



#### **IV. CONCLUSIONS**

Within the framework of the work, topology optimization program module and integrated computeraided design and engineering system were developed. Topology optimization of bracket and parametric design optimization of space waffle shell were carried out through ISCDE. The obtained results will be used in Russian space companies.

## ACKNOWLEGMENTS

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# MODELING OF HEAT TRANSFER IN BUILT-UP CURVILINEAR PLATE

I. Savel'eva, A. Zhuravskii Bauman Moscow State Technical University, Moscow, Russia

#### **I. INTRODUCTION**

One of the important direction of modern industry is creation of new materials based on additive technologies, i.e. technologies of buildup of various solid bodies. Chemical vapor deposition (CVD) is one of such technologies that represents the deposition of a film or a coating, i.e., a continuous layer, including a nanocrystalline material, on a cooled plate [1].

The current work considers buildup of the most general case: that of a curvilinear surface and includes special for this process boundary conditions. This work also shows modification that allows to consider diffusive transfer of material and linear change of curvature along the plate thickness. In the work, a numerical algorithm for finding the temperature profile at any instant of time has been constructed, and results of numerical calculation for different materials have been given.

#### **II. MATHEMATICAL MODEL**



Figure 1 – Curvilinear plate

In Fig. 1 curvilinear plate of thickness H is shown on whose surface (with curvature  $\kappa 0$ ) we have the chemical vapor deposition of material with a constant rate v. Let us adopt the assumption of constant temperatures of the gas and the cooling medium Tg and Tm. Then the temperature distribution in the curvilinear surface where the Ox axis is directed normally to the surface is described by the equation [7]

$$c^{(k)}\rho^{(k)}\frac{\partial T}{\partial t} = -\frac{\partial q^{(k)}}{\partial x} - 2\kappa(x)q^{(k)}$$
(1)

In case of the basic model we can assume mean curvature  $\kappa(x)$  and heat flux in the plate as a:

$$\kappa(\mathbf{x}) \approx \kappa(0) = \kappa_0 = (1/R_1 + 1/R_2)/2$$
 (2)

$$q^{(k1)} = -\lambda^{(k1)} \nabla T \tag{3}$$

Using (2), (3) and (4) we have equation of heat conduction in a curvilinear orthogonal coordinate system for plate and deposited material[8]:

$$c^{(k)}\rho^{(k)}\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda^{(k)}\frac{\partial T}{\partial x}\right) + 2\kappa_0 \left(\lambda^k\frac{\partial T}{\partial x}\right)$$
(4)

For equations (4) the ideal thermal contact condition will be of the following form:

$$\begin{cases} T(t, 0-0) = T(t, 0+0) \\ \lambda^{(1)} \frac{\partial T}{\partial x}|_{x=0-0} = \lambda^2 \frac{\partial T}{\partial x}|_{x=0+0} \end{cases}$$
(5)

The boundary conditions:

$$\begin{cases} \lambda^{(1)} \frac{\partial T}{\partial x}|_{x=-H} = \alpha_m (T(t, -H) - T_m); \\ \lambda^{(2)} \frac{\partial T(x)}{\partial x}|_{x=vt} = \alpha_g \left( T_g - T(t, vt) \right) - \\ -\varepsilon \sigma_0 T^4(t, vt) + Aq_1^g + c^{(2)} \rho^{(2)} v \left( T_g - T(t, vt) \right) + \rho^{(2)} v L^{(2)}. \end{cases}$$
(6)

To solve a heat-conduction equation (4) with boundary conditions (5,6), we will use the integrointerpolation method [9–12].

#### **III. CALCULATION RESULTS AND POSSIBLE MODIFICATIONS**

Let us analyse results by considering the problem of deposition of titanium nitride on a steel plate. We make calculations for different value of curvature  $\kappa 0$  and different speed of deposition v.



Figure 2 – Calculation of the distribution of the temperature field across the plate thickness at different mean curvatures of the plate  $\kappa_0 = 1,0,-1,m-1$  at the instant of time  $t = 2.5 \cdot 10^4$  s. The material of the plate is steel, the material of the coating is titanium nitride



Figure 3 – Calculation of the distribution of the temperature field across the plate thickness at different deposition rates  $v = 10^{-6}$ ,  $10^{-7}$ ,  $10^{-8}$ , m/s at the instants of time  $t = 2.5 \cdot 10^{3}$ ,  $2.5 \cdot 10^{4}$ ,  $2.5 \cdot 10^{5}$  s. The material of the plate is steel, the material of the coating is titanium nitride.

In Fig.2 we can see dependence of the temperature profile on the mean curvature  $\kappa_0$ . In the plate of bigger positive mean curvature the temperature in each section is higher and for smaller negative mean curvature the temperature in each section is lower. Note that for large values of mean curvature the onedimensional equation (1) is incorrect. The rate of chemical vapor deposition also exerts a substantial influence on the temperature field in the plate (Fig. 3): growth in the sputtering rate produces a substantial increase in the temperature.

Let us consider a modification of mathematical model that includes diffusive transfer of gas particles in plate. If we accept a hypothesis of intensive heat transfer between plate and particles, we can find the heat flux in the plate [14] as

and equation (4) can be transformed into system

\_\_\_\_\_ (8)

Addition of boundary condition, contact condition [15] and initial distribution to these equations allows us to include diffusive heat transfer in mathematical model.

The other way of model modification is considering of linear change of curvature along the plate thickness:

$$\kappa(x) = \left(\frac{1}{R_1 + x} + \frac{1}{R_2 + x}\right)/2 \approx \kappa_0 - x \cdot \kappa_x \tag{9}$$

Using formula (16) (except (2)) in equation (1) we can consider the plates at different mean and Gauss curvatures.

## **VI. NOTATION**

c(k), specific mass heat,  $J/(kg \cdot K)$ ;

 $\rho(k)$ , density, kg/m3;

 $\lambda(k)$ , thermal-conductivity coefficient, W/(m·K);

 $\kappa$ 0, mean curvature of the plate cross section, m-1;

H, thickness of curvilinear plate, m;

p1, p2, coefficients of local and nonlocal part of heat transfer;

a, characteristic size of influence zone, m;

A,ε, coefficient of absorption of the radiation and emissivity;

L(2), specific heat of the phase transition, J/kg;

 $q_1^g$ , density of the radiant flux of the gas, W/m2;

Tm, Tg, temperature of the cooling medium and gas, K;

 $\alpha$ m,  $\alpha$ g, coefficients of convective heat transfer, W/(m2·K);

Q(k) – concentration of particles, kg/m3;

D(k) – diffusion coefficient, m2/s;

 $\kappa_x = (2\kappa_0^2 - K^G)$  – negative linear part of  $\kappa(x)$  function, m-2;

KG – Gauss curvature, m-2.

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