TRIPLET EFFECT IN SUPERCONDUCTOR/FERROMAGNET MULTILAYERS. MATRIX CALCULATIONS

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It is known that in the layered superconductor(S)/ferromagnet(F) structures with a given direction of the F-layers magnetizations, the superconducting condensate includes the triplet pairs (besides the singlet ones), which are characterized by zero projection of the full spin, the spatial even, and odd in the imaginary time parities [1]. In the case of non-collinear magnetizations, the "long-range" triplet components with projection ±1 of the spin have generated in addition [1]. The triplet effect has been studied in details on the characteristics of superconductivity of the elemental, $F_0/S/F$ and $S/F_0/F$, structures, as well as on the phase diagrams of 0- and π-states of bilayers. In the general case, the problem on the manifestations of the triplet components in spectra of superconducting states, that characterize S/F multilayers [2], evidently arises. In order to solve this problem, the extension of the method, elaborated for the calculation the characteristics of critical states of $F_0/S_1/F_1/\ldots/S_N/F_N$ structures [2], is given in the present work for arbitrary in-plane magnetizations $\mathbf{M}_j = E_{ex}\,\mathbf{m}_j$ ($j = 0, 1,\ldots, N$; E_{ex} is the exchange energy and $\mathbf{m}_j$ the unit vector).

In the diffusive limit, the critical state of superconductivity of the S/F structure is described by the system of linearized Usadel equations for the singlet, $\Phi_{s,\omega}(x)$, and triplet, $\mathbf{\Phi}_{t,\omega}(x)$, anomalous Green functions [1, 3]:

$$\begin{cases}(-D(x)\partial_x^2 + 2|\omega|)\Phi_{s,\omega}(x) + 2iE_{ex}(\mathbf{m}(x),\mathbf{\Phi}_{t,\omega}(x)) = 4\pi T\,\widetilde{\lambda}(x)\sum\limits_{\omega>0}\Phi_{s,\omega}(x)\\ (-D(x)\partial_x^2 + 2|\omega|)\mathbf{\Phi}_{t,\omega}(x) + 2iE_{ex}\mathbf{m}(x)\Phi_{s,\omega}(x) = \mathbf{0}\end{cases}. \tag{1}$$

Here, the OX axis of the coordinate system is taken orthogonal to the layers, and directions are set relative to OZ axis so that the vectors $\mathbf{m}_j$ have the coordinate form ($0\ \sin\theta_j\ \cos\theta_j$); the vector-function $\mathbf{m}(x)$ is $\mathbf{m}_j$ for F_j-layer and $\mathbf{0}$ for S-layers. Further, the step functions $D(x)$, $\widetilde{\lambda}(x)$ take the values D_S, λ for S-layers and D_F, 0 for F-layers, respectively, where D_S, D_F are the diffusion coefficients, λ is the effective electron-electron interaction constant; $\omega \equiv \omega_n = \pi T(2n + 1)$ are the Matsubara frequencies ($n = 0, 1, 2,\ldots$); the system of units with $\eta = k_B = 1$ is used.

The system (1) has supplemented by Kupriyanov – Lukichev matching conditions at the S-F interfaces [5], and by the boundary ones at the surfaces $x = 0$ and $x = L$:

$$\partial_x\Phi_{s,\omega}(0) = \partial_x\Phi_{s,\omega}(L) = 0, \qquad \partial_x\mathbf{\Phi}_{t,\omega}(0) = \partial_x\mathbf{\Phi}_{t,\omega}(L) = 0. \tag{2}$$

The solution of the set of equations (1), as well as in the case considered earlier [2], can be get in the explicit matrix form: $\mathbf{Y}(x) = \mathbf{R}(x)\,\mathbf{Y}_0$. Here $\mathbf{Y}(x)$ is a vector-function of the superconducting state defined by the formula $\mathbf{Y}(x) = \mathbf{\Phi}_s \oplus \mathbf{\Phi}_s' \oplus \mathbf{\Phi}_{t3} \oplus \mathbf{\Phi}_{t3}' \oplus \mathbf{\Phi}_{t2} \oplus \mathbf{\Phi}_{t2}'$, $\mathbf{Y}_0 = \mathbf{Y}(0)$, and $\mathbf{R}(x)$ the matrizant of the system (1). The column vectors $\mathbf{\Phi}_s(x)$, $\mathbf{\Phi}_{t2}(x)$, $\mathbf{\Phi}_{t3}(x)$ are constituted by the respective functions $\Phi_{s,\omega}(x)$, $\Phi_{t2,\omega}(x)$, $\Phi_{t3,\omega}(x)$ ($\Phi_{t1,\omega}(x) = 0$ in view of $m_1(x) = 0$). Matrizant $\mathbf{R}(x)$ can be expressed by means of recurrence relations: $\mathbf{R}(x) = \mathbf{S}(x - x_i)\,\mathbf{P}_{SF}\,\mathbf{R}(x_i)$ if the plane $x = x_i$ is the left boundary of the S-layer, and $\mathbf{R}(x) = \mathbf{M}(\theta_i; x - x_i)\,\mathbf{P}_{FS}\,\mathbf{R}(x_i)$ in the opposite case. Here, $\mathbf{S}(x)$ and $\mathbf{M}(\theta_j; x)$ are the matrizants of the system (1) defined in the range of S- and F-layers, respectively,

$P_{FS}$and P_{SF} are the matrices of the contact conditions. The matrices $\mathbf{S}(x)$ and$\mathbf{P}_{FS}$, $\mathbf{P}_{SF}$ are expressed by the block-diagonal forms: $\mathbf{S}(x) = \mathrm{diag}[\mathbf{S}^+, \mathbf{S}^-, \mathbf{S}^-]$, $P_{FS(SF)} = \mathrm{diag}[P_{FS(SF)}, P_{FS(SF)}, P_{FS(SF)}]$, where the matrices $\mathbf{S}^+$, $\mathbf{S}^-$, P_{FS}, P_{SF} are defined in ref. 2. Further, it is easy to prove the formula $\mathbf{M}(\theta_j; x) = \mathbf{U}(-\theta_j)\mathbf{M}(x)\mathbf{U}(\theta_j)$, where

$$\mathbf{M}(x) = \begin{pmatrix} \mathrm{Re}(\mathcal{M}(x)) & i\,\mathrm{Im}(\mathcal{M}(x)) & \mathbf{0} \\ i\,\mathrm{Im}(\mathcal{M}(x)) & \mathrm{Re}(\mathcal{M}(x)) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathcal{N}(x) \end{pmatrix} \qquad (3)$$

and the rotation matrix

$$\mathbf{U}(\theta_j) = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1}\cdot\cos\theta_j & \mathbf{1}\cdot\sin\theta_j \\ \mathbf{0} & -\mathbf{1}\cdot\sin\theta_j & \mathbf{1}\cdot\cos\theta_j \end{pmatrix}. \qquad (4)$$

In the expression (3), the matrix $\mathrm{M}(x)$ corresponds to the singlet, $\mathbf{\Phi}_s$, and triplet, $\mathbf{\Phi}_{t3}$, components of the state function and has defined in ref. 2; the matrix $\mathbf{N}(\mathrm{x})$ is given by

$$\mathcal{N}(x) = \begin{pmatrix} \mathrm{diag}\left[\mathrm{ch}\left(\dfrac{x\sqrt{t(2n+1)}}{\xi_F}\right)\right] & \mathrm{diag}\left[\dfrac{\xi_F}{\sqrt{t(2n+1)}}\mathrm{sh}\left(\dfrac{x\sqrt{t(2n+1)}}{\xi_F}\right)\right] \\ \mathrm{diag}\left[\dfrac{\sqrt{t(2n+1)}}{\xi_F}\mathrm{sh}\left(\dfrac{x\sqrt{t(2n+1)}}{\xi_F}\right)\right] & \mathrm{diag}\left[\mathrm{ch}\left(\dfrac{x\sqrt{t(2n+1)}}{\xi_F}\right)\right] \end{pmatrix}. \qquad (5)$$

Here, $\xi_F = \sqrt{D_F / 2\pi T_S}$ with T_S is the bulk critical temperature of the superconductor (note that the characteristic length ξ_F does not depend on the exchange energy).

The substitution of the found solution $\mathbf{Y}(x)$ to the boundary conditions (2) leads to the system of linear uniform algebraic equations. Then, solving the characteristic equation of this system, we get the critical temperature, T_c, and, next, the eigenvector-function of the critical state.

As an example, let us consider the symmetrical $F_0/S_1/F_1 \ldots S_5/F_5$ structure with material parameters quoted in ref. 2 and with the F- and S-layers thicknesses $d_{Fj} = d_{S3} = 0.5\xi_S$, $d_{S1} = d_{S2} = d_{S4} = d_{S5} = 4.7\,\xi_S$, where ξ_S is the coherence length in the S-material. Fig. 1 shows two characteristics $T_c(\theta)$ (θ is the angle between $\mathbf{m}_1$ and $\mathbf{m}_0$ vectors), which calculated for two of various transitions from the ferromagnetic ($\mathbf{m}_5 = \ldots = \mathbf{m}_0$) to the antiferromagnetic ($\mathbf{m}_j = -\mathbf{m}_{j-1}$) orderings of the magnetizations. One of transitions is realized by the synchronous rotation of the odd magnetic moments, $\mathbf{m}_1 = \mathbf{m}_3 = \mathbf{m}_5 = \mathbf{m}(\theta)$ ($\mathbf{m}_4 = \mathbf{m}_2 = \mathbf{m}_0$), and other consists of the spiral motion of magnetizations, $\mathbf{m}_k(\theta) = \mathbf{m}(k\theta)$ (k = 0,…,5). As one can see from Fig. 1, both characteristics consist of two branches, $T^{(1)}(\theta)$ and $T^{(0)}(\theta)$, with crossover points, $\theta_{cr} \sim 80°$. The left and right branches correspond to the antisymmetric, with one node, and symmetric, without nodes, singlet components, respectively. The variation of the singlet component in the range of central S-layer with increasing of the angle θ from 0 to π has shown in Fig. 2a. As one can see from Fig. 2a, the deformations of the function $\Phi_s(x)$, as well as the decreasing of the value T_c, are small at $\theta \le \theta_{cr}$. Moreover, the singlet component has almost zero values in S_3-layer. At the crossover point, the function $\Phi_s(x)$ changes the symmetry, and Cooper's pairs fill the layer S_3. The appreciable growth of the singlet component at $\theta>\theta_{cr}$ occurs together with the increasing of the critical temperature. Meanwhile, the response of the triplet components to variation θ in the neighborhood of θ_{cr} is not radical. It is one of causes of the unusual effect when $\theta\to\theta_{cr}-0$: the almost full disappearing of the singlet and the sizeable suppression of Φ_{t3} components (see Fig. 2b). This means, that a thin S-layer can be the channel for superfluid electron pairs with ±1 projection of the spin.

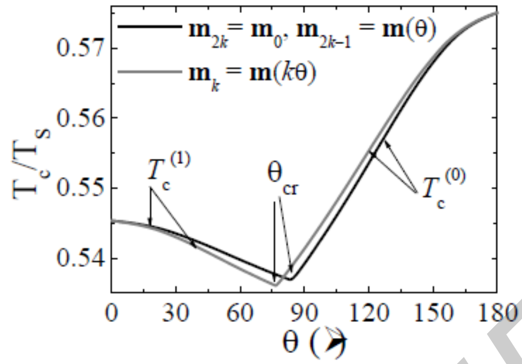

Fig. 1. The critical temperature of the structure $F_0/S_1/F_1/.../S_5/F_5$ versus the angle of "synchronous" (black line) and "spiral" (grey line) rotations of the magnetic F-layer moments

Fig. 2. The critical state functions in the range of the central S-layer of the structure $F_0/S_1/F_1/.../S_5/F_5$: (a) singlet components for the magnetic state $\mathbf{m}_{2k}=\mathbf{m}_0$, $\mathbf{m}_{2k-1}=\mathbf{m}(\theta)$; (b) singlet and triplet components for the magnetic state $\mathbf{m}_k=\mathbf{m}(k\theta_{cr})$

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