

**МАТЕРИАЛЫ**

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**PERIOD MULTIPLICATION AND CHAOTIC DYNAMICS  
IN A SEMICONDUCTOR WITH THE GUNN INSTABILITY**

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На основе дрейф-диффузионной модели разработана программа расчета процессов переноса и нелинейной динамики колебаний в GaAs полупроводниках с эффектом Ганна. Показано, что нелинейное взаимодействие характеризуется умножением периода колебаний и возникновением странных хаотических аттракторов.

*Key words:* nonlinear dynamics, arsenid gallium, strange attractor, chaotic oscillations, Gunn-effect model

**Introduction**

Self-oscillating systems are encountered in most branches of science and engineering. Gunn unstable semiconductor is one of the systems of this type, where dc voltage gives a rise to high-field domain dynamics and the well-known Gunn oscillations. The Gunn-effect devices, widely known as Gunn diodes, are capable of converting direct current power into microwave frequency power when they are coupled to the appropriate resonator. Typical applications for Gunn diode oscillators include local oscillators in the range from 10 GHz to above 120 GHz, voltage controlled oscillators, radar and communication transmitters.

Having found a system with a natural oscillation due to travelling-wave motion, it is natural to ask whether harmonic forcing would lead to chaos with spatial structure. During the last decade the nonlinear dynamics of periodically forced Gunn devices has been investigated [1–8]. One of the frequently observed phenomena is phase locking of the transit Gunn oscillation to the periodical forcing. Among the observed phenomena also were period doubling, and chaotic response. In this paper we describe the nonlinear dynamics obtained numerically for 3- $\mu\text{m}$  GaAs Gunn device.

**Description of the Gunn device model**

In this paper we consider the Gunn effect in a one-dimensional GaAs sample occupying  $0 < x < L$  with the cathode and the anode being  $x=0$  and  $x=L$ . The processes of electron transport in  $n^+ - n - n^+$  GaAs structures can be described by the drift-diffusion model, consisting of the current continuity and particle current relationships, and Poisson's equation

$$\frac{\partial n(x,t)}{\partial t} = -\frac{1}{e} \frac{\partial J(x,t)}{\partial x}, \quad (1)$$

$$J(x,t) = e \cdot n(x,t) \cdot v(E(x,t)) - e \cdot D(E(x,t)) \frac{\partial n(x,t)}{\partial x}, \quad (2)$$

$$\frac{\partial E(x,t)}{\partial x} = -\frac{e}{\varepsilon} (n(x,t) - n_0(x)), \quad (3)$$

where  $e$  is the electron charge,  $\varepsilon$  is the static dielectric constant,  $n_0(x)$  is the equilibrium electron density,  $J(x,t)$  is the total current density,  $n(x,t)$  is the electron carrier density,  $E(x,t)$  is the local electric field distribution,  $v(E)$  and  $D(E)$  are characteristics of the drift velocity and the diffusivity on electric field. The accurate cubic spline approximation of these characteristics was used in our simulation.

The device terminal current  $I(t)$  is partitioned into a particle current and a displacement current and is equal to

$$I(t) = \frac{S}{L} \int_0^L J(x,t) dx + C_{diode} \frac{dV(t)}{dt}, \quad (4)$$

where  $S$  is the cross-sectional area,  $L$  is the length of the transit region,  $C_{diode}$  is the static diode capacitance.  $V(t)$  is the voltage applied to a GaAs structure, consisting of the dc voltage  $V_{dc}$  and the external microwave signal with amplitude  $V_{ac}$  and frequency  $f_d$ :

$$V(t) = V_{dc} + V_{ac} \sin(2\pi f_d t). \quad (5)$$

To complete the mathematical description, specifications of initial conditions at  $t=0$ , and boundary conditions at both cathode ( $x=0$ ) and anode ( $x=L$ ) locations, are required. Initial conditions ( $t=0$ ) are:

$$n(x, 0) = n_0(x),$$

$$E(x, 0) = V_{dc} / L.$$

The boundary conditions are:

$$\partial^2 n(x, t) / \partial x^2 = 0 \text{ at } x=L.$$

$$n(0, t) = n_0(x), \text{ at } x=0,$$

$$\varphi_c(0, t) = V(t),$$

$$\varphi_a(L, t) = 0.$$

Poisson's equation (3) and the continuity equation (1) comprise a system of coupled nonlinear partial differential equations. They were integrated numerically using a Runge-Kutta scheme [9].

The application of an external voltage  $V_{dc}$  exceeding a high-field threshold value  $V_{th}$  causes current transit oscillation with frequency  $f_0$  in the external circuit. The external microwave signal essentially changes and complicates the oscillation dynamics.

### Numerical simulation

In all simulations to be discussed, we have fixed the following parameters of the  $n$ -GaAs sample:  $L=3 \mu\text{m}$ ,  $S=10^{-5} \text{cm}^2$ ,  $C_{diode}=0,036 \text{pF}$ , threshold voltage  $V_{th}=0,77 \text{V}$ , homogeneous doping density profile with  $n_0(x)=5 \cdot 10^{15} \text{cm}^{-3}$  and with doping notch  $0,25 \mu\text{m}$  length and  $3,5 \cdot 10^{15} \text{cm}^{-3}$  doping density, located  $0,3 \mu\text{m}$  from cathode. For dc bias below the threshold we find steady state, non-oscillatory behaviour. Above threshold, self-sustained periodic oscillations occur. The typical shape of the natural oscillation with frequency  $f_0 \approx 27,43 \text{GHz}$  is shown in figure 1 ( $A=V_{ac}/(V_{dc}-V_{th})=0$ ). The space-time electric field and electron concentration characteristics for this case are given in [6]. Figures 2 and 3 show the attractor and the Poincare section of this oscillation. To our surprising we

can see that they have non-trivial strange geometrical structure. Taking into consideration that the calculated correlation dimension is approximately  $\nu=1,08$  and the computed first Lyapunov exponent value is nearly 0.5 one can conclude that we deal with the oscillation which formally can be defined as chaotic [9–12]. Here chaotic refers to exponential divergence of nearby trajectories and strange means that the dimension of the attractor is not an integer. We can also see that, on the whole, the oscillation shape does not change. It can be supposed that a strong convergence during fairly short time intervals is outweighed by the divergence of nearby trajectories that occurs within other time intervals. This example shows that the evolutionary stability and chaotic dynamics are perfectly compatible.

Now, we turn to the study of the responses of this self-oscillatory system to periodically varying applied voltage. This problem is related to the operation of Gunn diodes inserted in a microwave resonant circuit [7]. In figure 4 one can see that the influence of the external forcing leads to the specific modulation of the natural Gunn oscillation. The resultant shape is a sequence of complicated asymmetrical oscillations, which seem almost periodic and similar to polar-modulated ones [6, 12]. Figure 5 shows that the increase of amplitude leads to the successive multiplication of natural oscillation period  $N$  times, where  $N=1, 2, 3...35$ . Denoting the period of the resultant oscillations  $T$ , we have that  $T \approx N \cdot T_d$ , where  $T_d = 1/f_d$ . The shape of the curve is similar to a staircase which length of steps (stable regions) is gradually decreases with growth of microwave amplitude  $A$ . The transition between next stable regions gives rise to narrow windows of non-periodic responses like shown in figure 5 and more complicated oscillations. The correlation dimension values of the resultant oscillations are less than two.

As we can see the competition between the natural oscillations due to the space-charge domain dynamics and the periodic forcing can result in low-dimensional fractal oscillations. It is important that the current forms and bifurcation processes obtained numerically agree closely with experimental data determined earlier using a millimetre-wave Gunn oscillator [7]. It gives the hope that the model used reflects the real mechanism of nonlinear interaction in Gunn devices. Figures 6, 7 show the attractor and the Poincaré sections of the current oscillation at  $A=0.605$  (see figure 4). The calculations showed that the first Lyapunov exponent was positive and the correlation dimension was about 1.95. It means that we also deal with the chaotic behaviour. Finally in figure 8 we present the return maps of this oscillation. To our surprise one can see that after fairly long period of comparative stability the oscillation becomes more irregular. The comparison of figures 8,*b* and 8,*c* shows that the part of the return map in figure 8,*b* marked  $R$  is practically the full return map shown in figure 8,*c*. So we can see that the used model demonstrates the some transience from one to another random state. Nevertheless, even by the end of the calculation time the oscillation has a high predictability.

## Conclusion

It was shown that the drift-diffusion transport model of the Gunn-effect structure could reproduce strange chaotic behaviour. Nevertheless, the time-dependence of the current keeps a very high level of predictability. It fails to detect the chaotic nature of the system. The examples given show that the evolutionary stability and chaotic dynamics are fairly well compatible. This phenomenon connected not only with the applying of the external sinusoidal signal, that leads to the suppression or maintenance of the travelling charge layers. It turned out that the transit Gunn oscillation also reproduces the low-dimensional strange chaotic behaviour. It is not an ordinary phenomenon and we connect it with the complex form of the drift velocity and diffusivity characteristics on electric field. However, we want to emphasize that the obtained results are only preliminary and do not give detailed information about reasons of such behaviour.

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## Abstract

A drift-diffusion Gunn effect model is used to analyse complex behaviour of the natural and driven Gunn oscillations. The results of the numerical simulation are presented. It was shown that Gunn devices might exhibit quite complicated nonlinear dynamics, such as period multiplication and strange chaotic attractors.

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