of the maximal tree L. Let n-simplexes* δ_0^n *and* δ_1^n *have a common face* δ_{01}^{n-1} *having the white inter part and* $\mathrm{Int}\delta_{01}^{n-1}$ *has no common points with the maximal tree* L. *Then we can connect the centers of* δ_0^n, δ_{01}^{n-1}, δ_1^n *by the broken line by the method considered in the proof above.*

2) We assume the following inductive hypothesis:

The generalized Poincare conjecture (*the main theorem*) *can be proved by the method considered in* [4] *for dimension n*–1 *i.e. the representation* $M^{n-1} = C^{n-1} \cup \{x_0\}$ *can be obtained by the algorithm from* 2), 1 *and by the Propositions* 3, 4, 5.

It is obvious for $n-1=2$ (see 5), 1) It is proved for $n-1=3$ in [4].

We choose a small ball $B^n(x_0)$ with the center in a vertex x_0 which is diffeomorphic to a small ball in $\mathbf{R}^n$ and call a *trace* of *k*-simplex $\delta^k \left(k = \overline{1,n}\right)$ with a vertex in x_0 its intersection $\bar{\delta}^{k-1}$ with the sphere $S^{n-1}(x_0)$ (smooth manifold) which is the boundary of $B^n(x_0)$. The sphere $S^{n-1}(x_0)$ is supposed to be transversal to all the *k*-simplexes $\left(k = \overline{1,n}\right)$ with the vertex x_0. Such a sphere $S^{n-1}(x_0)$ exists because of the smoothness of the triangulation of M^n [5,6]. All other vertexes of the triangulation are supposed to be out of $B^n(x_0)$. The ball $B^n(x_0)$ can be chosen in such a vay that every edge with the endpoint x_0 has only one point of the intersection with $S^{n-1}(x_0)$ and every *k*-simplex δ^k with the vertex x_0 has only one connected component $\bar{\delta}^{k-1}$ of $\delta^k \cap S^{n-1}(x_0)$. Let $Bs^k(x_0)$ be the set of black *k*-simplexes with x_0 as their vertex and $Bs(x_0) = \bigcup_{k=1}^{n} Bs^k(x_0)$. There exists one to one correspondence between the set of simplexes having a vertex (endpoint) x_0 and the set of their traces on $S^{n-1}(x_0)$ therefore all steps of the algorithm below bring to the corresponding steps on the sphere $S^{n-1}(x_0)$ and the converse is true. In particular, a process of the construction of a maximal tree $\bar{L}_1$ on the sphere $S^{n-1}(x_0)$ (see the Remark 2) brings to the construction of a tree L_1 connecting by intervals all the centers of the *n*-simplexes with x_0 as their vertex via the centers of some white their faces. Every such the face has x_0 as its vertex.

Proposition 6. *The complex* K^{n-1} *can be rebuilt in such a vay that* $Bs(x_0)$ *contains only one* 1*-simplex* x_0x_1.

Proof. We consider the smooth triangulation of $S^{n-1}(x_0)$ induced by all the simplexes with the vertex x_0 and apply to this triangulation the algorithm from 2), 1 taking any $(n-1)$-simplex $\bar{\delta}_0^{n-1}$ as initial one where $\bar{\delta}_0^{n-1}$ is the trace of δ_0^n with a vertex x_0. Let $\bar{\delta}_0^{n-1}$ be the trace on $S^{n-1}(x_0)$ of δ_1^n with a vertex x_0 where $\bar{\delta}_1^{n-1}$ has a common face with $\bar{\delta}_0^{n-1}$. We repaint δ_1^n black and apply to it the proposition 5 (the remark 4) obtaining the canonical polyhedron $\bar{\delta}_0^{n-1} \cup \bar{\delta}_1^{n-1}$ on $S^{n-1}(x_0)$. Further, we iterate the algorithm. Every step of the algorithm on $S^{n-1}(x_0)$ implies the transformation of $Bs(x_0)$ and K^{n-1} by the proposition 5 (the remark 4). The maximal tree $\bar{L}_1$ on $S^{n-1}(x_0)$ and the corresponding subtree L_1 have been constructed in the end. Further, free black simplexes on $S^{n-1}(x_0)$ and the corresponding free simplexes from $Bs(x_0)$ can be annihilated by the propositions 3, 4, 5. By the inductive hypothesis only one black point remains on $S^{n-1}(x_0)$ in the end. This point is the trace of an edge x_0x_1 which is isolated.

QED.

Remark 5. *It is clear that if we paint black one inner vertex in the canonical polyhedron then we get two black* *points on* $S^{n-1}(x_0)$ *in the end of the algorithm.*

3) We consider a small ball $B^n(x_1)$ with the center x_1 and the boundary $S^{n-1}(x_1)$ which is similar to $B^n(x_0)(S^{n-1}(x_0))$. The centers of all the *n*-simplexes having x_0x_1 as their edge belong to the subtree L_1 and the union of all the traces of this *n*-simplexes on $S^{n-1}(x_1)$ forms the canonical polyhedron on $S^{n-1}(x_1)$ having one black inner vertex (the trace of isolated edge x_0x_1). We apply the Proposition 6 (the Remark 5) to the $S^{n-1}(x_1)$ and $Bs(x_1)$. As a result $Bs(x_1)$ consists of two semi-isolated edges x_0x_1 and x_1x_2.

Further, we iterate the process getting a broken line $x_0x_1\cdots x_k$ and for $i=\overline{1,k-1}$ $Bs(x_i)$ consists of two black semi-isolated edges $x_{i-1}x_i$ and x_ix_{i+1}. We remark that the process of the annihilation of black simplexes in $Bs(x_i)$ cannot bring to an appearance of a black simplex having a generic point with $x_{j-1}x_j\ (j<i)$. Really, otherwise such a black simplex gives an opportunity to connect the endpoints x_{i-1} and x_i of the semi-isolated edge $x_{i-1}x_i$ by a black curve which is different from $x_{i-1}x_i$. As a result a black loop with the semi-isolated edge $x_{i-1}x_i$ as its part has been obtained and the loop is not contractible that is a contradiction to the simply connectivity of K^{n-1}.

The complex K^{n-1} is connected therefore the broken line $x_0x_1\cdots x_k$ contains all the possible black vertexes from K^{n-1} at some step of the algorithm *i.e.* we come to 5, 1.

By the method of mathematical induction the main theorem is true for every $n\in\mathbf{N}, n\neq 1$.

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