The number of integer polynomials whose discriminants are divided by a large prime power

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Let

\[ P(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0, \quad a_j \in \mathbb{Z}, \quad 0 \leq j \leq n, \]

is an integer polynomial of degree \( \deg P = n \) (this means \( a_n \neq 0 \)), the height \( H = H(P) = \max_{0 \leq j \leq n} |a_j| \leq Q \) and roots \( \alpha_1, \alpha_2, \ldots, \alpha_n \).

Then the discriminant \( D(P) \) of the polynomial (1) is equal to

\[ D(P) = a_n^{2n-2} \prod_{1 \leq i < j \leq n} (\alpha_i - \alpha_j)^2. \]

The expression (2) is often taken as a definition of the discriminant.

For \( 1 \leq v \leq n - 1 \) and a natural number \( Q > 1 \) introduce a class \( \mathcal{P}_n(Q, v) \) of polynomials

\[ \mathcal{P}_n(Q, v) = \{ P(x) \mid \deg P \leq n, 1 \leq D(P) < Q^{2n-2-2v} \}. \]

Denote \( \# \mathcal{P}_n(Q, v) \) the number of elements of the finite set \( \mathcal{P}_n(Q, v) \). In [1] was proven that

\[ \# \mathcal{P}_n(Q, v) > c_1(n)Q^{n+1 - \frac{n+2}{n}v}. \]

Estimates from above for the \( \# \mathcal{P}_n(Q, v) \) were received in [2] for \( n = 2 \) and \( n = 3 \).

Let \( |a|_p \) – \( p \)-adic norm of a natural number \( a \). Similarly to (3) define a class of polynomials

\[ \mathcal{P}^*_n(Q, v) = \{ P(w) \mid \deg P \leq n, |D(P)|_p < Q^{-2v} \}. \]

THEOREM 1. Let \( 2 \leq n \leq 4 \) and \( \varepsilon > 0 \). Then

\[ \# \mathcal{P}^*_n(Q, v) < Q^{n+1 - \frac{n+2}{n}v + \varepsilon}. \]

References


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