Ontology of Learning Problems in Boolean Algebra

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***Abstract*—The article describes an ontology of learning problems in course “Introduction to Boolean Algebra”. Nowadays the issue of educational problems classification in computer science is not studied enough. That is why the ontology based on new classification of learning problems. The classification includes Tollinger’s taxonomy (cognitive complexity) and structural complexity of the problems (number of input parameters and computational complexity). Depending on the values of these two complexities every learning problem belongs to one of three levels of complexity (easy, medium, difficult). Moreover, all the problems divided into four categories depending on their purpose. The ontology was developed with Protégé 4.3 and OWL DL. It includes over 50 classes and about 60 individual entities. The developed ontology will be useful for teachers to solve the issue of selecting learning problems.**

***Keywords*—ontology, learning problems, complexity, classification, taxonomy, computer science, informatics, Boolean algebra**

I. Introduction

There is an active process of collecting, analyzing and formalizing knowledge for their subsequent representation in the form of ontologies nowadays. Ontology is a hierarchy of concepts of the subject domain, relationships and rules between them. First ontologies were created in 1980s and nowadays a large number either of common ontologies or ontologies of specific areas have already been developed [1, 2, 3, 4, 5]. At the same time, it should be noted that this process is far from completion, because existing ontologies cover only a small part of knowledge areas.

In this article, the area of learning problems for the course “Introduction to the Boolean Algebra” is selected as the specific area for ontology. This choice can be explained by the growing interest in computer science lessons at school. The teacher must carefully select the material for lessons in order to get positive results in education. The ontology is developed for teachers teaching Boolean Algebra to help them to select learning problems more effective. This issue is faced by most teachers because it can be very difficult to find problems for students which would help them to learn the course. A well-chosen set of different tasks can help teachers achieve pedagogical goals such as explaining students how correctly interpret the condition of the problem, break it down into simple components, solve the problem in an effective way, etc.

At the same time, the course “Introduction to the Boolean Algebra” is important part of school informatics. The course “Introduction to the Boolean Algebra” is developed for students of tenth form. It allows students to obtain base knowledge of the Boolean algebra and it causes the development of logical thinking. As practice shows, the uptake of this course by students is difficult due to natural reasons as ambiguous interpretation of some language constructs - “or” in natural language can match either logical “or” or logical “excluding or”, the use of three-valued logic in life as opposed to two-valued logic in Boolean algebra. At the same time during problems solving students can work out all difficult moments and learn how they can use the material for practical purposes. Therefore a well-chosen set of problems is an important part of this course.

II. Domain analysis

From pedagogical point of view, all learning problems of this course can be divided into four categories according to their purpose:

- methodically important problems that must be solved on the blackboard by the teacher during the lesson;
- problems for independent work in the classroom, which help to work out solution algorithms during the lesson and don’t require checking by the teacher;
- homework problems, which are the most suitable for consolidating the material;
- test problems that allow teacher to check if students get the material well.

Nowadays various classifications of learning problems exist but they are not very suitable for such lessons as informatics. The most suitable for this area Tollinger’s taxonomy was taken [6, 7]. It allows to divide the problems into 5 categories according to the increase of their cognitive complexity and operational value.

Moreover, the specificity of the problem area of learning problems for the course “Introduction to Boolean Algebra” makes this classification of problems not suitable for their division, because it does not take into account the computational complexity of the problems. For example, two problems of the same category in Tollinger’s taxonomy with different number of input parameters, should be assigned to different complexity categories, since they require different time to solve them.

To solve the issue, a new classification of learning problems that takes into account both the cognitive complexity of the Tollinger’s taxonomy and the computational complexity, was developed. New classification of problems was tested in Advanced Educational Scientific Center (faculty) – Kolmogorov's boarding school of Moscow State University and described in the work "Classification of Learning Problems" in course "Introduction to Boolean algebra" Based on Tollinger's taxonomy" [8]. The results of the work were also published in the article in "Informatics in School" [9]. This classification was taken as a basis for creating ontology of learning problems. Development and implementation of the ontology is carried out as N.Bulgakova's master dissertation at the Faculty of Computational Mathematics and Cybernetics of Lomonosov Moscow State University.

During the domain analysis, a set of 60 problems was selected from different sources [10, 11, 12]. This set can be divided into two groups: formalization of the expressions and logical problems. To solve the problems of the first group, students must analyze the text written in natural language and write it in the form of a logical formula, possibly with a further transformation of the resulting formula into a shorter one. To do logical problems students must transform the problem into logical formula and do additional actions with received formula to answer the question posed in the problem.

All problems were classified according to their purpose and complexity according with developed classification. In order to make the ontology easier to extend and supplement with new problems, a formula was chosen. It based on two input parameters - cognitive complexity in Tollinger's taxonomy (K) and computational complexity (C). The cognitive complexity parameter is assigned the category number in Tollinger's taxonomy. The computational complexity depends on number of input parameters that must be taken into account during problem solving. The value of this parameter is expressed by a number, based on the following principles:

- if there are two structural elements in learning problem, then $C = 1$;
- if there are 3 or 4 elements, then $C = 2$;
- if there are 5 or 6 elements, then $C = 3$;
- if there are 7 or 8 elements, then $C = 4$;
- if there are more than 8 elements, then $C = 5$.

The resulting formula helps to automatically calculate the problem class:

- if $2K + C \leq 6$, then easy level of problem complexity;
- if $7 \leq 2K + C \leq 9$, then medium level of problem complexity;
- if $2K + C \geq 10$, then difficult level of problem complexity.

Thus each problem can be associated with an integer lattice point shown in the Fig. 1.

For classifying problems into 4 categories according to their purpose the following principles were used:

- methodically important problems should contain material unknown for students or demonstrate a new algorithm for problems solving;

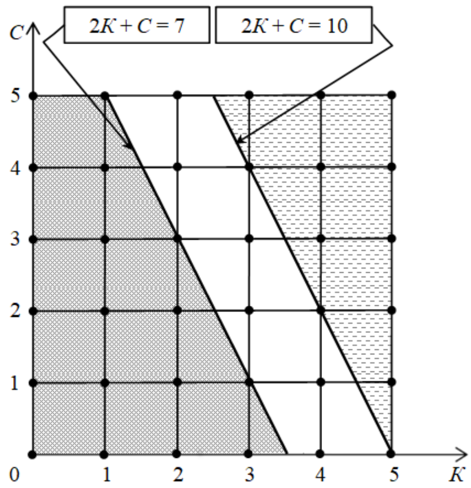

Figure 1. Levels of problem complexity

- problems for independent work in a classroom should have low computational complexity in contrast to problems for homework;
- test problems should show whether the material was learned by the students or not

Example 1. (Formalization problem) You are on duty today, or John, but not both.

The cognitive complexity of this problem is $K = 2$ (Tollinger's taxonomy, category 2.4. Analysis and synthesis), the number of input parameters is $C = 2$, so the resulting complexity is $2*2+2 = 6$ and this problem has easy level of complexity. By purpose this problem is for independent work in the classroom because it has low computational complexity.

Example 2. (Formalization problem) Fog outside or hoarfrost on the trees can be if and only if the thaw is on the street.

The cognitive complexity of this problem is $K = 2$ (Tollinger's taxonomy, category 2.4. Analysis and synthesis), the number of input parameters is $C = 3$, so the resulting complexity is $2 * 2 + 3 = 7$ and this problem has medium level of complexity. By purpose this problem is for homework because it allows students to consolidate new material.

Example 3. (Logical problem) Mrs. Black prepared a wonderful apple strudel for the birthday of little Henry. However, someone stole a treat. Three suspects were soon found, and each made two statements, but it is known that everyone lied at least one time, and exactly one of them is guilty.

A.
1) It was done by either B or I.
2) I agree with the second statement of B.
B.
1) I agree with the first statement of A.
2) I'm not guilty.
C.
1) I agree with the second statement of A.
2) It was done by A, and B helped him.
Who stole the strudel of Mrs. Black?

The cognitive complexity of this problem is $K = 5$ (Tollinger's taxonomy, category 5.2. Problem tasks), the number of input parameters is $C = 3$, so the resulting complexity is $2{*}5{+}3 = 13$ and this problem has difficult level of complexity. By purpose this problem is methodically important because it allows teacher to demonstrate that it is necessary to read the problem carefully and accurately understand its meaning accurately.

Example 4. (Logical problem) Five play cards. In one of the parties in the deck there were five aces: someone was cheating. That's what the players said (everyone once said the truth and once lied):

A.

1) I am innocent.

2) I do not know who was cheating.

B.

1) I am innocent.

2) Neither A nor D was cheating.

C.

1) The first statement of B is true.

2) B is cardsharper.

D.

1) B was cheating.

2) The second time B told the truth.

E.

1) A that day was one of the players.

2) The second statement of D is false.

So who is the cardsharper?

The cognitive complexity of this problem is $K = 5$ (Tollinger's taxonomy, category 5.5. Problems based on rational observations), the number of input parameters is $C = 5$, so the resulting complexity is $2 * 5 + 5 = 15$ and this problem has difficult level of complexity. By purpose this problem is for test because the solution of this problem doesn't require analyzing all the statements. In this problem it is necessary to find only one statement that exactly true to reduce computational complexity. Only students who understand solution algorithms of such problems can quickly come to the answer, while for other students it can take a long time.

III. Formalization of domain knowledge

After domain analysis and development of a new classification, the question of a formal description of the developed ontology appeared. The OWL (Web Ontology Language) language was chosen as the formal description language. This language was chosen because there are many ontology editors [13] with the ability to export ontologies written in OWL, for further work with them. OWL has three main modifications: OWL Lite, OWL DL (DL stands for "Description Logic"), OWL Full. Every modification has its own syntax, which defines the semantic power of language. At the same time, the OWL language version, known as OWL Lite, does not include the possibility of specifying a logical relationship between domain concepts. And the language OWL DL makes it possible to describe the logical relationship of the concepts of the domain, which allows to automatically classify learning problems by their purpose and cognitive and computational complexity. So, the OWL DL was chosen for developing the ontology. The main components of the ontology developed with OWL DL are the following:

1) classes - an abstract description of a group of objects. In OWL; there is a Thing subclass, from which other classes are inherited, this class includes all individuals existing in ontology;
2) instances - components of an ontology that are specific objects and are related to the lowest level of ontology;
3) attributes - object properties of classes and data properties.

The Protégé 4.3. ontology editor that allows to visualize the developed ontology and save it as OWL DL code was selected [14] as editor. Moreover, this editor provides the ability to specify instances at the level of classes and subclasses, to track the structure of classes, and has significant reference material.

Currently, the developed ontology includes more than 50 classes, including the Tollinger's taxonomy (33 classes), computational complexity (6 classes) and classification of problems by purpose (4 classes). The scheme of part of ontology is shown in Fig. 2.

In the ontology every learning problem represented as individual entity with description which consists of the problem itself and its solution. Each learning problem is assigned to the cognitive complexity class of Tollinger's taxonomy and to the class of computational complexity. The belonging to the class with level of resulting complexity displayed automatically according to the given logical formula.

In order to add a new problem to the ontology it is necessary to do the following:

1) enter a unique problem identifier;
2) assign the problem with one of the learning topics (tasks for formalization, table method of solving problems, logical reasoning method, etc.);
3) add the problem statement in Russian and in English, add the correct answer;
4) assign one of cognitive complexity classes of Tollinger's taxonomy;
5) assign one of computational complexity classes according to the number of input parameters;
6) assign one of the classes by problem purpose.

Acknowledgment

The described ontology of learning problems in course "Introduction to Boolean Algebra" based on developed classification can really help teachers to find problems for students. Ontology provides users (teachers of informatics) the following opportunities:

- obtain a set of problems with the required characteristics due to the structure of the ontology;
- establish relationship between objects of the ontology, methods of solving learning problems, and complexity of problems;
- add new problems by determining their complexity with developed classification.

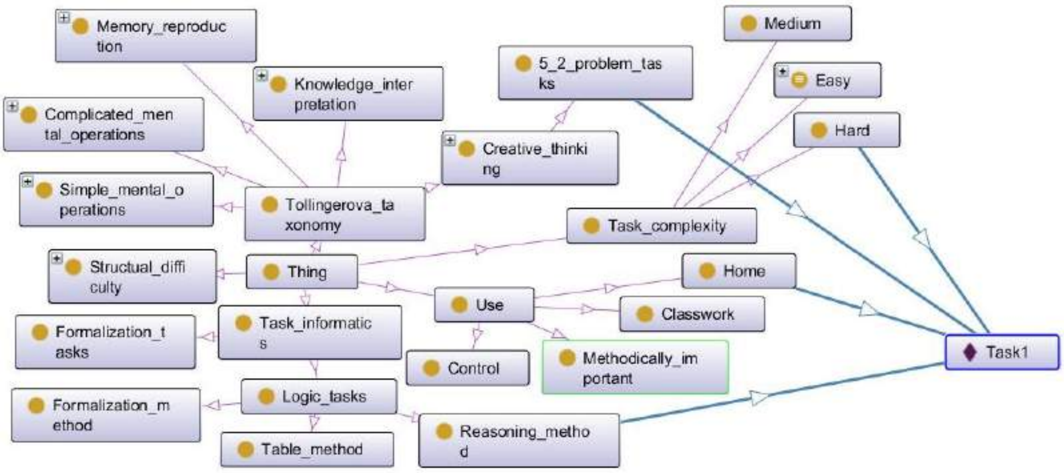

Figure 2. Structure of ontology

Completion of the ontology is planned for March 2018. Testing is preliminary scheduled for April-May 2018 in Advanced Educational Scientific Center (faculty) - Kolmogorov's boarding school of Moscow State University.

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ОНТОЛОГИЯ УЧЕБНЫХ ЗАДАЧ ПО АЛГЕБРЕ ЛОГИКИ

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В данной работе описывается онтология учебных задач по курсу "Введение в алгебру логики" предназначенному для школьников старших классов средней школы. Онтология разработана для учителей, ведущих данный курс в средней школе, и предназначена для решения проблемы подбора задач, с которой сталкивается большинство учителей. Для создания онтологии была проведена предварительная классификация задач по сложности. Для классификации использовалась таксономия Толлингеровой, которая позволяет оценить когнитивную сложность задачи, а также структурный анализ задачи, позволяющий определить вычислительную сложность задачи в зависимости от количества входных параметров. Кроме того, для каждой задачи определено её назначение и итоговая сложность, рассчитанная по специальной формуле. Онтология разработана с помощью формального языка OWL и редактора онтологий Protégé 4.3. В онтологии выделено более 50 классов, среди которых 33 отвечают за таксономию Толлингеровой, 6 за вычислительную сложность, 4 за назначение задачи. Задачи представляют собой индивидные сущности, включающие их описание, состоящее из текста условия и текста решения задачи. Для каждой задачи задана принадлежность к классу когнитивной сложности по Толлингеровой и к классу вычислительной сложности. Итоговая сложность задачи выводится автоматически согласно заданным логическим формулам. С учетом итоговой сложности и особенностей решения задачи определяется класс задачи по её назначению.