DUAL CONTROL OF MULTIDIMENSIONAL-MATRIX STOCHASTIC OBJECTS

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The main results in dual control theory are analyzed. The problem of the dual control of the multidimensional-matrix stochastic objects is formulated and the functional equations for its solution are given.

Introduction

Dual control theory deals with the control of objects whose characteristics are initially unknown. The controller's objectives in controlling such an object are twofold. One objective is control action to control of the object as well as possible based on current knowledge on object and the other is experiment with the object so as to learn about its behavior and control it better in the future. The dual control theory was developed by Russian scientist A.A. Feldbaum in 1960 [1] and published later in the monograph [2] (see also [3]).

I. Dual control of the deterministic objects

The system having the block diagram in the figure 1 was considered in the works [1, 2, 3].

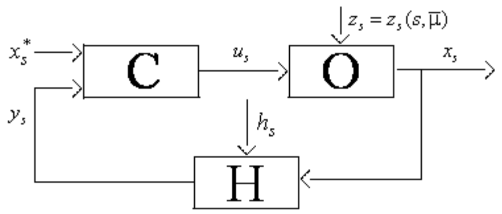

Рис. 1 – To dual control of the deterministic objects

In the figure 1 h_s is the sequence of independent random numbers in the measurement channell H of the output variable of the controlled object O, C is the controller. The controlled object O is deterministic and is described by function $x_s = F(z_s, u_s) = F(s, u_s, \bar{\mu})$, where $\bar{\mu}$ is a random vector with known priory probability density $f(\bar{\mu})$. It is assumed that the store of information about the vector $\bar{\mu}$ can be accumulated in the control process. The task consists of determining such a sequence of control actions $u_0, u_1, , ..., u_n$ that provides minimum to the total risk R: $R = \sum_{s=0}^{n} E\{W_s(s, x_s, x_s^*)\} = \sum_{s=0}^{n} R_s$, where W_s is loss function, x_s^* is the some known sequence, R_s is the mean specific risk, E is a symbol of the mathematical expectation. In the works [1,2,3] the functional equations is derived which allow in principle to determine the optimal sequence of the control actions. However, it is practically impossible to receive the solution of the equations in the analytical form. In the case of the numerical solving of the equations the difficulties connected with the necessity to remember and minimize the functions of a large number of variables are appeared, moreover, the number of variables increases with the increasing interval of the control. The equations are such that a reasonable choose of the method of the numerical integration can be performed in the case of determining the control action only at the penultimate time instant $n-1$ when the integrand can be received in analytical form. The reasonable choose of the method of the numerical integration in the following calculations seems extremely difficult, because the integrand is known in discrete set of points. In the paper [4] other approach to the solution to the problem is proposed. It consists of the separation of the problem into the problem of the estimation of the parameters of the controlled object and the problem of the optimal control on the base of received estimations. This approach is more effective compared with the approach of the works [1,2,3].

II. Dual control of the multivariate stochastic objects

The problem of dual control of the deterministic objects considered in [1,2,3] was generalized in paper [5] to the multivariate stochastic objects with the unknown parameters in the form of the vector random sequence. The approach of the paper [4] consisting of the separation of the problem into the problem of the estimation of the parameters and the problem of the optimal control was used to solution to this problem. The problem is formulated as following. The feedback system showed in the figure 2 is considered, where O is the controlled object, C is the controller, $\bar{y}_s = (y_{1,s}, y_{2,s}, ..., y_{p,s},)$ is the output vector of the object with p coordinates at the time instant s, $\bar{u}_s = (u_{1,s}, u_{2,s}, ..., u_{k,s},)$ is the vector of the control action with k coordinates at the time instant s , $\bar{x} = (x_1, x_2, ..., x_m)$ is the random vector with m coordinates, $\bar{x}_s = (x_{1,s}, x_{2,s}, ..., x_{m,s})$ is the random vector sequence with m coordinates at the time instant s, $\bar{g}_s = (g_{1,s}, g_{2,s}, ..., g_{l,s})$ is the vector of the 'driving action' with l coordinates at the time instant s.

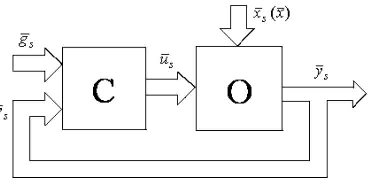

Рис. 2 – To dual control of the stochastic objects

Controlled object O at the time instant s is described by the conditional probability density $f(\bar{y}_s/\bar{x}_s, \bar{u}_s), s = 0, 1, 2, ..., n$. Such an object is stochastic and is called a regression object. The vector of the 'driving action' $\bar{g}_s$ is assumed, for simplicity, to be known deterministic sequence. The probability density $f(\bar{x})$ of the random vector $\bar{x}$ or the $(n+1)$-dimensional probability density $f(\vec{x}_n) = f(\bar{x}_0, \bar{x}_1, ..., \bar{x}_n)$ of the random sequence $\bar{x}_s$ are assumed to be known. The control vector $\bar{u}_s$ is restricted by the condition $\bar{u}_s \in U$, where U is a certain admissible region, for example, $U = \{\bar{u}_s : \bar{u}_s \geq 0, \psi_j(\bar{u}_s) \leq 0, s = 0, 1, ..., n, j = 1, 2, ..., l\}$, and $\psi_j(\bar{u}_s)$ are some scalar functions of the vector $\bar{u}_s$. It is assumed, that the controller C utilizes the all past information in the form of the measurements $\vec{u}_{s-1} = (\bar{u}_0, \bar{u}_1, ..., \bar{u}_{s-1})$, $\vec{y}_{s-1} = (\bar{y}_0, \bar{y}_1, ..., \bar{y}_{s-1})$ to form the control action $\bar{u}_s$ at the time instant s. It is assumed, that the controller has a random behavior and is described at the time instant s by the conditional probability density $\Gamma_s = f(\bar{u}_s/\vec{u}_{s-1}, \vec{y}_{s-1})$ called the strategy of the controller. The quality of the functioning of the system at the time instant s is evaluated by the loss function $W_s(\bar{y}_s, \bar{q}_s)$ depending on the output vector $\bar{y}_s$ and may on the vector of the 'driving action' $\bar{g}_s$. Mathematical expectation of the loss function $R_s = E\{W_s(\bar{y}_s, \bar{g}_s)\}$ is called mean specific risk. The sum of the mean specific risks on the $n+1$ time instants

$$R = E\{\sum_{s=0}^{n} W_s\} = \sum_{s=0}^{n} E\{W_s\} = \sum_{s=0}^{n} R_s \qquad (1)$$

is called the mean total risk. The system will be optimal when the mean total risk R will be minimal. The problem consists of determining the sequence of the strategies of the controller $\Gamma_s = f(\bar{u}_s/\vec{u}_{s-1}, \vec{y}_{s-1}), s = 0, 1, 2, ..., n$, minimizing the mean total risk R (1).

As it is known [1,2,3], the strategies of the controller $\Gamma_s = f(\bar{u}_s/\vec{u}_{s-1}, \vec{y}_{s-1}), s = 0, 1, ..., n$, minimizing the mean total risk R (1) are not randomized, i.e $\Gamma_s = \delta(\bar{u}_s - \bar{u}_s^*)$, δ is the unit impulse function, $\bar{u}_s^*$ is optimal control action. Provided the $\bar{x}$ is random vector, the sequence of the control actions $\bar{u}_n^*, \bar{u}_{n-1}^*, ..., \bar{u}_0^*$ is defined from the following functional equations:

$$f_n^*(\vec{u}_{n-1}, \bar{u}_n^*, \vec{y}_{n-1}) = \min_{\bar{u}_n \in U} \phi_n(\vec{u}_n, \vec{y}_{n-1}), \qquad (2)$$

$$f_{n-r}^*(\vec{u}_{n-r-1}, \bar{u}_{n-r}^*, \vec{y}_{n-r-1}) =$$
$$= \min_{\bar{u}_{n-r} \in U} (\phi_{n-r}(\vec{u}_{n-r}, \vec{y}_{n-r-1}) +$$
$$+ \int_{\Omega(\bar{y}_{n-r})} f_{n-r+1}^*(\vec{u}_{n-r}, \bar{u}_{n-r+1}^*, \vec{y}_{n-r}) \cdot$$
$$\cdot f(\bar{y}_{n-r}/\vec{u}_{n-r}, \vec{y}_{n-r-1}) d\Omega), r = 1, 2, ..., n, \qquad (3)$$

where

$$\phi_s(\vec{u}_s, \vec{y}_{s-1}) = \int_{\Omega(\bar{y}_s)} W_s(\bar{y}_s, \bar{g}_s) f(\bar{y}_s/\vec{u}_s, \vec{y}_{s-1}) d\Omega, \qquad (4)$$

$$f(\bar{y}_s/\vec{u}_s, \vec{y}_{s-1}) = \int_{\Omega(\bar{x})} f(\bar{y}_s/\bar{x}, \bar{u}_s) f_s(\bar{x}) d\Omega, \qquad (5)$$

$$f_s(\bar{x}) = \frac{f_0(\bar{x}) \prod_{i=0}^{s-1} f(\bar{y}_i/\bar{x}, \bar{u}_i)}{\int_{\Omega(\bar{x})} f_0(\bar{x}) \prod_{i=0}^{s-1} f(\bar{y}_i/\bar{x}, \bar{u}_i) d\Omega}. \qquad (6)$$

III. Dual control of the multidimensional-matrix stochastic objects

The representation of the scalar-vector or the vector-vector regression function of the many input variables by the polynomial of the degree more than one has a serious drawback such as absent of the strict theoretic formalization and algorithmic generality. In this paper the multidimensional-matrix approach to the description of the controlled object is proposed [6]. It does not have this drawback. It is the analogue to the scalar approach, generalizes the vector-matrix approach and owns the strict theoretic formalization and algorithmic generality. P-dimensional-matrix regression function $\bar{y}_s = \phi(\bar{x}, \bar{u}_s)$ of the q-dimensional matrix argument $\bar{u}_s$ describing the controlled object O in the form of the polynomial of the degree r has the following expression [7]:

$$\bar{y}_s = \sum_{i=0}^{r} {}^{0,iq}(\bar{x}_i \bar{u}_s^i).$$

So, we propose to consider the system in the figure 2, but we will consider the variables in the figure 2 as multidimensional-matrix. The theory in this case will be practically identical to the formulas (2) – (6).

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