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Experience in Statistical Forecasting of the Atmospheric Temperature

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Abstract:
In this paper, the problem of forecasting of quantitative features of the weather (atmospheric temperature, atmospheric pressure, direction and velocity of wind, relative humidity of the air and some other) is considered. The problem is formulated as a task of linear extrapolating a vector random sequence. Such forecast with statistical estimations of parameters of predictive algorithm was named as linear statistical forecast. Empirical data about precision of linear statistical forecast of the atmospheric temperature in comparison with precision of the official numerical forecast and climatological forecast is given and analyzed. The experience of forecasting shows that statistical forecast is more accurate than climatological forecast and a little less accurate than numerical forecast.

Keywords:
Extrapolation of a Random Sequence; Forecasting of the Atmospheric Temperature; Numerical Weather Forecast; Statistical Weather Forecast

1. INTRODUCTION

Analysis of the literary sources shows that the deterministic approach prevails in the problem of the weather forecasting. That approach is based on the hydrodynamic model of atmosphere expressed as differential equations which describe atmospheric processes. It is known as Numerical Weather Prediction (NWP) [1, 2] because the realization of hydrodynamic model is produced using numerical techniques. The NWP is so successful that "modern weather services completely rely on NWP models for their operational weather forecasting over time scales ranging from a few hours to a few weeks" [2].

There is a certain level of interest in, and attention to, in statistical techniques of the weather forecast [3, 4], including research separate from professional meteorologists [5]. In recent years, professionals in the field of weather forecasting come to the conclusion that, perhaps, the deterministic approach has reached its limits of predictability, and there has been a trend towards greater use of statistical techniques [2]. One of the ways to use statistical methods is in the servicing of the output of numerical weather forecast models. This pertains to the statistical post-processing of model output, data assimilation, ensemble forecasting techniques and others [2, 6].

There are some examples of "independent" or "pure" application of statistical methods in weather forecasting [5, 7]. However, they are difficult to reproduce for comparison because of different formulations, predictands, solution methods, regions considered, insufficiently detailed description of the algorithms used and their parameters, *etc*.

Purely theoretical developments available in the statistical literature often do not lead to any practical application and comparison (to prediction algorithms and estimation of their parameters, performed calculations in relation to meteorology, comparative analysis of the results of the calculations).

In author's opinion, statistical methods in weather forecasting and climate change deserve more attention and development. This paper outlines the use of "pure" application of statistical methods to predict the temperature of the atmosphere. The problem is considered as a task of extrapolating a random sequence. The analysis of the results for the statistical forecasting of the atmospheric temperature at the Minsk weather station will be compared with the available results of other weather forecasts.

2. ALGORITHM OF LINEAR STATISTICAL EXTRAPOLATING OF VECTOR RANDOM SEQUENCE

The following algorithm is based on the work of A. N. Kolmogorov [3] regarding the interpolation and extrapolation of stationary random sequence.

The problem of linear extrapolation of scalar stationary random sequence $\gamma(t)$ with zero mean value $E(\gamma(t)) = 0$, according to A. N. Kolmogorov [3], consists in the selection under given $s > 0$ and $m \geq 0$ such real coefficients a_i, in which a linear combination;

$$L = a_1\gamma(t-1) + a_2\gamma(t-2) + \ldots + a_s\gamma(t-s) \tag{1}$$

gives the most exact approximation to the random variable $\gamma(t+m)$. As measure of the accuracy of the approximation is an accepted value

$$\sigma^2 = E(\gamma(t+m) - L)^2 = B(0) - 2\sum_{s=1}^{n} B(m+s)a_s + \sum_{p=1}^{n}\sum_{q=1}^{n} B(p-q)a_p a_q, \tag{2}$$

where $B(k) = E(x(t+k)x(t))$, E is the mathematical expectation. If the moments of the second order $B(k)$ are known, that can be obtained such values of the coefficients a_s, under which σ^2 reaches the lowest value $\sigma_E^2(n,m)$.

In Kolmogorov [3] the algorithm used for the calculation of these coefficients (extrapolating algorithm) is not presented, and attention is focused on the analysis of value $\sigma_E^2(n,m)$.

This problem has such particularities as 1) linearity of the algorithm, 2) scalar and stationary sequence, 3) extrapolation on one fixed moment of time $t+m$, 4) enough general types of sequences.

The first of these three allows the execution of a generalized problem in different ways and the fourth is attractive for use in practical applications.

The solution of the problem generalized on vector, non-stationary, random sequence with extrapolation on any set of moments in time is given below [8].

The vector (n-variate) random sequence $\gamma(t_i) = (\gamma_1(t_i), \ldots, \gamma_n(t_i))$, $i = 1, 2, \ldots$, is considered. The observed part of sequence is denoted as two-dimensional $(n \times s)$-matrix

$$\xi = (\xi_{i_1,i_2}) = (\gamma_{i_1}(t_{i_2})), i_1 = \overline{1,n},\ i_2 = \overline{1,s}, \tag{3}$$

and the forecasted part as two-dimensional $(n \times (k_2 - k_1 + 1))$-matrix

$$\eta = (\eta_{i_1,i_2}) = (\gamma_{i_1}(t_{i_2+s+k_1-1})), i_1 = \overline{1,n}, i_2 = \overline{1,k_2-k_1+1}, 1 \leq k_1 \leq k_2. \tag{4}$$

Here s is the number counts of the observed part of the sequence, k_1, k_2 , and these are accordingly the minimum and maximum number of time periods in the forecast. The task of extrapolating is that upon observation $x = (x_{i_1,i_2})$ of the matrix ξ and in order to find estimation $\widehat{y} = (\widehat{y}_{j_1,j_2})$ the matrix η must be solved in a manner that minimizes the average risk under quadratic loss function,

$$r = E(^{0,2}(\widehat{y} - y)^2). \tag{5}$$

Here $^{0,2}(\widehat{y} - y)^2 = {}^{0,2}((\widehat{y} - y)(\widehat{y} - y))$ is the $(0,2)$-convolute square of matrix $(\widehat{y} - y)$ [8].

A decision on that problem is a linear estimation based on observations $x = (x_{i_1,i_2})$:

$$\widehat{y} = A_\eta + {}^{0,2}({}^{0,2}(R_{\eta,\xi}\,{}^{0,2}D_\xi^{-1})(x - A_\xi)), \tag{6}$$

and the variance matrix of the estimation is defined by expression,

$$D_{\eta/\xi} = D_\eta - {}^{0,2}({}^{0,2}(R_{\eta,\xi}\,{}^{0,2}D_\xi^{-1})R_{\xi,\eta}). \tag{7}$$

Here $A_\xi = E(\xi)$, $A_\eta = E(\eta)$, are the expected mathematical solutions, $D_\xi = E(^{0,0}(\xi - A_\xi)^2)$, $D_\eta = E(^{0,0}(\eta - A_\eta)^2)$, are the matrixes of variance, $R_{\xi,\eta} = E(^{0,0}((\xi - A_\xi)(\eta - A_\eta)))$ the mutual matrix of variance of the matrixes ξ and η accordingly, $^{0,2}D_\xi^{-1}$ –$(0,2)$-inverse to D_ξ matrix [8], $R_{\eta,\xi} = R^T_{\xi,\eta}$, T is a symbol for the transpose of matrix $R_{\xi,\eta}$ in accordance with substitution T [8],

$$T = \begin{pmatrix} i_1, i_2, i_3, i_4 \\ i_3, i_4, i_1, i_2 \end{pmatrix}. \tag{8}$$

The necessary proof is found in the work [9]. Marking in equation (6) $C = {}^{0,2}(R_{\eta,\xi}\,{}^{0,2}D_\xi^{-1})$, we will get a more high-speed algorithm

$$\widehat{y} = A_\eta + {}^{0,2}(C(x - A_\xi)), \tag{9}$$

where C is defined as decision following multidimensional-matrix equation

$$^{0,2}(CD_\xi) = R_{\eta,\xi}\,. \tag{10}$$

In algorithm (9), (10), in contrast with algorithm (6), (7), we avoid inverting the matrix D_ξ, but herewith we lose the possibility of the calculation for the *a posteriori* matrix of variance $D_{\eta/\xi}$ from the estimation (7).

We will name the matrices $A_\xi, A_\eta, D_\xi, D_\xi, R_{\eta,\xi}$ parameters of the extrapolation algorithm. In element form these parameters are defined as follows:

$$A_\xi = (a_{\xi,i_1,i_2}) = (E(\xi_{i_1,i_2})), i_1 = \overline{1,n_\xi},\; i_2 = \overline{1,s_\xi}, \tag{11}$$

$$A_\eta = (a_{\eta,j_1,j_2}) = (E(\eta_{j_1,j_2})), j_1 = \overline{1,n_\eta},\; j_2 = \overline{1,s_\eta}, \tag{12}$$

$$D_\xi = (d_{\xi,i_1,i_2,i_3,i_4}) = (E(\overset{\circ}{\xi}_{i_1,i_2}\overset{\circ}{\xi}_{i_3,i_4})), i_1 = \overline{1,n_\xi},\; i_2 = \overline{1,s_\xi}, i_3 = \overline{1,n_\xi},\; i_4 = \overline{1,s_\xi}, \tag{13}$$

$$D_\eta = (d_{\eta,j_1,j_2,j_3,j_4}) = (E(\underset{j_1,j_2}{\overset{\circ}{\eta}}\underset{j_3,j_4}{\overset{\circ}{\eta}})), j_1 = \overline{1,n_\eta},\ j_2 = \overline{1,s_\eta}, j_3 = \overline{1,n_\eta},\ j_4 = \overline{1,s_\eta}, \tag{14}$$

$$R_{\xi,\eta} = (r_{\xi,\eta,i_1,i_2,i_3,i_4}) = (E(\underset{i_1,i_2}{\overset{\circ}{\xi}}\underset{i_3,i_4}{\overset{\circ}{\eta}})), i_1 = \overline{1,n_\xi},\ i_2 = \overline{1,s_\xi}, i_3 = \overline{1,n_\eta},\ i_4 = \overline{1,s_\eta}. \tag{15}$$

The circle overhand symbol marks the centered random variable, for instance,

$$\underset{i_1,i_2}{\overset{\circ}{\xi}} = \xi_{i_1,i_2} - a_{\xi,i_1,i_2}. \tag{16}$$

The parameters of the algorithm can be received by moment functions from a random sequence. Let the mean value $A_\gamma(t) = (a_{\gamma,i}(t)) = (E(\gamma_i(t)))$ and the covariance function $R_\gamma(t,u) = (r_{\gamma,i,j}(t,u)) = (E(\underset{i}{\overset{\circ}{\gamma}}(t)\underset{j}{\overset{\circ}{\gamma}}(u)))$ of the vector random process $\gamma(t)$ be known. Then necessary matrix parameters can be received on the following formulas:

$$A_\xi = (a_{\xi,i_1,i_2}) = (a_{\gamma,i_1}(t_{i_2})), i_1 = \overline{1,n},\ \ i_2 = \overline{1,s}, \tag{17}$$

$$A_\eta = (a_{\eta,i_1,i_2}) = (a_{\gamma,i_1}(t_{i_2+s+k_1-1})), i_1 = \overline{1,n}, i_2 = \overline{1,k_2-k_1+1}, \tag{18}$$

$$D_\xi = (d_{\xi,i_1,i_2,i_3,i_4}) = (E(\underset{i_1,i_2}{\overset{\circ}{\xi}}\underset{i_3,i_4}{\overset{\circ}{\xi}})) = (E(\underset{i_1}{\overset{\circ}{\gamma}}(t_{i_2})\underset{i_3}{\overset{\circ}{\gamma}}(u_{i_4})) = (r_{\gamma,i_1,i_3}(t_{i_2},u_{i_4})),$$

$$i_1,i_3 = \overline{1,n}, i_2,i_4 = \overline{1,s}, \tag{19}$$

$$D_\eta = (d_{\eta,i_1,i_2,i_3,i_4}) = (E(\underset{i_1,i_2+s+k_1-1}{\overset{\circ}{\eta}}\underset{i_3,i_4+s+k_1-1}{\overset{\circ}{\eta}})) =$$

$$= (E(\underset{i_1}{\overset{\circ}{\gamma}}(t_{i_2+s+k_1-1})\underset{i_3}{\overset{\circ}{\gamma}}(u_{i_4+s+k_1-1}))) = (r_{\gamma,i_1,i_3}(t_{i_2+s+k_1-1},u_{i_4+s+k_1-1})),$$

$$i_1,i_3 = \overline{1,n}, i_2,i_4 = \overline{1,k_2-k_1+1}, \tag{20}$$

$$R_{\xi,\eta} = (r_{\xi,\eta,i_1,i_2,i_3,i_4}) = (E(\underset{i_1,i_2}{\overset{\circ}{\xi}}\underset{i_3,i_4+s+k_1-1}{\overset{\circ}{\eta}})) =$$

$$= (E(\underset{i_1}{\overset{\circ}{\gamma}}(t_{i_2})\underset{i_3}{\overset{\circ}{\gamma}}(u_{i_4+s+k_1-1}))) = (r_{\gamma,i_1,i_3}(t_{i_2},u_{i_4+s+k_1-1})),$$

$$i_1,i_3 = \overline{1,n}, i_2 = \overline{1,s}, i_4 = \overline{1,k_2-k_1+1}. \tag{21}$$

For a stationary random process $\gamma(t)$ we have the variables, $A_\gamma(t) = (a_{\gamma,i})$, $R_\gamma(t,u) = R_\gamma(t-u) = R_\gamma(u-t)$, and from the previous formulas take the following type:

$$A_\xi = (a_{\xi,i_1,i_2}) = (a_{\gamma,i_1}), i_1 = \overline{1,n},\ \ i_2 = \overline{1,s}, \tag{22}$$

$$A_\eta = (a_{\eta,i_1,i_2}) = (a_{\gamma,i_1}), i_1 = \overline{1,n},\ i_2 = \overline{1,k_2-k_1+1}, \tag{23}$$

$$D_\xi = (d_{\xi,i_1,i_2,i_3,i_4}) = (r_{\gamma,i_1,i_3}(t_{i_2} - u_{i_4})), i_1,i_3 = \overline{1,n},\ i_2,i_4 = \overline{1,s}, \tag{24}$$

$$D_\eta = (d_{\eta,i_1,i_2,i_3,i_4}) = (r_{\gamma,i_1,i_3}(t_{i_2+s+k_1-1} - u_{i_4+s+k_1-1})), i_1,i_3 = \overline{1,n}, i_2,i_4 = \overline{1,k_2-k_1+1}, \tag{25}$$

$$R_{\xi,\eta} = (r_{\xi,\eta,i_1,i_2,i_3,i_4}) = (r_{\gamma,i_1,i_3}(t_{i_2} - u_{i_4+s+k_1-1})), i_1,i_3 = \overline{1,n}, i_2 = \overline{1,s}, i_4 = \overline{1,k_2-k_1+1}. \tag{26}$$

In practice the parameters of the algorithm, in accordance with statistical substitution rule, can be replaced their estimations.

The practical application of the algorithm (9) and (10) consists of two stages: 1) obtain estimates of the parameters of the prediction algorithm and, 2) direct forecasting. The estimates can be received from a long prehistory of a random sequence. For direct forecasting a relatively short prehistory is used, directly converging to moment of the forecasting.

Obtaining estimates of the parameters in the prediction algorithm for non-stationary sequences is associated with great difficulty. Therefore its use should be limited to stationarity in the covariance function sequences, but allowing for the possibility of non-stationarity in the mathematical expectation.

3. AN EMPIRICAL ANALYSIS OF THE ACCURACY OF PREDICTING THE TEMPERATURE OF ATMOSPHERIC AIR

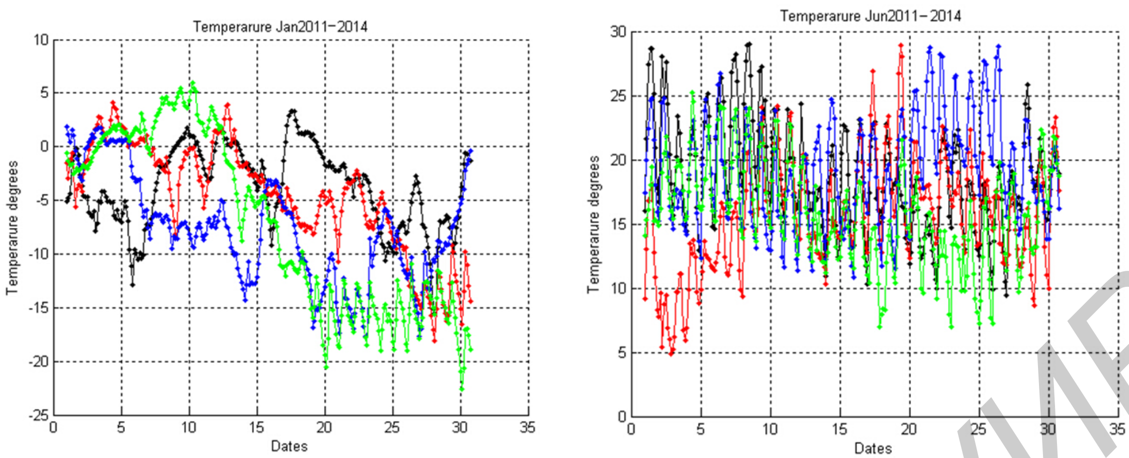
The quantitative features of the weather (the temperature of the atmosphere, atmospheric pressure, direction and velocity of the wind, relative humidity of the air, and any other) are measured at meteorological stations with a periodicity of three hours beginning at 0000 UTC/GMT (Greenwich Time). Using these measurements one can consider as the realization of a vector random sequence. As presented above algorithm (9), (10) was used for marginal (separate) forecasting of the air temperature at the meteorological station 26850 Minsk (algorithm (9), (10) with $n = 1$). The air temperature was assumed stationary in the covariance function during a certain period, but for the mathematical expectation it was allowed non-stationary. A visual depiction of the random sequences of the atmospheric temperature there is on the figure 1 for 4 years.

The parameters of the algorithm were calculated using a prehistory of not less than 9 years. The actual numbers of those years are provided in Table 1 in the last line for each year. For example, meteorological data for the previous 11 years was used to calculate the forecast for the period of January – April 2011.

In Mukha [9, 10] there are some details of obtaining estimates of the parameters of the prediction algorithm (9), (10).

The prehistory from the 112 values was used for forecasting, and the prediction was also performed at a length of 112 values (in (9), (10) $k_1 = 1$, $k_2 = s = 112$). We will mark such a forecast as 112/112 in values, or 336/336 in hours, or 14/14 in days.

The results of the forecasting were grouped by month. The results were included in the month when the forecast was made for days in that month. Table 1 shows the numbers of forecasts by month (top line of table for each year). For example, in January 2011 27 forecasts were performed, in February – 23 forecasts, *etc*. The last string in Table 1



(a) (b)

Figure 1. The graphs of the atmospheric temperature at the meteorological station 26850 Minsk in January and in June 2011–2014 years.

Table 1.

	January	February	March	April	May	June	July	August	September	October	November	December
2011	27	23	29	26	22	24	26	19	27	26	24	26
	11	11	11	11	10	10	9	10	10	10	10	10
2012	27	26	25	26	22	24	22	23	27	29	28	30
	12	12	12	12	11	11	10	11	11	11	11	11
2013	24	26	23	27	24	25	26	30	27	28	30	30
	13	13	13	13	12	12	11	12	12	12	12	12
2014	29	28	30	27	29	30	28	29	29	31	30	31
	14	14	14	14	13	13	12	13	13	13	13	13
Total	107	103	107	106	97	103	102	72	81	83	82	86

shows the total number of predictions, which were calculated in each month cumulatively over the four years. In total there were 1278 predictions made.

The retrospective forecasting made it possible to obtain the actual forecast error. For a fixed length of prediction times, a sample average module error (AME) of each statistical forecast was calculated:

$$s_{\mathrm{st},i} = \frac{1}{k}\sum_{j=1}^{k} |t_{\mathrm{for},j} - t_{\mathrm{ac},j}|, i = \overline{1,26}, \tag{27}$$

where $t_{\mathrm{for},j}$ – the predictive value of the atmospheric temperature at the time j, $t_{\mathrm{ac},j}$ – the actual value of the atmospheric temperature at the time j, k – the number of the forecasts of the bottom values in Table 1.

In order to compare with available forecasts from the Russian Hydrometeorological Center (Gismeteo, http://www.gismeteo.ru) the sample average module error was not calculated for all 112 values, but for day and night (maximum day temperature and minimum night temperature, respectively). For the all of the 26 forecasts, daytime and nighttime values of AME can be calculated as shown in the formula (27). According to the last formula the corresponding AME of the Gismeteo forecats were calculated (forecast Gismeteo is the numerical model forecast that they use, and according to its authors, one of the best predictive models). Calculated AME are presented as graphs in **Figure 2**, **Figure 3**. The graphs relate to the case when the average value of the predicted sequence (temperature) is considered as non-stationary in the given time period.

The graphs of AME in **Figure 2**, **Figure 3** are very irregular. For the simplification of the comparison, a cubic approximation using the least squares method was performed (LSE-approximation). The cubic LSE-approximation of the AME in **Figure 2**, **Figure 3** are continuous. The square marker in **Figure 2**, **Figure 3** are located at midday.

Figure 2 shows the AME of statistical, climatological and Gismeteo forecasts which was calculated for all 1278 forecasts in the years 2011–2014 (annual AME of the forecast). We see that in the short-term, the statistical forecast is more accurate compared with the climatological forecast and less accurate compared with the Gismeteo forecast. However, the difference between statistical and Gismeteo forecasts doesn't exceed 1.5 °C. This error is not an issue since home thermometers can have an error bar up to 2.0 °C. For the long-term, the statistical forecast are possibly more accurate compared with the forecasts Gismeteo.

Figure 3 shows the AME for individual months of the year (month's AME of the forecast). These results are more ambiguous. The statistical short-term forecast remains less accurate compared with the forecast Gismeteo for all months of the year. At the same time, for a number of months we observe greater accuracy for the long-term statistical forecasting, for example, in January, April, June, especially in August, September, November. The month's AME of the climatological forecast wasn't shown in fig. 3. It is approximately horizontal line as in **Figure 2**.

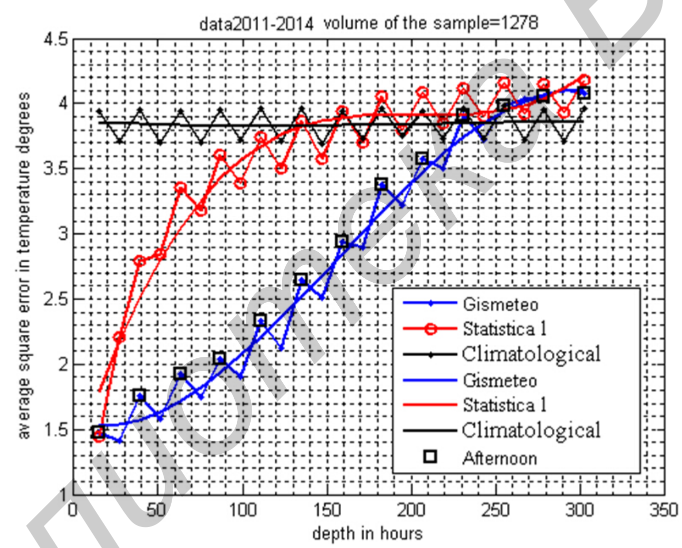
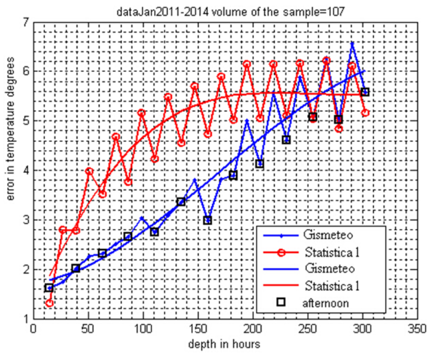
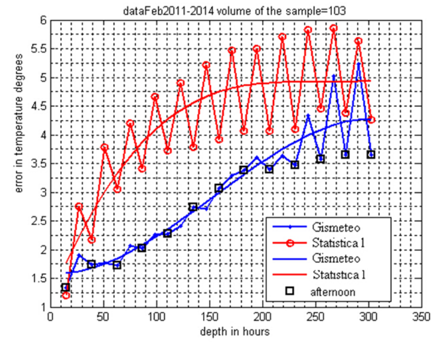


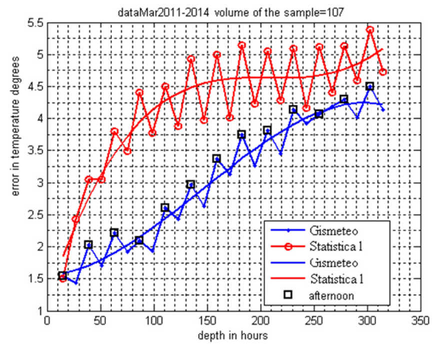
Figure 2. The graphs of annual sample average module error of the forecast of the temperature.


dataJan2011-2014 volume of the sample=107
error in temperature degrees
depth in hours
Gismeteo
Statistica 1
Gismeteo
Statistica 1
afternoon

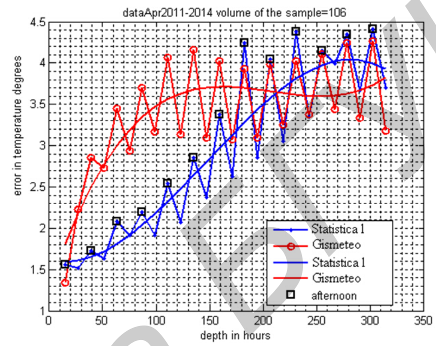
(a)


dataFeb2011-2014 volume of the sample=103
error in temperature degrees
depth in hours
Gismeteo
Statistica 1
Gismeteo
Statistica 1
afternoon

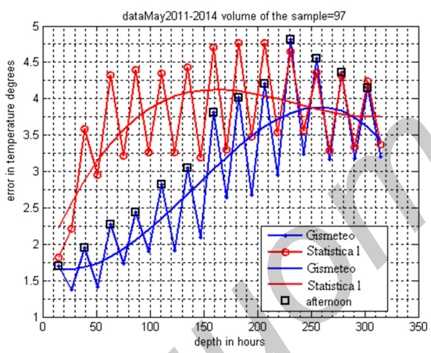
(b)


dataMar2011-2014 volume of the sample=107
error in temperature degrees
depth in hours
Gismeteo
Statistica 1
Gismeteo
Statistica 1
afternoon

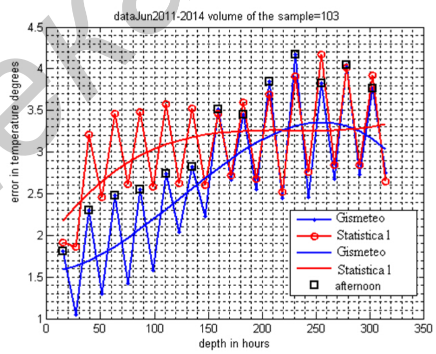
(c)


dataApr2011-2014 volume of the sample=106
error in temperature degrees
depth in hours
Statistica 1
Gismeteo
Statistica 1
Gismeteo
afternoon

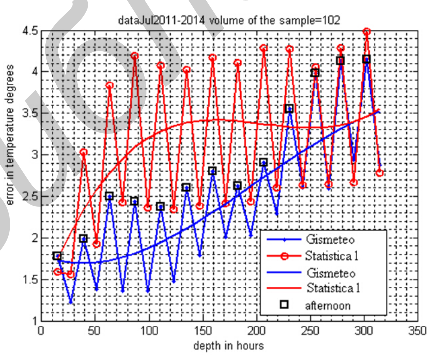
(d)


dataMay2011-2014 volume of the sample=97
error in temperature degrees
depth in hours
Gismeteo
Statistica 1
Gismeteo
Statistica 1
afternoon

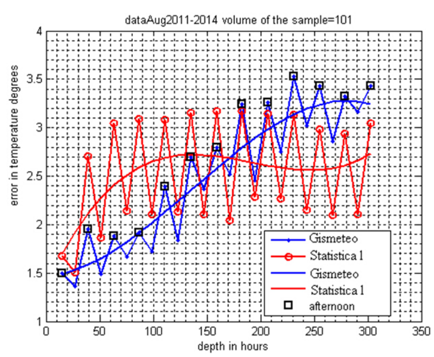
(e)


dataJun2011-2014 volume of the sample=103
error in temperature degrees
depth in hours
Gismeteo
Statistica 1
Gismeteo
Statistica 1
afternoon

(f)


dataJul2011-2014 volume of the sample=102
error in temperature degrees
depth in hours
Gismeteo
Statistica 1
Gismeteo
Statistica 1
afternoon

(g)


dataAug2011-2014 volume of the sample=101
error in temperature degrees
depth in hours
Gismeteo
Statistica 1
Gismeteo
Statistica 1
afternoon

(h)

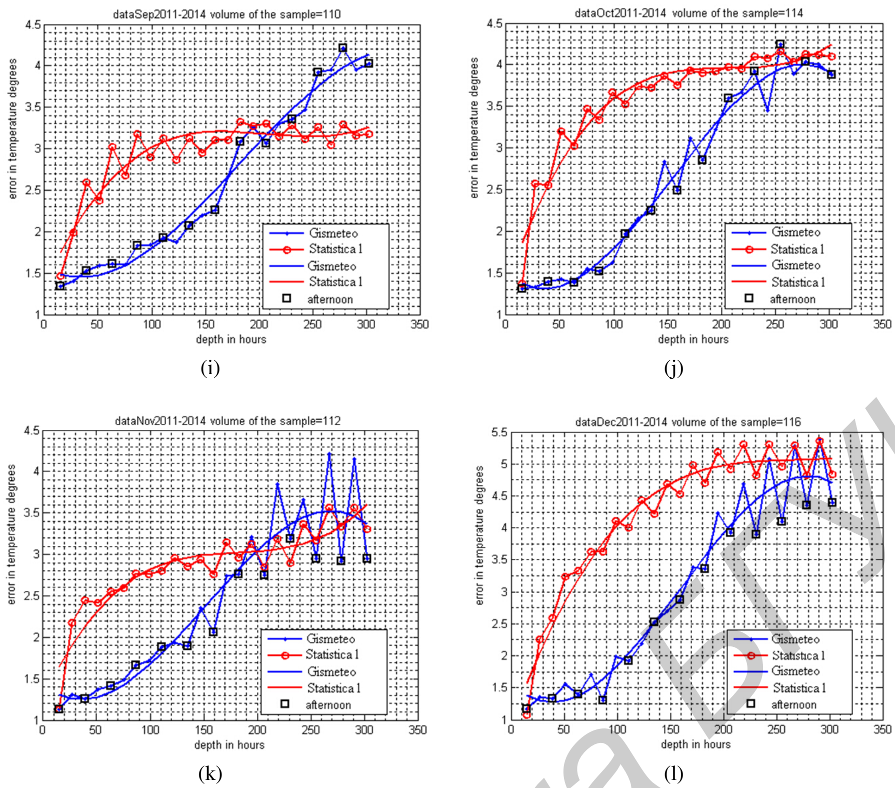


Figure 3. The graphs of month’s sample average module error of the forecast of the temperature.

4. SUMMARY AND CONCLUSIONS

Thus, the original algorithm of linear statistical extrapolation of vector random sequence on any set of moments in time has been proposed. This algorithm will be able used to forecasting in any knowledge domain. In this paper it has been used to forecasting of the atmospheric temperature. As a result the place of statistical forecast was determined between the numerical Gismeteo forecast and climatological forecast. In the short-term (less than one week), the statistical forecast is more accurate compared with the climatological forecast but on the whole a little less accurate compared with the numerical Gismeteo forecast. In the long-term (more than one week), both the statistical and the climatological forecasts are approximately equivalent, but the numerical Gismeteo forecast maybe less accurate.

Due to satisfactory accuracy, low cost and flexibility to the resources of the statistical forecast it can be recommended for use along with numerical forecast. Statistical forecasting can be implemented in local automated meteorological complexes and meteorological stations.

It makes sense to continue the study of the statistical approach in directions of accumulation of data for estimating the parameters of the algorithm, the calculation of the monthly forecast errors, long-term forecasting.

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