Remembering Lotfi Zadeh

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Lotfi Zadeh February 4, 1921 – September 6, 2017

Recently, world science has suffered a serious loss. The great scientist of our time, the founder of a number of major scientific trends in the theory of control, applied mathematics electrical engineering, computer science, artificial intelligence, the Father of Fuzzy Logic, Professor Lotfi Zadeh passed away on 6 September 2017. He was part of a cohort of very few pioneering scientists who generate new, original scientific ideas and form the basic scientific paradigms that are changing our world.

Professor L.Zadeh was the creator of fuzzy set theory and linguistic variables, fuzzy logic and approximate reasoning, possibility theory and soft computing, information granulation and generalized uncertainty theory, z-numbers and generalized constraints, etc. His ideas and theories not only opened a new era in the development of science free from restrictions of narrow disciplines to enable interdisciplinary synergies. They highly contributed to the emergence of new information and cognitive technologies, brought about the arrival of effective industrial applications, such as fuzzy controllers and computers, fuzzy chips and networks, fuzzy recognition and clustering systems, and so on [1].

Zadeh's origin and amazing personal destiny made him a convinced internationalist. In his interviews he said that «the question really is not whether I'm American, Russian, Iranian, Azerbaidjani or anything else. I've been shaped by all these people and I feel quite comfortable among all of them» [1]. He called himself «an American, mathematically oriented, electrical engineer of Iranian descent, born in Russia» [2]. This phrase can seem strange for young reader. Zadeh's birthplace is Novkhany near Baku. Why Russia? Where began the «Russian trace» of his biography?

The youth of Zadeh's parents, his birth and childhood came at the turning points of our history: disintegration of the Russian Empire and the emergence of the USSR. From 1859 to 1917 Baku was the center of Baku province of the Russian Empire. At the crossroad of the XIX-XXth centuries it was considered as the fastest growing city of all Russia. The main reason for the rapid growth of economic activity was a large oil production. At the beginning of the XXth century more than half of the world oil output came from Baku region. It worthily enjoyed the glory of «Russian Texas». Engineers and specialists from the most developed countries, including Germany, United Kingdom, USA, France, Sweden, worked at Baku province and introduced the most advanced industrial technologies of those times at the Baku plants.

So a multinational city emerged, the center of different cultures and religions that rapidly became an economic capital of Russian South. During the «oil fever» period, it grew rapidly, attracting rich people, businessmen and simple adventurers from all over the world. The Rothschild and Nobel companies prospered... This «oil fever» weakened, but did not stop even after the Great Russian revolution. In this wonderful city, «Paris of the East», Zadeh's parents met each other. It happened already in the years of great social cataclysms, Civil War in Russia and USSR formation. Between 1918 and 1920 there was chaos in Baku, one power (musavatists) succeeded another one (commissars), Turkish and German troops occupied the city. In April 1920, the Red Army broke into Baku, and the Azerbaijan Soviet Socialist Republic was organized.

Lotfi Asker Zadeh was born in Novkhany, Baku Region,

Soviet Azerbaijan, on **February 4, 1921** as Lotfi Aliaskerzadeh [3]. His father, Rahim Aliaskerzade, an Iranian Azeri from **Ardabil**, was a journalist, the foreign correspondent for the newspaper Iran in Baku. Rahim also had a good own business – wholesale matches. His Russian Jewish mother, Feiga (Fanya) Korenman, from **Odessa**, studied medicine and became a pediatrician. After the wedding she also obtained the status of Iranian citizen.

So his mother tongue was Russian. In 1928 Lotfi was enrolled in the Russian elementary school No 16 in Baku. Even 80 years after he kept fond memories about his school [4]: «at dawn of the Soviet era, what was extolled was science, scientists and engineers». Under this influence, Lotfi decided at an early age to become an engineer. This decision was the core of his outlook on life.

In Baku Lotfi completed three years of elementary school. It is not surprising that Russian culture and literature highly influenced the young boy (Figure 1).

At the beginning of 1930's ethnic Azerbaijanis with foreign passports were faced with a choice – obtain Soviet citizenship or leave the USSR. In 1931 Zadeh's parents decided to move to Iran, his father's homeland, taking him with them. Ten-year's Lotfi was enrolled in American Alborz College in Tehran, which was a Presbyterian missionary school.



Figure 1. Lotfi Zadeh in his youth

During his student days at the Albortz college, Lotfi met Fay, his future wife, who was the student at the women's branch of Alborz college. She also had Russian roots (her family was from Dvinsk, an old city on the Western Dvina, nowadays Latvian Daugavpils). According to her words, in Tehran period of Zadeh's life «in his room on the shelves along the wall a library of about 2000 volumes in Russian was collected» [5]. Young Lotfi read Tolstoy and Dostoevsky, Turgenev and even Shakespeare (translated into Russian).

After graduating from American college, Lotfi Zadeh passed the exams to the University of Tehran and placed third in the entire country. In 1942, he graduated from the University of Tehran with a degree in electrical engineering. Then Lotfi worked with his father, who did business with American military commanders in Tehran as a supplier of construction materials. Despite a rather high income, Lotfi decided to leave behind a comfortable life in Tehran and immigrate to the United States to fulfill his dream of a career in the academic world (see Figure 2).

Lotfi Zadeh left Tehran early in 1944 traveling to the United States by air and sea. He arrived in New York in July 1944 and moved to Cambridge after spending the summer months working at the International Electronics Corporation.



Figure 2. A chronological representation of Zadeh's educational and academic trajectory in the USA

Zadeh entered the Massachusetts Institute of Technology in 1944 as a graduate student and received his MS degree in electrical engineering from MIT in February 1946 (Figure 3).



Figure 3. Newly-made master Lotfi Zadeh with his parents

His parents came to the United States a little later and settled in New York. Lotfi did not want to be away from them and decided to move to Columbia University. There he was lucky to find the position of instructor in electrical engineering. After spending three years as instructor, he obtained his PhD degree in 1949 under the supervision of Professor J. Ragazzini. The thesis was concerned with the frequency analysis of timevarying networks.

In 1950's Zadeh's scientific interests shifted from classical electrical engineering to systems analysis and information science. Already in 1950 he published a significant paper «An extension of Wiener's theory of prediction», co-authored with Professor Ragazzini. This work found application in designing finite-memory filters; today it is considered classical. In 1952 Zadeh again together with Ragazzini proposed the z-transform method for discrete systems. Nowadays this method is also viewed as classical one; it is widely used in digital signal processing. In 1953 developed a new approach to non-linear

filtration and constructed a hierarchy of non-linear systems, which was based on the Volterra-Wiener presentation. Thus, the fundamentals of optimal non-linear processors to detect useful signals in noise were formulated.

In 1950's L.A.Zadeh became very interested in probability theory and its application to decision analysis. He met H.Robbins, a brilliant mathematician, and R.Bellman, the father of dynamic programming, who later became his close friends. In 1956-1957 he was a visiting member of the Institute for Advanced Study in Princeton, New Jersey. There he was inspired by a course of logic taught by S.Kleene.

In 1954 Lotfi Zadeh was promoted to the rank of Associate Professor and he received a full professor rank in 1957.

Zadeh taught for ten years at the Columbia University. In January 1959 Professor J.Whinnery, the Chair of Electrical Engineering Department at the University of California, proposed him to move to Berkeley. There were pros and cons. After weighing it, in July 1959 the 38-year-old Lotfi Zadeh with his family (Figure 4) started a long journey by car from New York to Berkeley.



Figure 4. Lotfi Zadeh with his mother Feiga and wife Faina (Fay). San Francisco, 1960

Professor Lotfi Zadeh joined the Department of Electrical Engineering at the University of California, Berkeley, in 1959 and served as its chairman from 1963 to 1968. In 1963 he published an important book [6], co-authored by Prof. Desoer, where a new state-based approach to linear system theory was described. This book has survived 4 editions. Its ideas and results were the sources of various modern approaches in systems analysis and automatic control. Nowadays the state space approach is widely used in system engineering ranging from industrial robots to space guidance control.

Thus, in mid-sixties, Professor L.A.Zadeh had already become a leading scientist in the field of systems theory, automatic control theory, and their applications. However, an innovation spirit proper to Lotfi Zadeh did not allow him to rest on his laurels. In 1965 44-year-old Prof. L.A.Zadeh published in Information and Control a main scientific work of his life – the pioneering paper «Fuzzy Sets» [7]. This work is of great historical significance. It opened a new scientific area that induced a powerful resonance all over the world and generated an enormous flow of publications. This flow is not exhausted up to now.

A basic Zadeh's idea was simple: real human reasoning based on natural language cannot be adequately modeled in the framework of classical mathematical methods. The introduction of *fuzzy set* – a class with vague boundaries, described by membership function, provided a suitable basis for developing more flexible approach to reasoning, decision-making and modeling of complex humanistic systems. The behavior of such systems is characterized rather by linguistic variables than ordinary numeric variables.

In the above mentioned paper «Fuzzy Sets» L.Zadeh defined the concept of fuzzy set and its ordinary level sets, suggested various ways of specifying intersection and union operations, introduced pseudo-complementation operation, and new unary operations – concentration and dilatation? Fuzzy relations, their composition and projections were defined. The extension principle was formulated, and mappings of fuzzy sets were considered. Fuzzy sets with fuzzy membership functions called fuzzy sets of type 2 were introduced. Fuzzy restrictions and translations rules for fuzzy propositions were proposed.

The development of fuzzy models for complex systems to bridge the gap between classical logic and intuition, creation of innovative formal approaches allowing adaptation of strict mathematics to real human ways of everyday thinking and communication – there were novel keynote scientific problems formulated by Lotfi Zadeh.

In fact, such an unexpected and sharp turn over of Zadeh's scientific interests from «honorable strict science» to nonclassical «vague science» had been a risky step that put in danger his further scientific career. Primarily, the ideas of fuzzy sets and fuzzy logic were rather coldly received by Western scientists, including Americans. For instance, one of the most brilliant Zadeh's ex-students, R.Kalman, the author of the well-known filter, had to say [4]: «Fuzzification is a kind of scientific permissiveness, it tends to result in socially appealing slogans unaccompanied by the discipline of hard scientific work and patient observation... These proposals could be severely, even brutally criticized from a technical point of view».

This primary negative reaction to fuzzy logic and linguistic variables had a rational explication. At first, from ancient times, Western philosophy and logic was based on the principle of clear boundaries and the law of excluded middle. Yes or no, true or false, good or bad, who is not our friend, is our enemy, and other similar expressions of hard opposition are the children of classical binary logic. The adoption of intermediate truth values and grades of truth by many-valued and fuzzy logics was a break-down for this perfect, certain world. At second, during the ages it was a deep-seated tradition in science to accord much more respect to numbers than to words. The introduction and use of linguistic variables put this tradition in question.

The first two papers on fuzzy sets by Lotfi Zadeh, «Fuzzy Sets» [7] and «Shadows of Fuzzy Sets» [8] were published almost at the same time in the USA and Soviet Union. The first Zadeh's translator into Russian V.L.Stefanuk confirms that Lotfi himself selected adequate words for Russian translation. He wished that his new ideas on fuzzy sets and fuzzy logics were known both in the West and in the East.

To perform it in the period of «iron curtain», Professor Zadeh decided to disseminate his works in the East, particularly, in Soviet Union and Japan.

In this fortunate for him 1965, Lotfi and Fay Zadeh visited Soviet Union two times [5]: the first one they were in Moscow in May by invitation of Popov Radio-Electronics and Automatics Society and the second time he was invited by the Soviet Academy of Sciences to attend in September a six-days congress on automatic control. After the opening ceremony in September 20, 1965 at Odessa Opera House, the congress was held on board of Admiral Nakhimov's cruise ship (Figure 5) on the route Odessa – Batumi and back.



Figure 5. Professor L.Zadeh with the participants of the Congress on board of Admiral Nakhimov's ship. Black Sea, September, 1965

This congress was attended by more than a thousand Soviet scientists and about 60 foreign guests, including Lotfi Zadeh and his wife Fay. On the board of «Admiral Nakhimov» L.A.Zadeh delivered his first talk on fuzzy set theory in Russian.

In the memoirs of Academician N.N.Moisseev [9], it was mentioned that this Zadeh's talk provoked a great interest and was favorably received by the audience, although in the remarks of pure mathematicians some condescending notes were guessed due to rather informal mode of presentation and non-usual terminology. So in 1960-1970's the concept of fuzziness in applied mathematics was better received by Soviet scientists than their American colleagues.

It was after these trips to the USSR that L.Zadeh found good personal acquaintances and kind friends among Soviet scientists. In particular, Academicians N.N.Moisseev, R.V.Gamkrelidze, V.A.Ilyin, G.S.Pospelov, V.I.Siforov, Ya.Z.Tsypkin became his friends. Nikita Moisseev himself remembered with pleasure the International School on Optimal Control in Dubrovnik two years after the first Zadeh's visit in Moscow [9]. He delivered lectures as an invited professor and shared a picturesque cottage over the Adriatic Sea with the families of two other lecturers – Lotfi Zadeh and Richard Bellman. Every evening the lecturers were sitting together over the sea and discussed on mathematics. Their mutual sympathy and close scientific views served an additional stimulus for friendship.

Just Moisseev and Pospelov were the first outstanding Soviet scientists, who highly appreciated and promoted Zadeh's works. The Russian translation of the book «Linear System Theory: The State Space Approach» was published in 1970 under the editorship of G.S.Pospelov.



Figure 6. The first Zadeh's book on linguistic variables and fuzzy sets was in Russian

In 1974 an important volume «Mathematics Today» appeared (edited by N.N.Moisseev); it contained the Russian translation of Zadeh's foundational work «Outline of a New Approach to the Analysis of Complex Systems and Decision Processes». Also Academician Nikita Moisseev initiated the Russian translation of Zadeh's papers on linguistic variables and suggested their publication in the form of monograph in the book series «Mathematics: New Trends in Foreign Science». This book «The Concept of a Linguistic Variable and Its Application to Approximate Reasoning». edited by N.Moisseev and S.Orlovsky, was published in 1976 by the famous publishing house Mir [10]. For a long time, this thin book in Russian (Figure 6) remained the only Zadeh's monograph on fuzzy sets and linguistic variables.

Also in 1976, a new volume (collection of foreign papers) «Issues of Decision Analysis and Decision-Making Procedures» was published in Russian with a preface by G.S. Pospelov. The translated version of the paper by R.E.Bellman, L.A.Zadeh «Decision-Making in a Fuzzy Environment» was included into it.

In his turn, in 1978 Lotfi Zadeh invited N.N. Moisseev and M.A.Aizerman to join the editorial board of newly formed International Journal of Fuzzy Sets and Systems.

A funny coincidence: a well-known Soviet scientist from the Institute for Control Studies of the Soviet Academy of Sciences, Prof. M. Aizerman was also born in Dvinsk like Fay Zadeh.

In 1966 Lotfi and Fay Zadeh again visited the USSR. The future «Father of Fuzzy Logic» participated at the XVth International Congress of Mathematicians in August 16-26. The first time this congress was held in Moscow. The opening ceremony took place in the Grand Kremlin Palace, and regular sessions were held in high-rise building of Moscow State University.

This year, together with R.Bellman and R.Kalaba, L.Zadeh published a paper [11] on the use of fuzzy sets in abstraction and pattern classification in prestigious Journal of Mathematical Analysis and Applications.

In late 1960's Lotfi Zadeh published such papers as «Probability Measures of Fuzzy Events» [12], «Fuzzy Algorithms» [13], «Note on Fuzzy Languages» [14]. He was always interested in probability theory and searched for natural ways of its extension. In [12] Zadeh introduced the notion of a fuzzy event. Usually an event is seen as a precisely specified collection of points in the sample space. By contrast, in everyday experience one frequently encounters situations in which an «event» is ill-defined or fuzzy. Zadeh cited as examples of ill-defined events: «It is a warm day», «x is approximately equal to 5», «In twenty tosses of a coin there are several more heads than tails». These expressions are fuzzy because of the imprecision of the meaning of the underlined words. He generalized the mathematical expressions for mean, variance and entropy in probability theory in case of fuzzy events. In his opinion, there are many concepts and results in probability theory, information science and related fields which admit of such generalization.

A new conceptual framework for decision-making in the case of fuzzy goals and fuzzy constraints was proposed in the earlier mentioned paper «Decision-Making in a Fuzzy Environment» [15]. The most important feature of this framework is its symmetry with respect to goals and constraints – a symmetry that erases the differences between them and makes it possible the specification of fuzzy goals and constraints in the same set of alternatives. Here fuzzy decision is obtained as a convolution of fuzzy goals and fuzzy constraints.

This confluence principle was detailed by considering three cases: intersection, product and convex combination of fuzzy

goals and constraints. It was also shown that the case where the goals and the constraints were defined as fuzzy sets in different spaces could be easily reduced to the previous case as they would be defined in the same space. Furthermore, the authors illustrated the new decision-making framework by examples of multi-stage decision processes, stochastic systems in a fuzzy environment and systems with implicitly defined termination time.

In «Similarity Relations and Fuzzy Orderings» [16] two basic kinds of fuzzy relations were defined. The degrees of similarity and preference were introduced. A fuzzy similarity as generalization of the notion of equivalence is a reflexive, symmetric and transitive fuzzy relation. A fuzzy ordering is a fuzzy relation which is transitive. In particular, a fuzzy partial ordering is a fuzzy ordering which is anti-symmetric and reflexive. At last, fuzzy linear ordering meets the extended condition of linearity: for any two alternatives x, y either x is preferred to y with a degree $\mu > 0$ or inversely y is preferred to x with a degree $\mu > 0$. Various properties of fuzzy similarity and fuzzy ordering relations were investigated and, as an illustration, an extended version of Szpilrajn's theorem was proved.

Zadeh was deeply interested in the problems of natural and artificial languages that stimulated his studies on semantics. His main semantic question was «Can the fuzziness of meaning be treated quantitatively, at least in principle? In the paper «Quantitative Fuzzy Semantics» [17] he gave an affirmative answer to this question. In the section «Meaning» of this paper he formulated the basics: «We consider two spaces: a universe of discourse U and a set of terms T, which play the roles of names of subsets of U. Let the generic elements of T and U be denoted by x and y, respectively. Then the meaning M(x)of a term x is given a fuzzy subset of U characterized by a membership function $\mu(y \mid x)$ which is conditioned on x. For instance, if we take a color palette, then the meaning of «red» M(red), is a fuzzy subset of U».

In the following section «Language», Zadeh defined a language L a fuzzy binary relation in $T \times U$ that is characterized by the membership function $\mu L : T \times U \rightarrow [0, 1]$.

Another semantic-oriented paper which was appeared in 1972, concerned the concept of linguistic hedge. A basic idea suggested in [18] was that a linguistic hedge such as «very», «more», «more or less», «much», «essentially», «slightly» etc. may be viewed as a nonlinear operator which acts on the fuzzy set representing the meaning of its operand.

Among Zadeh's works in 1970's, four seminal papers are of special concern: the already mentioned «Outline of a New Approach to the Analysis of Complex Systems and Decision Processes» [19], «The Concept of Linguistic Variable and Its Application to Approximate Reasoning» [20], as well as «Fuzzy Logic and Approximate Reasoning» [21] and «Fuzzy Sets as a Basis for a Theory of Possibility» [22].

In [19] Zadeh's *Principle of Incompatibility* was formulated: «As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics». And further: «the key elements in human thinking are not numbers, but labels of fuzzy sets, that is, classes of objects in which the transition from membership to membership is gradual rather than abrupt. Indeed, the pervasiveness of fuzziness in human thought processes suggests that much of the logic behind human reasoning is not the traditional two-valued or even multi-valued logic, but the logic with fuzzy truths, fuzzy connectives, and fuzzy rules of inference».

Three main features of the proposed new approach were noticed:

- 1) use of linguistic variables in place of or in addition to numerical variables;
- characterization of simple relations between fuzzy variables by conditional statements;
- 3) characterization of complex relations by fuzzy algorithms.

In particular, if x and y are linguistic variables, the conditional statements describing the dependence of y on x can be written in the form: «If x is small then y is very large», «If xis not small and not large then y is not very large», and so on. A fuzzy algorithm [13] is an ordered sequence of instructions (like a computer program) in which some of the instructions may contain labels of fuzzy sets, e.g. «Reduce x slightly if yis large», «If x is small then stop; otherwise increase x by 2».

Besides, a compositional rule of inference was proposed and the notion of «Computation of the Meaning of Values for a Linguistic Variable» was introduced. Fuzzy relational and behavior algorithms, in particular, algorithm Behavior, algorithm Oval, algorithm Intersection, algorithm Obstacle and others were constructed.

«The Outline of a New Approach...» was really a landmark paper. It served as a foundation of fuzzy control: on its basis E.Mamdani developed the first fuzzy controller.

Another keynote paper [20] contained a basic definition of linguistic variables: «by a linguistic variable we mean a variable whose values are words or sentences in a natural or artificial language». In more specific terms, a linguistic variable LV is characterized by a quintuple $\langle L, T(L), U, G, M \rangle$, where L is the name of the variable, T(L) is the term-set of L, that is the collection of its linguistic values; U is a universe of discourse, G is a syntactic rule which generates the terms in T(L) and M is a semantic rule which associates with each linguistic value X its meaning M(X). Here M(X) denotes a fuzzy subset of U. The meaning of a linguistic value Xis characterized by a compatibility function $c: U \rightarrow [0, 1]$, which associates with each u in U its compatibility with X.

In this paper, the examples of term-sets were specified for *Age, Appearance, Truth, Probability*, etc. «The specification of *Truth* as a linguistic variable with values such as *true, very true, completely true, not very true, untrue*, etc., leads to what is called *fuzzy logic*. By providing a basis for approximate reasoning, that is, a mode of reasoning which is neither exact nor very inexact, such logic may offer a more realistic framework for human reasoning than the traditional two-valued logic». Basic logical connectives for fuzzy logic were

specified. An example of approximate Modus Ponens rule was given

It was shown that probabilities, too, can be treated as linguistic variables with values such as *likely*, *very likely*, *unlikely*, etc.

The paper entitled «Fuzzy Logic and Approximate Reasoning» [21] was published in 1975 and was the first Zadeh's great publication with reflection on fuzziness in logic (the short paper of 1974 [23] can be mentioned only in a historical retrospective).

The term «fuzzy logic» is used in this paper to describe an imprecise logical system, in which the truth-values are fuzzy subsets of the unit interval with linguistic labels such as true, false, not true, very true, more or less true, quite false, very false, etc. Linguistic truth values are not allowed in traditional logical systems, but are routinely used by humans in everyday discourse. The truth-value set is assumed to be generated by a context-free grammar, with a semantic rule providing a means of computing the meaning of each linguistic truth-value as a fuzzy subset of [0,1]. Since is not closed under the operations of negation, conjunction, disjunction and implication, the result of an operation on truth-values requires, in general, a linguistic approximation. As a consequence, the truth tables and the rules of inference in fuzzy logic are inexact and depend on the meaning associated with the primary truthvalue true as well as the modifiers very, quite, more or less.

In [21] L.Zadeh summarized: «Perhaps the simplest way of characterizing fuzzy logic is to say that it is a logic of approximate reasoning. As such, it is a logic whose distinguishing features are:

- i fuzzy truth-values expressed in linguistic terms with modifiers;
- ii imprecise truth tables;
- iii rules of inference whose validity is approximate rather than exact.

In these respects, fuzzy logic differs significantly from standard logical systems ranging from the classical Aristotelian logic to inductive logics and many-valued logic with set-valued truth-values».

In the paper «Local and Fuzzy Logics» [24] the authors emphasized that «Fuzzy logic is local, i.e. both the truth values and their conjunctions such as "AND", "OR" and "IF-THEN" have variable rather than fixed meanings. This is the reason why fuzzy logic can be viewed as a local logic. Hence, the inference process has a semantic character rather than a syntactic one: in FL, the conclusion depends on the meaning assigned to the fuzzy sets that appear in the set.

Consequently, fuzzy logic is the result of a double weakening of the basic laws of classical logic. On the one hand, the principle of bivalence and the law of excluded middle are rejected, that gives rise to a multi-valued logic and, finally, to a membership function that allows us to interpret the predicates. On the other hand, the variability of the meaning related both to truth values and connectives, makes the logical inference imprecise. Later on, in 1994 L.Zadeh already noticed that «The term fuzzy logic is actually used in two different senses. In a narrow sense, fuzzy logic is a logical system which is an extension of multi-valued logic and is intended to serve as a logic of approximate reasoning. But in a wider sense, fuzzy logic is more or less synonymous with the theory of fuzzy sets» [25].

Nowadays, a broad concept of fuzzy logic includes fuzzy sets and linguistic variables (specifically, linguistic truth values), fuzzy relations and approximate reasoning, fuzzy rules and fuzzy constraints, test-score (experience-based) semantics and generalized uncertainty theory, etc. It encompasses a variety of soft formal methods and tools for fuzzy control, pattern recognition, natural language processing, and so on. In some sense, it implements the engineering approach to logical modeling.

The really close acquaintance of Russian specialists in Artificial Intelligence, including some authors of this paper, with Professor L.Zadeh happened at the International Workshop on AI in Repino near Leningrad in 1977. This scientific meeting took place in a small resort on the shore of the Gulf of Finland and was in its own way unique. The leading experts in AI in Western Europe and North America, mainly its founding fathers such as J.McCarthy, M.A.Arbib, J.E.Hayes, C.Hewitt, D.B.Lenat, D.Michie, N.J.Nilsson, R.S.Shank, L.A.Zadeh, joined forces with their Soviet counterparts to give answers to new scientific challenges in AI (Figure 7). At this workshop Professor Zadeh gave a presentation on approximate reasoning. In 1960-1970's L.A.Zadeh was sending to Moisseev and G.S.Pospelov the preprints of his papers from Berkeley (the University of California memos). It considerably contributed to the formation of Soviet specialists in the field of fuzzy sets and their applications. When reading them, it seemed that a large team was working on a new topic.

Lotfi Zadeh always treated Soviet scientists very well and willingly talked with them in Russian. In an informal conversation with him, we noted the importance of fuzzy reasoning for intelligent systems, thanked him for these preprints and congratulated with good disciples. How great was our surprise, when Lotfi said that he had no post-graduate students in the field of fuzzy sets for a long time. He had post-graduate students only in the areas of system science and automatic control. The reason was that fuzzy sets and systems not only did not find financial support, but were under an unofficial ban. Then no one knew that everything would change in a few years, not even years, but months later, and fuzzy technologies, especially, fuzzy controllers, will begin their triumphal march through Japan and the whole world.

The paper by Lotfi Zadeh «A Theory of Approximate Reasoning», as other contributions of this Workshop, was published in the volume «Machine Intelligence 9» [26], preface by Academician A.P.Ershov, edited by J.E.Hayes, D.Michie and L.I.Mikulich.

After this wonderful meeting, some scientific trips of professors from the Academy of Sciences of the USSR to Berkeley were organized (Figure 8).

A seminal paper «Fuzzy Sets as a Basis for a Theory of



Figure 7. The International Workshop on Artificial Intelligence in Repino near Leningrad (April 18-24, 1977). From right to left: L.Zadeh is participating in a discussion, J.McCarthy, computer scientist known as the father of AI, V.I. Varshavsky, Soviet classic in the field of collective behavior of automata, D.A.Pospelov, founder of AI in Soviet Union



Figure 8. The Soviet scientists from the Academy of Sciences of the USSR after the workshop in Berkeley (1977). From right to left: L.Zadeh, Ya.Z. Tsypkin, M.A.Aizerman, E.Jury, A.A.Dorofeuk

Possibility» [22] was published in 1978 in the first issue of Fuzzy Sets and Systems. Here the main Zadeh's thesis was as follows: «When our main concern is with the meaning of information rather than with its measure, the proper framework for information analysis is possibilistic in nature than probabilistic one. What is needed for such an analysis is not probability theory but an analogous and yet different. This theory might be called the theory of possibility. The mathematical apparatus of the theory of fuzzy sets provides a natural basis for the theory of possibility, playing a role which is similar to that of measure theory in relation to the theory of probability».

In [22] Zadeh introduced the concept of possibility distribution function via membership function of fuzzy set and considered it as an interpretation of fuzzy restriction. The possibility measure can be easily constructed by possibility distribution with using max-normalization. To a large extent, possibility theory is comparable to probability theory because it is based on set-functions. However, the possibility measure is a modification of probability measure: the two first axioms of classical measure–boundary condition and monotonousness axiom– are preserved, but the additivity axiom is replaced by max-axiom (the either-or condition).

The possibility-probability consistency principle was proposed to explain a weak connection between possibility and probability.

Marginal possibility distributions and conditional possibility distributions were also studied in [22]. Moreover, the possibility distributions of composite and qualified propositions were introduced. Conditional translation rules of type I and type II were proposed.

In brief, possibility theory is an uncertainty theory aimed at handling of incomplete information. It differs from the probability theory by the use of a pair of dual set functions (possibility and necessity measures) instead of only one. Besides, it is not additive and makes sense on ordinal structures.

According to [27], possibility theory is one of the most promising off-springs of fuzzy sets that can bridge the gap between artificial intelligence and statistics. Possibility theory clarifies the role of fuzzy sets in uncertainty management and explains why probability degrees, viewed as frequency or betting rates, can be used to derive membership functions.

The concepts of possibility theory were successfully applied in PRUF (Possibilistic Relational Universal Fuzzy) – a meaning representation language for natural languages [28]. In addition to approximate reasoning, PRUF can be employed as a language for the representation of imprecise knowledge and as a means of precisiation of fuzzy propositions expressed in a natural language.

In the opinion of B.Turksen [29], by the late 1970's, Lotfi Zadeh and his followers essentially developed the foundation of applied fuzzy mathematics.

Many Zadeh's papers in 1980's were dedicated to applications of fuzzy sets, fuzzy logic, possibility theory in artificial intelligence. In[30] it was stressed that «Management of uncertainty is an intrinsically important issue in the design of expert systems because much of the information in the knowledge base of a typical expert system is imprecise, incomplete or not totally reliable. Fuzzy logic subsumes both predicate logic and probability theory, and makes it possible to deal with different types of uncertainty within a single conceptual framework. In fuzzy logic, the deduction of a conclusion from a set of premises is reduced, in general, to the solution of a nonlinear program through the application of projection and extension principles. This approach to deduction leads to various basic syllogisms which may be used as rules of combination of evidence in expert systems. Among them are the intersection/product syllogism, the generalized modus ponens, the consequent conjunction syllogism, and the majorpremise reversibility rule».

In [31] this fuzzy syllogistic topic was continued: «A fuzzy syllogism in fuzzy logic is defined as an inference schema in which the major premise, the minor premise, and the conclusion are propositions containing fuzzy quantifiers. A

basic fuzzy syllogism in fuzzy logic is the intersection/product syllogism. Furthermore, it is noticed that syllogistic reasoning in fuzzy logic provides a basis for reasoning with dispositions, that is, with propositions that are preponderantly but not necessarily always true. It is also shown that the concept of dispositionality is closely related to the notion of usuality and serves as a gateway to what might be called a theory of usuality, a theory that may eventually provide a computational framework for commonsense reasoning».

The theory of disposition was outlined in [32, 33]. The basic idea was that «commonsense knowledge may be viewed as a collection of dispositions, that is, propositions with implied fuzzy quantifiers». Typical examples of dispositions are: *Icy roads are slippery. Tall men are not very agile. What is rare is expensive*, etc. It is understood that, upon restoration of fuzzy quantifiers, a disposition is converted into a proposition with explicit fuzzy quantifiers, e.g., *Tall men are not very agile* \rightarrow *Most tall men are not very agile*.

Since traditional logical systems do not provide methods for representing the meaning of propositions containing fuzzy quantifiers, such systems are unsuitable for dealing with commonsense knowledge. An appropriate computational framework for dealing with commonsense knowledge is provided by fuzzy logic, which is the logic underlying approximate reasoning. A summary of the basic concepts and techniques underlying the application of fuzzy logic to knowledge representation was given in [34].

In 1990's Professor L.A.Zadeh perceived a new burst of creative energy that was resulted in opening innovative scientific areas–Soft Computing and Information Granulation. In 1994 two pioneering papers on Soft Computing [35,36] appeared. This Zadeh's initiative is closely related to the emergence of hybrid systems in computer science and AI.

In biology, hybridization is considered as the most powerful form of integration, when in one organism the various hereditary features are merged. By analogy, a hybrid system in Computer Science includes two or more heterogeneous subsystems, integrated by a shared goal or joint actions, although these subsystems can have both different nature and specification languages. In brief, hybrid computer systems use two or more various computer technologies

According to Zadeh [35,36], the basis of soft computing is that unlike the traditional hard computing, soft computing is aimed at an accommodation with pervasive imprecision of real world. The guiding principle of soft computing is to exploit the tolerance for imprecision and uncertainty to achieve tractability, robustness, low solution cost and better rapport with reality. The role model for soft computing is the human mind.

Soft computing is not a single methodology. Moreover, it is not a simple collection of methodologies, but their partnership. The principal partners in this juncture are fuzzy logic, neurocomputing, genetic algorithms, probabilistic reasoning and chaos theory. Here fuzzy logic is mainly concerned with imprecision and approximate reasoning, probabilistic reasoning – with uncertainty and propagation of beliefs, neural network – with learning, genetic algorithm – with search and optimization, and chaos theory – with nonlinear dynamics.

In essence, fuzzy logic, neurocomputing, genetic algorithms, etc. are complementary and synergistic rather than competitive technologies. For this reason, it is advantageous to use them in combination.

The 1st EUFIT (European Congress on Fuzzy and Intelligent Techniques) took place in September 7-10, 1993 in Aachen (Germany). The arrival of soft computing was the reason to rename it, beginning from the 2nd EUFIT (the abbreviation remained the same) into European Congress on Intelligent Techniques and Soft Computing. After the creation of Soviet Association for Fuzzy Systems in 1991 (from 1993 it became known as Russian Association for Fuzzy Systems), we regularly participated at both IFSA and EUFIT congresses and rather often met L.A.Zadeh. In particular, we have good memories of the 6th and 7th International Fuzzy Systems Association World Congresses in Rio de Janeiro (Figure 9) and Prague in 1995 and 1997 respectively.



Figure 9. The IFSA-95 Conference in Rio de Janeiro. From right to left: L.A.Zadeh, A.N.Averkin, D.A.Pospelov, A.P.Ryjov, G.Klir, T. Yamakawa

In Rio Professor L.Zadeh made a presentation «New Frontiers of Fuzzy Logic» and in Prague he was awarded the B.Bolzano Medal by the Academy of Sciences of the Czech Republic «For outstanding achievements in fuzzy mathematics». By the way, he became a Foreign Member of Russian Academy of Natural Sciences (Computer Sciences and Cybernetics Section) in 1992.

Computing with Words (CW) is a methodology in which words are used in place of numbers for computing and reasoning. In [37] it was argued that «fuzzy logic plays a pivotal role in CW and vice-versa. Thus, as an approximation, fuzzy logic may be equated to CW».

In two first decades of XXIst century such initiatives of the Father of Fuzzy Logic as an Information Granulation Theory and a non-traditional Granular Mathematics program seem to be of primary concern. In 1979 his work entitled «Fuzzy Sets and Information Granularity» was published, where information granules were introduced [38].For some time this work remained imperceptible. The situation changed in 1997 when L.Zadeh formulated some fundamentals of the Theory of Fuzzy Information Granulation (TFIG) in his seminal paper «Toward a Theory of Fuzzy Information Granulation and its Centrality in Human Reasoning and Fuzzy Logic» [39].

The term «granule» is originated from Latin word *granum* that means grain, to denote a small particle in the real world. In [39] L.Zadeh specified *granule* as «a collection of objects which are drawn together by indistinguishability, similarity, proximity or functionality». There are various classifications of granules: crisp and fuzzy granules, information and knowledge granules, time and space granules, etc.

Specifically, the following Zadeh's granulation principle [40] is worth mentioning: «To exploit the tolerance for imprecision, employ the coarsest level of granulation, which is consistent to allowable level of imprecision».

Granulation is a basic property of human cognition. There are three basic concepts that underlie human cognition: granulation, organization and causation. Informally, granulation involves decomposition of whole into parts; organization involves integration of parts into whole; and causation involves association of causes with effects.

The TFIG is inspired by the ways in which humans granulate information and reason with it. However, the foundations of TFIG and its methodology are mathematical in nature. The point of departure in TFIG is the concept of a generalized constraint. A granule is characterized by a generalized constraint which defines it. The principal types of granules are: possibilistic, veristic and probabilistic.

In Zadeh's opinion, «the fuzzy information granulation may be viewed as a mode of generalization which may be applied to any concept, method or theory. Related to fuzzy granule, there are the following principal modes of generalization:

- 1) *Fuzzification* (f-generalization). In this mode of generalization a crisp set is replaced by a fuzzy set.
- 2) *Granulation* (g-generalization). In this case, a set is partitioned into granules.
- 3) *Randomization* (r-generalization). In this case, a variable is replaced by a random variable.
- Usualization (u-generalization). In this case, a proposition expressed as X is A is replaced with: usually (X is A)».

The TFIG provides a reliable basis for computing with words [37]. The point of departure in CW is the observation that in a natural language words play the role of labels of fuzzy granules. In computing with words, a proposition is viewed as an implicit fuzzy constraint on an implicit variable. The meaning of a proposition is the constraint which it represents. In CW, the initial data set (IDS) is assumed to consist of a collection of propositions expressed in a natural language. The result of computation, referred to as the terminal data set (TDS), is likewise a collection of propositions expressed in a natural language. To infer TDS from IDS the rules of inference in fuzzy logic are used for constraint propagation from premises to conclusions.

There are two main rationales for CW. First, computing with words is a necessity when the available information is not precise enough to justify the use of numbers. And second, computing with words is advantageous when there is a tolerance for imprecision, uncertainty and partial truth that can be exploited to achieve tractability, robustness, low solution cost and better rapport with reality.

The 1st International Conference on Soft Computing and Computing with Words in System Analysis, Decision and Control was held in Antalya, Turkey, on June 6-8, 2001. It was dedicated to the 80th anniversary of Professor L.A.Zadeh. The jubilee himself gave a lecture «A critical view of the foundations of control and decision». He pointed out that «CW opens the door to a potentially radical enlargement of the role of natural language in science and, in particular, in information processing, decision and control».

Even in 2000's after his 80th anniversary Professor L.A.Zadeh continued to work actively. He extended CW to computing with perceptions [40-42] developed the theory of generalized constraints [39] and the generalized uncertainty theory [43], introduced the concept of fuzzy validity [44], proposed an extended fuzzy logic based on fuzzy geometry and fuzzy transforms [45,46], as well as the restriction-centered theory of truth [47], with truth values interpreted as fuzzy restrictions, the theory of Z-numbers [48], i.e. two-fold numbers, where the first component is a restriction, and the second component is a measure of reliability (certainty) of the first one.

In 2000-2010's Lotfi Zadeh was a Professor of Graduate School and served as the director of BISC (Berkeley Initiative in Soft Computing). The BISC program of UC Berkeley was positioned as a leading organization of fundamental and applied investigations on soft computing. An important part of its activities was to hold meetings and conferences.

The BISCSE'05 International Conference was held in the University of California, Berkeley, in November 2-5, 2005. Here SE stands for Special Event in Honor of Professor Zadeh, which was organized in occasion of the 40th anniversary of his seminal paper on fuzzy sets. The event was well attended by most of the pioneers in fuzzy logic, and by prominent researchers and practitioners around the world who were interested in expanding the knowledge frontiers by the use of fuzzy logic and soft computing methods.

This event was a good occasion to congratulate the Father of Fuzzy Logic and to give diplomas of honorary doctor of Ulyanovsk State Technical University and Tver State University (Figure 10).

At the World Congress of the The International Fuzzy Systems Association in June 2007, Cancun, Mexico, 86-year



Figure 10. Professor L.A.Zadeh receives the honorary doctorat from Ulyanovsk State Technical University. Three Presidents of Russian Association for Fuzzy Systems and Soft Computing (in different years) – Ildar Batyrshin, Nadezhda Yarushkina and Alexander Yazenin congratulate the Father of Fuzzy Logic on the 40th anniversary of Fuzzy Sets. Berkeley. November 2, 2005.

old L.A.Zadeh gave the hour-long plenary talk «Fuzzy Logic as the Logic of Natural Languages». As always, he was glad to talk on the sidelines with participants from Russia (Figure 11).



Figure 11. Lotfi Zadeh and vice-president of Russian Association for Fuzzy Systems and Soft Computing Valery Tarassov at IFSA-2007 World Congress in Cancun, June 19, 2007

In a few months we talked again to Professor L.Zadeh, already in Europe, at the 5th EUSFLAT Conference in Ostrava.

The 6th World Conference on Soft Computing dedicated to 95th anniversary of L.A,Zadeh took place in Berkeley in May

22-25, 2016. Ildar Baryrshin and Vadim Stefanuk attended this conference as advisory co-chairs and made their presentations. It was the last meeting with Lotfi Zadeh.



Figure 12. Vadim Stefanuk, a pioneer of AI in the USSR, at Zadeh's office in Berkeley: the last meeting. May, 2015

The scientific strength of the theory is largely determined by the possibilities of its further evolution and extension, the resonance it causes in the scientific community. Fuzzy set theory has not been the only model that has been introduced to deal with imprecise and uncertain information. During the past five decades a lot of new models have been proposed to mathematically tackle incomplete information. Some models are extensions of fuzzy set theory and others use a different path.

The following set theories were created after Zadeh's paper on fuzzy sets [7]: Type-2 fuzzy set theory and L-fuzzy set theory, Flou set theory and L-flou set theory, Interval-valued fuzzy set theory and Probabilistic set theory, Intuitionistic fuzzy set and Twofold fuzzy set theory, Grey set theory and Soft set theory, Toll set theory and Bipolar fuzzy set theory, Vague set theory and Theory of imprecise probabilities, Rough set theory, Fuzzy rough set theory and Rough fuzzy set theory, etc. (see [49,50]).

The role of Lotfi Zadeh in the modern world is not limited only to scientific achievements that opened the age of innovative fuzzy technologies. His extraordinary destiny, his own graded membership to different nations and cultures, comfortably carried through the whole life, that had directed his international activities, made a valuable contribution to the formation of planetary scientific community of XXI century. It accelerated the emergence of a new synergistic scientific vision that supposes a symbiosis of eastern and western traditions.

And yet, it is great that last travel of Lotfi Asker Zadeh was his way back to Baku. He returned 86 years later to his hometown of a new country which is proud of a great man.

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ВСПОМИНАЯ ЛОТФИ ЗАДЕ

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Статья коллектива авторов посвящена памяти профессора Лотфи Заде – выдающегося ученого, внесшего крупный вклад в развитие теории управления и прикладной математики, информатики и искусственного интеллекта.

Лотфи Заде входил в когорту немногочисленных ученых-первооткрывателей, которые генерируют новые, оригинальные научные идеи и формируют базовые научные парадигмы, изменяющие наш мир. Он был основоположником теории нечетких множеств и лингвистических переменных, «отцом» нечёткой логики и приближённых рассуждений, родоначальником теории возможности и обобщенной теории неопределённости, создателем концепций гранулярных и мягких вычислений, вычислений со словами и перцептивными оценками, автором теории Z-чисел.

Его идеи и теории не только открыли новую эпоху в развитии научной мысли, свободную от ограничений узких научных направлений и способствующую их синергизму. Они внесли весомый вклад в развитие новых информационных и когнитивных технологий, привели к созданию эффективных промышленных технологий, таких как нечёткие компьютеры и процессоры, нечёткие регуляторы, нечёткие системы кластеризации и распознавания, и многие другие.

Роль Л.Заде в современном мире не ограничивается его достижениями в области науки и технологий. Его необычная собственная биография, вся творческая жизнь и международная деятельность внесли неоценимый вклад в формирование единого общепланетарного научного сообщества в эру «сетевого интеллекта», способствовали становлению нового научно-технического мировоззрения, предполагающего симбиоз восточных и западных культур.

Авторы были лично знакомы и нередко встречались с «Отцом нечеткой логики». Поэтому они стремились не только описать основные этапы жизненного пути и научного творчества Лотфи Заде, но и рассказать о многих обстоятельствах и эпизодах его жизни, связанных с Советским Союзом и Россией, а также поделиться воспоминаниями о личных встречах с великим ученым и замечательным человеком.