# RF MEMS OSCILLATOR DESIGN FOR INTEGRATION WITH SOC USING ORDER REDUCTION METHODS 

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#### Abstract

The recurring tendency of system-on-chip (SoC) to integrate analog or mixed-signal components, and soon micro electromechanical systems (MEMS), on the same chip with digital systems implies the need for new modeling and co-simulation techniques. This is because the current modeling techniques consume an enormous amount of resources: numerical computation, time and designer's effort. We suggest implementing behavioral modeling of MEMS devices and applying methods of order-reduction (MOR) to these models to save time and effort in both modeling stage and simulation. In this paper, we implemented RF MEMS oscillators behavioral models (mechanical, electrical, and mathematical) and utilized singular values decomposition (SVD) and orthogonaltriangular decomposition (QRD) methods to these models for the purpose of order reduction and rank approximation of the input system. This resulted in improving the simulation time and system resources. Nevertheless, an IP softcore was recieved in VHDL code for each of these devices to be co-simulated in SOC platform.


## Introduction

The integration of MEMS as blocks in system-on-chip poses many new challenges to design engineers. SoC already embed typical sub-systems such as DSP, RAM, MPEG cores, etc. and they may soon include MEMS [1]. The absence of proper MEMS design methodology for integration in SoC is a major problem. Because of the physical nature of MEMS devices, their behavioral models consist of a huge number of mathematical partial differential equations (PDE), which consumes long time and computation. However, any slight change in the design scheme, which is inevitable in the design process, leads to changes in the behavioral model, and consequently, to more time and effort.

MEMS oscillators constructed on the basis of MEMS resonators are widely used nowadays in computing and communication systems, which started to appear in the market in the past decade as low-cost moderate performance alternatives in many applications including communication, military, industrial, and consumer applications.

In this paper, we introduce an implementation of MEMS oscillators behavioral models (including mechanical, electrical, and mathematical modeling), and apply singular values decomposition (SVD) and orthogonal triangular decomposition (QRD) to these models as methods of orderreduction (MOR), for the purpose of speeding up simulation using a more reduced-order model (ROM) which represents the MEMS device behavior with small loss in precision.

Currently, such methods have many different applications ranging from image processing and compression, molecular dynamics, small angle scattering, information retrieval, to electric circuit analysis and others. So, we decided to implement them to MEMS, since they are suitable for overdetermined systems (as mentioned in section 3).

## I. MEMS Device Modeling

Modeling of MEMS devices progressed from methods which depend on finite element and boundary element analysis to joint modeling, as in the case of MEMS + tool from CoventorWare, which offers the MEMS model as a black box presentation into MATLAB for further processing, simulation, and optimization. In our case, we performed modeling of these devices depending on their behavior in MATLAB/Simulink. We used Simscape and SimElectronics libraries in Simulink.

The mechanical model of a MEMS oscillator device are represented by 2 identical enforced mass-spring-damper systems coupled by a spring, while the mathematical model is represented by the equation: $F=M \cdot x^{\prime \prime}+B \cdot x^{\prime}+K \cdot x$ where F is the applied force, M is a mass moving relative to the force, B is the damping constant, and the spring constant $K=k+2 . k_{c}$ is the sum of the system springs and the coupling. The electrical model is an RLC equivalent series circuit of the mechanical one with $\mathrm{F}=\mathrm{V}$ (Voltage), $\mathrm{M}=\mathrm{L}, \mathrm{B}=\mathrm{R}, \mathrm{K}=$ $1 / \mathrm{C}$, and $\mathrm{x}^{\prime}($ velocity $)=\mathrm{i}$ (current). The output is a resonating movement with resonance frequency $f_{0}=\frac{1}{2 \pi} \sqrt{\frac{K}{M}}=\frac{1}{2 \pi} \sqrt{\frac{k+2 \cdot k_{c}}{M}}$.

## II. Simulation

Each of the above mentioned models for the device (which are of type SISO single-input single-output) is simulated using 2 sets of input signals: Gaussian(arbitrary) and sinusoidal. Each simulation is 100 sec and was accomplished on a Dell computer with Intel Core 2 Duo T9300 2.5 GHz processor, and 3 GB memory, on an operating system 32-bit Windows 7. After each simulation process, we construct the input matrix $\mathrm{A}(m \times n)$ of order r where $\mathrm{m}=5000$ and $\mathrm{n}=10$ and $\mathrm{r}<=$ n. A consists of multiple input vectors $a_{i}$, where $\mathrm{i}=1$ to n . Each vector $a_{i}$ consists of m input force values with constant values for $B_{i}$ and $K_{i}$ throughout the vector. Each $a_{i}$ is constructed for
the following values of B and K respectively: $\mathrm{B}=$ $0.1 ; 0.5 ; 1 ; 1.5 ; 2 ; 2.5 ; 3 ; 3.5 ; 4 ; 4.5 \mathrm{~K}=10 ; 50 ; 100$; 150; 200; 250; 300; 350; 400; 450.

In the following section, we apply orderreduction methods on matrix A in order to get a reduced-order matrix. Next, we simulate the models with both the original input matrix and the reduced one.

## III. Order-Reduction Methods

We choose to apply SVD and QRD methods on all models of the oscillator, since they are suitable for solving overdetermined systems of linear equations: $A \times x=b$, just like in the case of MEMS, where A is the input $(m \times n)$ matrix and $\mathrm{m}>\mathrm{n}$; i.e. with more equations than unknowns, and b is the output matrix of dimension $(m \times n)$. This is due to the need to repeatedly take measurements in order to minimize errors. For further information about these methods, we refer the reader to references [2, 3].

Based on SVD, A is decomposed into U, S, and V. In MATLAB, we use: $[\mathrm{U}, \mathrm{S}, \mathrm{V}]=\operatorname{svd}(\mathrm{A})$ where U and V are left and right singular vectors, and S is diagonal singular values (importance coeffcients), and x can be found by: $x=A^{-1} \times b$, where $A^{-1}=$ $V \times S^{-1} \times U^{T}$. The reduced matrix A of order $\mathrm{k}<$ r is: $A_{k}=U(: ; 1: k) \times S(1: k ; 1: k) \times V^{t}(1: k ;:)$

Also, QRD was applied: $[\mathrm{Q}, \mathrm{R}]=\mathrm{qr}(\mathrm{A})$ where $A=Q \times R$, and $x=Q^{T} \times R^{-1} \times b$, and the reduced model is $A=Q(:, 1: k) \times R(1: k,:)$. In every simulation, the values of x were checked from all methods and they were always equal. Moreover, the reduced matrices were calculated for all orders k from 1 to r .

## IV. Simulation with reduced models

After implementing MOR, each model is resimulated but this time with the reduced input matrix A of rank $\mathrm{k}<\mathrm{r}$, where $\mathrm{k}=1$ to r (usually it's supposed to simulate for the optimal order, but for comparison reasons, we decided to simulate for each order k from 1 to r ), and the new simulation time is calculated for each process.

In our system, the order $r$ of the original system was always 10 , i.e $\mathrm{r}=\mathrm{n}=10$. In the case of MOR usage, the optimal order k was equal to 1 .

We define the error as the percentage of the difference between the outputs of the reduced model $b_{k}$ of order k and the original model $b_{o}$ of order r , divided by output of the original one: er $=$ $\frac{\left(b_{k}-b_{o}\right)}{b_{o}} \times 100$.

Since we have 3 models with 2 types of inputs, then simulation was repeated 6 times with each MOR, SVD and QRD. In Fig. 1 and Fig. 2 the results of the simulation processes were illustrated. In Fig.1, the simulation time decrease is shown from the original system and reduced models from SVD and QRD implementation. Same for Fig.2, the error
between these reduced models in percentage to the original is shown.


Рис. 1 - Simulation time decrease in all experiments


Рис. 2 - Error \% in reduced models

## V. Conclusions

In this paper, MEMS oscillator is modeled in Simulink using its mechanical, electrical and mathematical behavioral models. In order to improve simulation, and decrease time, SVD and QR methods were applied. Also, optimal reduced orders were defined for the input sets. In Fig.1, simulation time was decreased for MEMS in the range of 2.5 to $50 \%$ of the simulation time of the original system, while keeping minimized error which ranged from 0 to $10 \%$. As for memory resources, decrease of memory usage was by $90 \%$, from 400,000 bytes to 40,000 bytes. However, it was also noticed that using the mathematical model was the most appropriate for modeling the behavior of MEMS. In the end, an IP-core was recieved using Matlab HDL coder, which will be used in the cosimulation with SOC in ModelSim from Xilinx in the coming work.

## VI. Reference List

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