# Resonant conversions of Majorana neutrinos in three generations

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Within three avor approximation the motion of the Majorana neutrino ux in condensed matter and intensive magnetic eld displaying the geometrical phase is examined. It is assumed that the neutrinos possess the dipole magnetic moment which value is close to the current experimental value. The evolution equation is found. All the resonance transitions of the electron neutrinos ux are probed.

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# 1. Introduction

The existence of the neutrino masses and the mixing between di erent neutrino types has been con rmed by the experimental observation of neutrino oscillations. These neutrino properties result in transitions between neutrino avors, which are called neutrino avor oscillations. Neutrino avor oscillations are known to occur even in vacuum. However, neutrinos can interact with other fermions, which a background matter is made of, by exchanging of virtual particles. This kind of interaction, although it is quite weak, can signi cantly in uence the process of neutrino avor oscillations culminating in the resonance enhancement of the transition probability. Despite neutrinos are electrically neutral particles, nothing prevents them to have nonzero multipole moments (dipole magnetic and electric moments, anapole moments and so on), which are induced by radiative corrections. The existence of the neutrino multipole moments (MM's) leads to interaction with the external electromagnetic eld. Owing to this interaction the particle spin precession in an external electromagnetic eld could be observed. In that case, left polarized neutrinos, which exist in the standard model (SM), could convert to right polarized particles being sterile neutrinos. If this process takes place within one neutrino generation, it is called neutrino spin oscillations. When neutrinos change both the avor and the polarization in an external electromagnetic eld then these transitions are named neutrino spin- avor oscillations (see, for review Ref. [1].

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Investigation of MM's of massive neutrinos, propagating in dense matter and in large magnetic fields, is of great importance for elementary particle physics, as well as for neutrino astrophysics and cosmology. The neutrino self-energy and its dispersion relation are modified in these conditions, and such modifications have been studied extensively in the literature [2–4]. Due to smallness of the neutrino MM's this interaction becomes essential in the case of intensive magnetic fields only. Such fields are present in many astrophysical sites such as the Sun, supernovae, neutron stars, and white dwarfs. Magnetars being young neutron stars could generate magnetic fields which may even reach the values of  $10^{16}$  G. Moreover, magnetic fields as high as  $10^{22} - 10^{24}$  G have been hypothesized to exist during the electroweak phase transition of the early universe [5].

The effects of neutrino dipole magnetic moment is also of interest in connection with the analysis of helioseismological observations [6]. The neutrino spin-flavor oscillations in a magnetic field may also explain the supernova problem providing an efficient mechanism of energy transfer from a proto neutron star [7].

In the present work, amongst the neutrino MM's, we will be interested in the dipole magnetic. In the framework of the minimally extended SM with right-handed neutrino singlets added, the diagonal magnetic moment of the neutrino appeared to be very small [8]

$$\mu_{\nu_l \nu_l} = 10^{-19} \ \mu_B \left( \frac{m_{\nu_l}}{\text{eV}} \right), \tag{1}$$

where  $\mu_B$  is the Bohr magneton. Some extensions of the SM predict substantially larger values for the neutrino DMM's (for a review, see [9]). So, when we are going to discuss the observable effects for the neutrino in electromagnetic field we should chose the definite SM extension predicting rather large the DMM value but which does not contradict to current experimental data. For a such SM extension we shall use the left right symmetric model (LRM) that is based on the  $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$  gauge group [10]. Note that investigation of the effects connecting with the neutrino DMM's is a way to define the neutrino nature (Dirac or Majorana). There are two fundamental differences between the Dirac and Majorana neutrinos: (i) Majorana neutrinos can have only transition magnetic moments having the property  $\mu_{ll'} = -\mu_{l'l}$ ; (ii) the right-handed Majorana neutrinos are not sterile, but interact as right-handed Dirac antineutrinos.

It is worth noting that most of papers studying the behavior of the neutrino beam in the magnetic field are limited by a two-flavor approximation. Within the three-neutrino generations this problem was investigated in Ref.[11] for the Dirac neutrinos. The goal of the present work is to consider the behavior of the Majorana neutrinos in an intensive magnetic field within three neutrino generations. As an example of such a field we shall consider the Sun's magnetic fields. In so doing of special interest are the magnetic fields of the solar sunspots which will be the source of the solar flare (SF). The SF formation starts from pairing big sunspots of opposite polarity (coupled sunspots — CS's). The magnetic flux ~  $10^{24}$  Gauss · cm<sup>2</sup> [12] erupting from the solar interior accumulates within the sunspots giving rise to the stored magnetic field. In that case, the magnetic field value for the CS's could be increased from ~  $10^4$  Gs up to ~  $10^5$  Gs and upwards.

In next section we find the evolution equation of the neutrino beam which travels through region of the CS's in preflare period. In section III we establish all possible resonance conversions of the neutrino system under study and derive the expression for the survival probability of the electron neutrino. Finally, in section IV, some conclusions are drawn.

## 2. Evolution equation in three-neutrino approximation

Since we are going to take into account the interaction of the neutrinos with the electromagnetic fields, the neutrino system under consideration must include both the left-handed and right-handed neutrinos. In the Majorana neutrino case the right-handed states are not sterile as it happens with Dirac neutrinos, but interact as right-handed Dirac antineutrinos. By virtue of this fact the  $\nu_{lR}$  is often denoted by  $\overline{\nu}_{lR}$ , or only  $\overline{\nu}_{l}$ , and called electron antineutrino. Further, for the right-handed neutrinos we shall use the designation  $\overline{\nu}_{lR}$ .

We shall reason that magnetic fields in which the neutrino beam travels exhibit the geometrical phase  $\Phi(z)$ 

$$B_x \pm iB_y = B_\perp e^{\pm i\Phi(z)},\tag{2}$$

which is defined by the simple expression

$$\Phi(z) = \frac{\alpha \pi}{L_{mf}} z. \tag{3}$$

So, we assume that the magnetic fields are in existence over a distance  $L_{mf}$  and twist by an angle  $\alpha \pi$ .

In the flavor basis the evolution equation is defined by the expression

$$i\frac{d}{dz}\begin{pmatrix}\nu_{eL}\\\nu_{\mu L}\\\nu_{\tau L}\\\overline{\nu}_{eR}\\\overline{\nu}_{\mu R}\\\overline{\nu}_{\tau R}\end{pmatrix} = \mathcal{H}^{M}\begin{pmatrix}\nu_{eL}\\\nu_{\mu L}\\\nu_{\tau L}\\\overline{\nu}_{eR}\\\overline{\nu}_{\mu R}\\\overline{\nu}_{\tau R}\end{pmatrix},$$
(4)

where

$$\mathcal{H}^{M} = \mathcal{U} \begin{pmatrix} \Sigma & 0 \\ 0 & \Sigma \end{pmatrix} \mathcal{U}^{-1} + \begin{pmatrix} I_{L} & \mathcal{M}^{M} \\ -\mathcal{M}^{M} & I_{R} \end{pmatrix},$$
(5)

$$\Sigma = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix}, \qquad I_L = \begin{pmatrix} V_{eL} + \Phi/2 & 0 & 0 \\ 0 & V_{\mu L} + \dot{\Phi}/2 & 0 \\ 0 & 0 & V_{\tau L} + \dot{\Phi}/2 \end{pmatrix},$$
$$I_R = \begin{pmatrix} -V_{eL} - \dot{\Phi}/2 & 0 & 0 \\ 0 & -V_{\mu L} - \dot{\Phi}/2 & 0 \\ 0 & 0 & -V_{\tau L} - \dot{\Phi}/2 \end{pmatrix},$$
$$\mathcal{M}^M = \begin{pmatrix} 0 & \mu_{e\mu} B_{\perp} & -\mu_{e\tau} B_{\perp} \\ -\mu_{e\mu} B_{\perp} & 0 & \mu_{\mu\tau} B_{\perp} 0 \\ \mu_{e\tau} B_{\perp} & -\mu_{\mu\tau} B_{\perp} & 0 \end{pmatrix}, \qquad \mathcal{U} = \begin{pmatrix} \mathcal{D} & 0 \\ 0 & \mathcal{D} \end{pmatrix},$$
$$\mathcal{D} = \exp(i\lambda_7 \psi) \exp(i\lambda_5 \phi) \exp(i\lambda_2 \omega) = \begin{pmatrix} C\omega c_{\phi} & s_{\omega} c_{\phi} & s_{\phi} \\ -s_{\omega} c_{\psi} - c_{\omega} s_{\psi} s_{\phi} & c_{\omega} c_{\psi} - s_{\omega} s_{\psi} s_{\phi} & c_{\psi} c_{\phi} \\ s_{\omega} s_{\psi} - c_{\omega} c_{\psi} s_{\phi} & -c_{\omega} s_{\psi} s_{\phi} & c_{\psi} c_{\phi} \\ \end{pmatrix},$$

 $\psi = \theta_{23}, \ \phi = \theta_{13}, \ \omega = \theta_{12}, \ s_{\psi} = \sin \psi, \ c_{\psi} = \cos \psi, \ \text{and so on, the } \lambda$ 's are Gell-Mann matrices corresponding to the spin-one matrices of the SO(3) group,  $V_{eL}$  ( $V_{\mu L}, V_{\tau L}$ ) is a matter potential describing interaction of the  $\nu_{eL}$  ( $\nu_{\mu L}, \nu_{\tau L}$ ) neutrinos with a dense matter,

$$V_{eL} = \sqrt{2}G_F(n_e - n_n/2), \qquad V_{\mu L} = V_{\tau L} = -\sqrt{2}G_F n_n/2, \qquad \dot{\Phi} = \frac{d\Phi}{dz},$$

and  $n_n$  is a neutron density.

In order to obtain the survival probabilities of definite flavor neutrinos, we can turn to the Hamiltonian (5) and find all possible resonant transitions in a flavor basis immediately. However, even though we work with the three component neutrino wave function  $\Psi^T = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})$  the physical implications will be far from transparent [13]. For this reason we shall rotate the flavor basis in such a way to make the physical meanings more obvious. In so doing, when the mixing angles  $\psi$  and  $\phi$  tend to zero, our results must convert into those obtained within the two-flavor approximation. This could be arranged by the following transformation

$$\begin{pmatrix} \nu_1^{M} \\ \nu_2^{M} \\ \nu_3^{M} \\ \overline{\nu}_1^{M} \\ \overline{\nu}_2^{M} \\ \overline{\nu}_3^{M} \end{pmatrix} = \mathcal{U}' \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \overline{\nu}_{eR} \\ \overline{\nu}_{\mu R} \\ \overline{\nu}_{\tau R} \end{pmatrix},$$
(6)

where

$$\mathcal{U}' = \begin{pmatrix} \mathcal{D}' & 0\\ 0 & \mathcal{D}' \end{pmatrix}, \qquad \mathcal{D}' = \exp(-i\lambda_5\phi) \exp(-i\lambda_7\psi) = \begin{pmatrix} c_\phi & 0 & s_\phi\\ -s_\phi s_\psi & c_\psi & c_\phi s_\psi\\ -s_\phi c_\psi & -s_\phi & c_\phi c_\psi \end{pmatrix}.$$

In this basis the Hamiltonian  $\mathcal{H}'^M$  has the form

$$\mathcal{H}^{\prime M} = \begin{pmatrix} \mathcal{B}_v + \Lambda & \mathcal{M} \\ -\mathcal{M} & \mathcal{B}_v + \overline{\Lambda} \end{pmatrix},\tag{7}$$

where

$$\mathcal{B}_{v} = \begin{pmatrix} -\delta^{12}c_{2\omega} & \delta^{12}s_{2\omega} & 0\\ \delta^{12}s_{2\omega} & \delta^{12}c_{2\omega} & 0\\ 0 & 0 & \delta^{31} + \delta^{32} \end{pmatrix}, \qquad \Lambda = \begin{pmatrix} V_{eL}c_{\phi}^{2} & 0 & V_{eL}s_{2\phi}/2\\ 0 & 0 & 0\\ V_{eL}s_{2\phi}/2 & 0 & V_{eL}s_{\phi}^{2} \end{pmatrix},$$
$$\overline{\Lambda} = \begin{pmatrix} -V_{eL}c_{\phi}^{2} - V_{\mu L}(1 + s_{\Phi}^{2}) - \dot{\Phi} & 0\\ 0 & -2V_{\mu L} - \dot{\Phi} & 0\\ -V_{eL}s_{2\phi}/2 & 0 & -V_{eL}'s_{\phi}^{2} - V_{\mu L}(1 + c_{\Phi}^{2}) - \dot{\Phi} \end{pmatrix},$$
$$\mathcal{M} = \begin{pmatrix} B_{\perp} & 0 & 0\\ 0 & B_{\perp} & 0\\ 0 & 0 & B_{\perp} \end{pmatrix} \times$$
$$0 \qquad \mu_{e\mu}c_{\psi}c_{\phi} + \mu_{e\tau}s_{\psi}c_{\phi} + \mu_{\mu\tau}s_{\phi} \qquad \mu_{e\mu}s_{\psi} - \mu_{e\tau}c_{\psi}\\ 0 & 0 & B_{\perp} \end{pmatrix} \times$$

$$\times \begin{pmatrix} 0 & \mu_{e\mu}c_{\psi}c_{\phi} + \mu_{e\tau}s_{\psi}c_{\phi} + \mu_{\mu\tau}s_{\phi} & \mu_{e\mu}s_{\psi} - \mu_{e\tau}c_{\psi} \\ -\mu_{e\mu}c_{\psi}c_{\phi} - \mu_{e\tau}s_{\psi}c_{\phi} - \mu_{\mu\tau}s_{\phi} & 0 & -\mu_{e\mu}c_{\psi}s_{\phi} - \mu_{e\tau}s_{\psi}s_{\phi} + \mu_{\mu\tau}c_{\phi} \\ -\mu_{e\mu}s_{\psi} + \mu_{e\tau}c_{\psi} & \mu_{e\mu}c_{\psi}s_{\phi} + \mu_{e\tau}s_{\psi}s_{\phi} - \mu_{\mu\tau}c_{\phi} & 0 \\ \delta^{ik} = \frac{m_i^2 - m_k^2}{4E}. \end{cases}$$

From Eq.(7) it follows that at such a choice of the basis the resonance conditions will not contain the angle  $\psi$  while the  $\psi$ -dependence will be transported to the resonance widths and the oscillation lengths.

# 3. Neutrino resonance conversions

In this section we shall find all the resonance conversions of the neutrino beam traveling through the region of the coupled sunspots being the source of the solar flares. For the resonance conversion to take place, there is a need to be fulfilled the following demands: (i) the resonance condition must be completed; (ii) the resonance width must be nonzero; (iii) the neutrino beam must travel a distance comparable with the oscillation length. In order to verify the fulfillments of these demands we should make the numerical estimates. With this aim in view, the experimental bounds on the values of the multipole moments should be used.

Considering Majorana three-neutrino mixing and using the solar and KamLAND data [14] along with Rovno [15], TEXONO [16] and MUNU [17] constraints the following results have been obtained

$$\sqrt{|\mu_{12}|^2 + |\mu_{23}|^2 + |\mu_{31}|^2} < 1.8 \times 10^{-10} \mu_B \qquad (90\% \text{ CL}).$$
(8)

Investigating the resonance conversions of the neutrino system we shall infer that the resonance localization places are arranged rather far from one another what enables us to treat them as independent ones. Let us begin with the  $\nu_1^{\prime M} \leftrightarrow \nu_2^{\prime M}$  transitions. In this case the resonance condition, the resonance width and the oscillation length are given by the expressions

$$-2\delta^{12}c_{2\omega} + V_{eL}c_{\phi}^2 = 0, \qquad (9)$$

$$\Gamma(\nu_1^{\prime M} \leftrightarrow \nu_2^{\prime M}) \simeq \frac{\sqrt{2}\delta^{12}s_{2\omega}}{G_F},\tag{10}$$

$$L_{\nu_1'^M \nu_2'^M} = \frac{2\pi}{\sqrt{(2\delta^{12}c_{2\omega} - V_{eL}c_{\phi}^2)^2 + (\delta^{12}s_{2\omega})^2}}.$$
(11)

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Comparing the expressions (9) - (11) with those for the  $\nu_1^{\prime D} \leftrightarrow \nu_2^{\prime D}$  resonance we make sure that they coincide with each other. Note, that from Eq.(11) follows that the oscillation length achieves maximum value at the resonance and the relation

$$\Gamma(\nu_1^{\prime M} \leftrightarrow \nu_2^{\prime M}) = \frac{2\sqrt{2\pi}}{G_F(L_{\nu_1^{\prime M}\nu_2^{\prime M}})_{max}}$$
(12)

takes place. It should be stressed that the connection between the maximum value of the oscillation length and the resonance width holds for any resonance conversion. It is clear that the resonance under question belongs to the kind of the matter-induced resonances. When  $\phi = 0$  then the expressions (9)-(11) convert to the corresponding expressions for  $\nu_{eL} \leftrightarrow \nu_{\mu L}$  resonance conversion found in two flavor approximation (Micheev-Smirnov-Wolfenstein — MSW resonance [18],[19]). As the analysis shows in the Sun conditions this resonance will be fulfilled before the convective zone and, as a result, has no bearing on the solar flares.

Further we shall consider the  $\nu_1^{\prime M} \leftrightarrow \overline{\nu}_2^{\prime M}$  resonance. The resonance condition and the maximum value of the oscillation length are as follows

$$-2\delta^{12}c_{2\omega} + V_{eL}c_{\Phi}^2 + V_{\mu L}(1+s_{\Phi}^2) + \dot{\Phi} = 0$$
(13)

$$(L_{\nu_1'^M \overline{\nu}_2'^M})_{max} \simeq \frac{2\pi}{\mu_{12}^M B_\perp},$$
 (14)

where

$$\mu_{12}^M = \mu_{e\mu}c_{\psi}c_{\phi} + \mu_{e\tau}s_{\psi}c_{\phi} + \mu_{\mu\tau}s_{\phi}$$

From comparing the foregoing expressions with those of the SM [11] one may make the conclusion that the conditions of observing the  $\nu_1^{\prime M} \leftrightarrow \overline{\nu}_2^{\prime M}$  and  $\nu_1^{\prime D} \leftrightarrow \overline{\nu}_2^{\prime D}$  resonances are little different from each other. Then, we may use the results of the work [11] and argue that  $\nu_1^{\prime M} \leftrightarrow \overline{\nu}_2^{\prime M}$  resonance may also occur only at the cost of magnetic field and, as a result, this resonance falls into the kind of the magnetic-induced resonances. The value of  $(\delta^{12})_{min} \simeq 10^{-12}$  eV entering into the resonance condition (13) could be compensated by the twisting frequency  $\dot{\Phi}$  only. For example, at  $B_{\perp} = 10^5$  Gs the value of  $\dot{\Phi}$  being equal to  $-10\pi/L_{mf}$  ensures the existence of the  $\nu_1^{\prime M} \leftrightarrow \overline{\nu}_2^{\prime M}$  resonance.

Setting  $\psi = \phi = 0$  in Eqs.(13),(14), we get the resonance condition and the oscillation length found in the two flavor approximation (FA) for the  $\nu_{eL} \leftrightarrow \overline{\nu}_{\mu R}$  resonance. Therefore, we may argue that the  $\nu_{eL} \leftrightarrow \overline{\nu}_{\mu R}$  resonance is an analog of the  $\nu_1^{'M} \leftrightarrow \overline{\nu}_2^{'M}$  resonance in the two FA.

Since the resonance width of the  $\nu_1^{\prime M} \leftrightarrow \overline{\nu}_1^{\prime M}$  resonance is equal to zero then it is not observed.

Further we consider the  $\nu_1^{\prime M} \leftrightarrow \nu_3^{\prime M}$  resonance. In the Hamiltonian  $\mathcal{H}^{\prime M}$  the quantity  $\Sigma = \delta^{31} + \delta^{32}$  is present. Since it offers the dominant term then the  $\nu_3^{\prime M}$  state is decoupled from the remaining ones. As a result the  $\nu_1^{\prime M} \leftrightarrow \nu_3^{\prime M}$  oscillation controlled by the  $\Sigma$  term could be simply averaged out in the final survival probability for neutrinos of any flavor. For the analogous reasons the  $\nu_1^{\prime M} \leftrightarrow \overline{\nu}_3^{\prime M}$  resonance is not observed too.

For the analogous reasons the  $\nu_1'^M \leftrightarrow \overline{\nu}_3'^M$  resonance is not observed too. Now we proceed to investigation of the resonance conversions of the  $\nu_2'^M$ . It is obvious that the  $\nu_2'^M \rightarrow \nu_3'^M$  transition is forbidden. The same is also true for the  $\nu_2'^M \rightarrow \overline{\nu}_3'^M$  transition. So, we should consider only the  $\nu_2'^M \rightarrow \overline{\nu}_1'^M$  transitions. The resonance condition and the maximal oscillation length are defined by

$$2\delta^{12}c_{2\omega} + V_{eL}c_{\Phi}^2 + V_{\mu L}(1 + s_{\Phi}^2) + \dot{\Phi} = 0, \qquad (15)$$

$$(L_{\nu_2'^M \overline{\nu}_1'^M})_{max} \simeq \frac{2\pi}{(\mu_{e\mu} c_{\psi} c_{\phi} + \mu_{e\tau} s_{\psi} c_{\phi} + \mu_{\mu\tau} s_{\phi}) B_{\perp}}.$$
(16)

Since in the region of the CS's the matter potential is much more less than  $\delta^{12}$  then this resonance may be realized only at the cost of the magnetic field. For example, setting  $B_{\perp} = 10^6 Gs$ , we get  $L_{mf} \simeq (L_{\nu_2'^M \overline{\nu}_1'^M})_{max} \simeq 10^8$  cm at the twist frequency  $3\pi/L_{mf}$ . It is clear that the  $\nu_{\mu L} \leftrightarrow \overline{\nu}_{eR}$  resonance represents the analogs of the  $\nu_2'^M \rightarrow \overline{\nu}_1'^M$  resonance in the two FA.

By virtue of the fact that the resonance widths of the  $\nu_2^{\prime M} \to \overline{\nu}_2^{\prime M}$  and  $\nu_3^{\prime M} \to \overline{\nu}_3^{\prime M}$  resonances are equal to zero these resonances are forbidden.

Let us show that the formulas of the three neutrino generations convert into well known ones of the two FA. Remind since we have assumed the resonance regions are well separated then we could interpret the resonances independently from each other. Next, for our purpose, we need an expression for the probability of oscillatory transitions between two neutrino states. As such, we take the expression which corresponds to the most simple case when  $n_e, n_n, \dot{\Phi}$  are constant

$$\mathcal{P}_{\nu_{\alpha}\leftrightarrow\nu_{\beta}} = \sin^2\theta_{eff}\sin^2\left(\frac{\pi z}{L_{\nu_{\alpha}\nu_{\beta}}}\right),\tag{17}$$

where  $\sin^2 \theta_{eff}$  and  $L_{\nu_{\alpha}\nu_{\beta}}$  are expressed in terms of the elements of the effective Hamiltonian matrix  $\mathcal{H}$  as

$$\sin^2 \theta_{eff} = \frac{4\mathcal{H}_{\alpha\beta}^2}{4\mathcal{H}_{\alpha\beta}^2 + (\mathcal{H}_{\beta\beta}^2 - \mathcal{H}_{\alpha\alpha}^2)^2}, \qquad L_{\nu_{\alpha}\nu_{\beta}} = \frac{2\pi}{\sqrt{4\mathcal{H}_{\alpha\beta}^2 + (\mathcal{H}_{\beta\beta}^2 - \mathcal{H}_{\alpha\alpha}^2)^2}}.$$
 (18)

The probabilities we shall be necessary are as follows

$$\mathcal{P}_{\nu_{1}^{\prime M}\nu_{2}^{\prime M}} = \frac{(2\delta^{12}s_{2\omega})^{2}}{(2\delta^{12}c_{2\omega} - V_{eL}c_{\phi}^{2})^{2} + (2\delta^{12}s_{2\omega})^{2}} \sin^{2} \left\{ \frac{\sqrt{(2\delta^{12}c_{2\omega} - V_{eL}c_{\phi}^{2})^{2} + (2\delta^{12}s_{2\omega})^{2}}}{2} \right\},$$
(19)  
$$\mathcal{P}_{\nu_{1}^{\prime M}\overline{\nu}_{2}^{\prime M}} = \frac{(2\mu_{12}^{M}B_{\perp})^{2}}{[-2\delta^{12}c_{2\omega} + V_{eL}c_{\phi}^{2} + V_{\mu L}(1 + s_{\phi}^{2}) + \dot{\Phi}]^{2} + (2\mu_{12}B_{\perp})^{2}} \times \\ \times \sin^{2} \left\{ \frac{\sqrt{[-2\delta^{12}c_{2\omega} + V_{eL}c_{\phi}^{2} + V_{\mu L}(1 + s_{\phi}^{2}) + \dot{\Phi}]^{2} + (2\mu_{12}^{M}B_{\perp})^{2}}}{2} \right\}.$$
(20)

Let us assume that we determined all the transition probabilities in the bases of the  $\Psi'$  states. Then, taking into account the flavor content of these states, we can find the survival probabilities of interest to us. For the electron neutrinos we obtain

$$\mathcal{P}^{M}_{\nu_{e}\nu_{e}} = 1 - \left\{ c^{2}_{\phi} (\mathcal{P}_{\nu_{1}^{\prime M}\nu_{2}^{\prime M}} + \mathcal{P}_{\nu_{1}^{\prime M}\overline{\nu}_{2}^{\prime M}}) + s^{4}_{\phi} s^{2}_{\psi} \mathcal{P}_{\nu_{1}^{\prime M}\nu_{2}^{\prime M}} + s^{2}_{\phi} s^{2}_{\psi} \mathcal{P}_{\nu_{2}^{\prime M}\overline{\nu}_{1}^{\prime M}} \right\}.$$
(21)

When in (21) we put  $\phi$  and  $\psi$  equal to zero, then, as would be expected, these expressions convert into the electron neutrino survival probability found in two FA

$$\mathcal{P}_{\nu_e\nu_e} = 1 - \left\{ \mathcal{P}_{\nu_{eL}\nu_{\mu L}} + \mathcal{P}_{\nu_{eL}\nu_{\mu R}} \right\}.$$
(22)

Using Eq.(21) we could find the discrepancy between the predictions obtained within the two FA and the three neutrino generations

$$\Delta \mathcal{P}^{M}_{\nu_e \nu_e} = \mathcal{P}^{M}_{\nu_e \nu_e} - \mathcal{P}^{M}_{\nu_e \nu_e} \bigg|_{\psi = \phi = 0}.$$
(23)

### 4. Conclusions

The aim of this work was to investigate the behavior of the neutrino flux in condensed matter and intensive magnetic field within three neutrino generations. The investigations have been accomplished both for the Majorana and for Dirac neutrinos. We have assumed that the neutrinos possess the dipole magnetic moment while the magnetic field has a twisting nature. For the description of the magnetic field twisting the simple model with the geometrical phase  $\Phi(z)$  being equal to  $\exp\{\alpha \pi/L_{mf}\}$  has been used. As the example of the magnetic fields we have considered fields of the coupled sunspots which are the sources of the solar flares. In order to render the results physically more clear we have passed to the basis in which the resonance conditions do not depend on the angle  $\theta_{23}$  while the  $\theta_{23}$ -dependence has been transported on the resonance widths and the oscillation lengths.

All the possible resonance conversions of the neutrinos entering into the beam have been examined. In spite of similar behavior of the Majorana and Dirac neutrinos there is the principal difference between these cases. It lies in the fact that in the Dirac neutrino case all magnetic-induced resonances transfer active (left-handed) neutrinos into the sterile (right-handed) ones while in the Majorana neutrino case he right-handed neutrinos are active. So, if the neutrino exhibits the Majorana nature, then the solar electron neutrino flux moving through the region of the coupled sunspots could be converted into the active right-handed neutrinos ( $\overline{\nu}_{eR}$ ,  $\overline{\nu}_{\mu R}$ ,  $\overline{\nu}_{\tau R}$ ) which could be detected by the terrestrial observer. Emergence of the  $\overline{\nu}_{eL}$  neutrinos, as an example, could be recorded through the inverse  $\beta$ -decay reaction which is at the heart of the antineutrino detectors. As far as the Dirac neutrino case is concerned, the magnetic-induced resonances convert the  $\nu_{eL}$  neutrinos into the sterile  $\nu_{lR}$  neutrinos. Threfore only decreasing the number of  $\nu_{eL}$  could be observed when the solar neutrino flux passes the coupled sunspots region. As regards the phenomena of depleting the solar electron neutrino flux, the observation of decreasing the  $\beta$ -decay rates of some elements during the SF's [20–23] may be speculated to be its experimental confirmation.

It might be well to point out that investigation of the neutrino fluxes which are emitted from the stellar objects will enable us to deduce information not only about such neutrino properties as multipole moment values and their nature (Dirac or Majorana) but about stellar object structure as well.

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