MATHEMATICAL DESCRIPTION OF PLANAR LINEAR STEPPING MOTOR

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The subject of the article is spatial two-coordinate linear stepping motors with air support. The paper presents the results of construction of the equations displaying physical processes, occuring in the appropriate coordinate stepping electric driver.

For complex motions with several degrees of freedom this principle of construction of coordinate systems allows to propose the concept of the construction of multi-coordinate drives. The basic idea of the concept consists in constructive integration of mobile parts of several coordinates in one execution multi-coordinate system. The integration of constructive elements assumes division of channels of management by a complex motion and modular fulfillment of active elements of the electromechanical coordinate device. Using this approach it is possible to replace mechanical constraints by electromagnetic, which are controlled by means of electronic management from digital computer. The increasing of functional opportunities of a drive can be achieved by means of number and combination of typical electromechanical modules, unification of control of a complex motion on all axes simultaneously.

Multi-coordinate drive based on electromechanical modules can realize rotary, linear and complex motions in Cartesian, cylindrical and spherical coordinate system without kinematic transformations; as result any required trajectories can be realized at microprocessor control with deep reduction and scaling of a motion.

Electromechanical multi-coordinate systems are intended for the realizing of complex motions on several axes simultaneously without mechanical elements of transformation of a motion by means of ample opportunities of planar linear stepping motors (PLSM) with air bearing. Such PLSM allows proposing the concept of the multi-coordinate electric drive which consists in constructive integration of mobile parts of several coordinates in one execution multi-coordinate system controlled by the digital computer.

The base module constructions of linear, rotary and planar types are primary elements for building of coordinate system. Combining of them into complex modules we can get a spatial motion system which realizes complex movements with given properties. In the paper we'll consider some types of positioning systems and construction of program motions.

The basic problem of mathematical modelling of multi-coordinate systems is a problem of construction of required program motions by definition of control actions. This problem in mathematical formulation is reduced to the choosing of parameters of differential equations of a motion or to the defining of a unknown parts of the differential equations using the condition of existence of the given particular solutions. In more general formulation the problem of construction of required program motions is reduced to the constructing of the differential equation system using a priori known properties of required motions, which are described by this system. Generally speaking, the solution of this problem is not unequivocal, and that allows constructing the required motions using additional conditions for electromechanical coordinate system.

Planar linear stepping motor (PLSM) with separated coordinates and air-magnetic bearings is shown on Fig. 1; it provides motions along ortogonal X and Y axes.

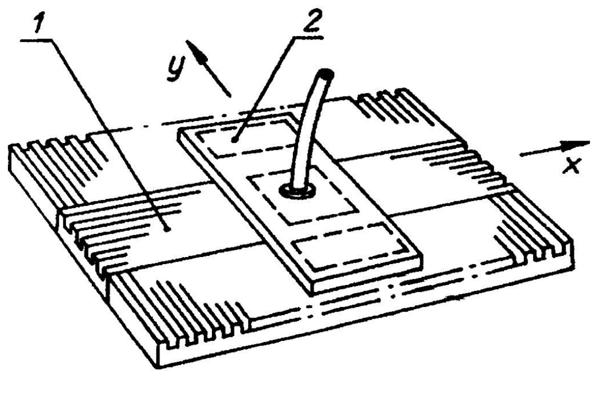

Рис. 1 – Planar linear stepping motor with separated coordinates: 1 – stator; 2 – inductor

The motionless stator 1 has three ortogonal to each other teeth zones. Inductor 2 consists of three groups of electromagnets (marked with dashed lines) which are separated in space and united by one frame. The inductor's teeth structures are formed on the block poles according to three teeth zones of stator. The gap between inductor and stator is made by pressed air transferred through capillary holes. Planar linear stepping motor (PLSM) with combined coordinates is shown on Fig. 2.

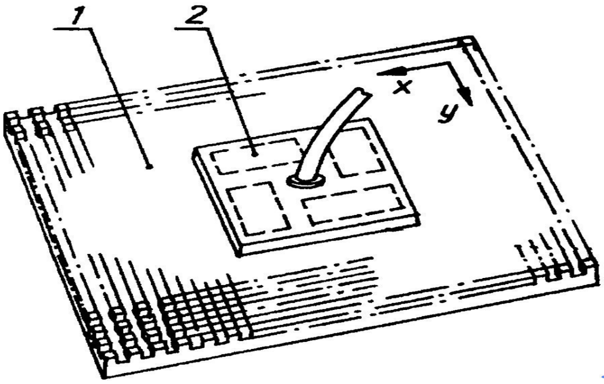

Рис. 2 – Planar linear stepping motor with combined coordinates: 1 – stator; 2 – inductor

To get the basic laws for electromechanical coordinate systems on a basis PLSM we can use various mathematical models, which describe stability, quality of motion, velocity and range of working speeds, and the dynamics of a system [1, 2]. The mathematical model of the two-coordinate stepping electric drive presented on Fig. 1 and Fig. 2 with biphase excitation and three degrees of freedom (ortogonal linear x, y and angular φ – rotation of inductor in a plane of a stator) can be written as following system [3, 4] (1), where:

- m – distributed mass (the load and the inductor);
- $\beta_x, \beta_y, \beta_\varphi$ – factors describing viscous friction;
- F_{cx}, F_{cy}, $F_{c\varphi}$ – resistance forces on x, y, φ accordingly;
- φ – coordinate describing angular displacement of inductor in the plane of a motion;
- $F(i_{1x}, i_{2x}, \varphi, t$, $F(i_{1y}, i_{2y}, \varphi, t$, $M(t)$ – the force characteristics of a drive accordingly on coordinates x, y, φ;
- r, i, ψ – resistance, current and interlinkage of phase windings correspondingly;
- $F_x(t)$, $F_y(t)$ – external mechanical influence on coordinates x and y;
- I – moment of inertia of distributed mass relatively to ortogonal axis to the plane of motion;
- U_{1x}, U_{2x}, U_{1y}, U_{2y} – laws of voltage in phase windings (functions of control action).

The systems with similar structure of the equations can be written for any multi-coordinate drive based on a PLSM [5]. The combined equations (1), which describes physical processes within stepping electric drive, is complete mathematical model of the considered device. The various representations of this model are used for mathematical research depending on the particular purpose. In our case for the researching of dynamics of coordinate system and for the constructing of program motions it is convenient to use the complete mathematical model (1) in the form:

$$x_i^{''} = f_i(x, x^{'}, t), i = 1, ..., n; \tag{2}$$

where $x(x_1, ...x_n)$ – vector of generalized coordinates of system; $x^{'}(x_1^{'}, ...x_n^{'})$ – vector of generalized velocities of system.

Using the elementary modules realizing one-coordinate and two-coordinate motions, flexible industrial systems can be created; their motion systems are the base of robotics for technological equipment and allow realizing any motions in 3D space.

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$$\begin{aligned}
&m\frac{d^2x}{dt^2} + \beta_x\frac{dx}{dt} + F_{cx} = F(i_{1x}, i_{2x}, \varphi, t) + F_x(t);\\
&m\frac{d^2y}{dt^2} + \beta_y\frac{dx}{dt} + F_{cy} = F(i_{1y}, i_{2y}, \varphi, t) + F_y(t);\\
&I\frac{d^2\varphi}{dt^2} + \beta_\varphi\frac{d\varphi}{dt} + F_{c\varphi} = M(t);\\
&r_{1x}i_{1x} + \frac{d\psi_{1x}}{dt} = U_{1x}(x, \varphi, t); r_{2x}i_{2x} + \frac{d\psi_{2x}}{dt} = U_{2x}(x, \varphi, t);\\
&r_{1y}i_{1y} + \frac{d\psi_{1y}}{dt} = U_{1y}(y, \varphi, t); r_{2y}i_{2y} + \frac{d\psi_{2y}}{dt} = U_{2y}(y, \varphi, t).
\end{aligned} \tag{1}$$