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A SIMPLE ALGORITHM FOR MULTIBEAM PROPAGATION COMPENSATION IN GNSS CONSUMER EQUIPMENT BASED ON A MULTICHANNEL ANTENNA SYSTEM

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Аннотация. Предложен простой алгоритм, обеспечивающий компенсацию отраженных от местных предметов сигналов в аппаратуре потребителей глобальных навигационных спутниковых систем на базе многоканальной антенной системы. Алгоритм включает оценивание временного положения максимума отраженного сигнала на выходе согласованного фильтра, амплитудно-фазового распределения на элементах антенной системы и выборочной корреляционной матрицы процессов на ее выходе по выходным сигналам согласованного фильтра до и в окрестности максимума по заданному числу периодов повторения навигационного сигнала, адаптивную регуляризацию корреляционной матрицы, вычисление вектора весовых коэффициентов и компенсацию отраженного сигнала. Приведены результаты математического моделирования.

Abstract. A simple algorithm is proposed that provides compensation for signals reflected from local objects in the equipment of consumers of global navigation satellite systems based on a multi-channel antenna system. The algorithm includes the estimation of the temporal position of the maximum of the reflected signal at the output of the matched filter, the amplitude-phase distribution on the elements of the antenna system and the sample correlation matrix of the process at its output by output signals of a matched filter and in the region of maximum for a given number of repetition periods of the navigation signal, the adaptive regularization of the correlation matrix, the computation of the weight vector and the compensation of the reflected signal. The results of mathematical modeling are presented.

Problem statement

We consider the equipment of GPS GNSS consumers with a simple (3-5 elements) multi-channel antenna system in conditions of multipath propagation. The cconsumer equipment operates autonomously, the orientation angles of the normal to the antenna system in space are unknown.

The multipath propagation model for a single navigation satellite system is shown in fig. 1. The signal from the navigation satellite reaches the receiving antenna system from two directions: direct from satellite and from a local object (for example, a building). The path difference for these waves is $\Delta d > 0$, complex reflection coefficient $\dot{K} = |K| e^{j\pi}$, where $|K|$ is the reflection coefficient module. It is considered that the phase of the reflected wave changes to π.

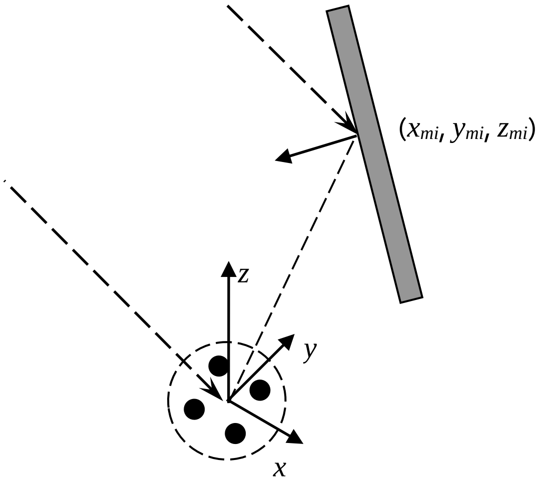


Fig.1. Multipath propagation Model

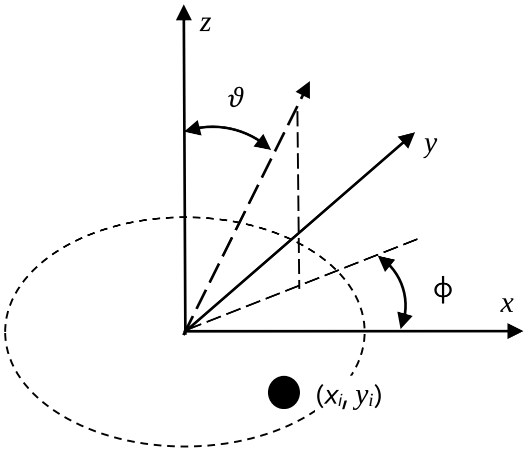
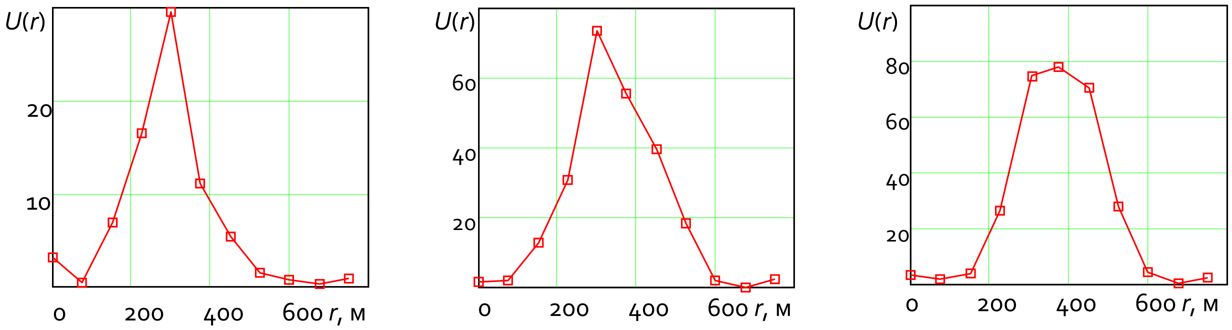


Fig.2. Antenna coordinate system

The angular positions of the forward and reflected signal in the coordinate system of the receiving antenna system are (θ_s, φ_s) and $(\theta_{mi}, \varphi_{mi})$ respectively, where θ is the angle between the normal to the plane of the antenna system and the direction of the signal; φ is the angle between the direction to the source in the plane of the antenna system and the specified axis (fig. 2.). Coordinates of receiving elements of the antenna system are (x_i, y_i), $z_i = 0$. Coordinates of the mirror point are (x_{mi}, y_{mi}, z_{mi}).

The presence of a reflected signal leads to distortion of the pseudo range measurement and deterioration of navigation definitions. An example of distortion of pseudorange's measurements is illustrated in fig. 3, which shows the signals at the output of the matched filter $|K|$ =0 (no reflections), $|K|$=0,5; 0,8 as a function of the range in the vicinity of the maximum. The re-reflection point was 150 m away from the consumer equipment and was at an angle of about 45° relative to the direction of the navigation signal source.



a) $|K|$=0 b) $|K|$=0,5 c) $|K|$=0,8

Fig. 3. View of the signal at the output of the matched filter

As can be seen from the figure, the presence of reflections leads to a shift in the peak of the compressed signal at the output of the matched filter, which can be up to 80...100 meters. When $\Delta d > \frac{c}{2\Delta f_0}$ where Δf_0 is the width of the navigation signal spectrum, the peaks due to the forward and reflected signals will be separated, and the first peak must be selected to evaluate the pseudorange.

The purpose of the work is to substantiate the algorithm for compensation of multipath propagation errors in the Autonomous operation of consumer equipment with a multi - channel receiving system.

Substantiation of the algorithm

Received signal from one of the visible navigation satellites at the input of the i- receiving element can be represented as

$$\dot{Y}_i(t) = \left(A \cdot \dot{S}(t-t_0) e^{j\frac{2\pi}{\lambda}\gamma_i(\theta_s,\varphi_s)} + \dot{K} \cdot A \cdot \dot{S}\left(t - t_0 - \frac{\Delta d}{c}\right) e^{j\frac{2\pi}{\lambda}\Delta d} e^{j\frac{2\pi}{\lambda}\gamma_i(\theta_{mi},\varphi_{mi})} \right) e^{j\beta_0} + N_i(t), \tag{1}$$

where A is the amplitude of the navigation signal; β_0 is the initial phase of the navigation signal determined by the time of passing the route and parameters of the ionosphere; $\dot{S}(t)$ is the complex envelope of the navigation satellite signal; $\gamma_i(\theta, \varphi) = \frac{2\pi}{\lambda}(x_i \sin\theta\cos\varphi + y_i \sin\theta\sin\varphi)$ is the function that determines the difference of the electromagnetic waves coming from the direction of the θ, φ for i^{th} element of the antenna system and the origin (0); $N_i(t)$ is internal noise i^{th} receiving channel. Expression (1) does not include signals from other satellites . Justification of the algorithm: the effect (for this algorithm – additional noise) must be evaluated separately.

Receiving channel signals after matched filtering are the following

$$\dot{U}_i(t) = F_{\text{сф}}(\dot{Y}_i(t)), \tag{2}$$

where $F_{\text{сф}}(\bullet)$ is the matched filtering operator. There are almost coincident highs (peaks) for the channel signals (2) corresponding to the approximate delay time of the reflected signal (figure 3).

Select the observation interval $T_{obs} = M\tau_{ns}$, where τ_{ns} is the period of the navigation signal; M - the number of periods and generate $m = \overline{1,M}$ discrete implementations of the signal at the output of the matched filter, where Δt is the sampling period. It is assumed that the sampling rate is taken with a margin - $F_{\text{S}} = (2...8)\Delta f_0$.

For each implementation, we determine the maximum number in each receiving channel

$$n_{i,m}^{\max} = \arg\max_{n} |\dot{U}_{i,m}(n \cdot \Delta t)|, \tag{3}$$

where n is the reference number for the delay time within one period of repetition of the navigation signal. Averaging it over the receiving channels are the following:

$$n_m^{\max} = \frac{1}{I}\sum_{i=1}^{I} n_{i,m}^{\max} . \tag{4}$$

where *I* is number of receiving channels.

Knowing the position of the maxima in each repetition period, we find:

- estimation of the amplitude-phase distribution of the direct signal on the elements of the antenna system

$$\mathbf{f} = \frac{1}{M \cdot L}\sum_{m=1}^{M}\sum_{\ell=1}^{L}\frac{\mathbf{u}_{m,\ell}}{\dot{U}_{1,m}((n_m^{\max} - \ell)\cdot\Delta t)}, \tag{5}$$

where L is the number of samples used for evaluation;

$\mathbf{u}_{m,\ell} = \begin{pmatrix} \dot{U}_{1,m}((n_{\max} - \ell)\cdot\Delta t) \\ ... \\ \dot{U}_{I,\text{m}}((n_{\max} - \ell)\cdot\Delta t) \end{pmatrix}$ - the vector of samples of complex amplitudes of signals at the outputs of the receiving channels for the observation number m and sample number ℓ.

Normalization in (5) is used to eliminate the initial phase of the navigation signal;

- estimation of the correlation matrix of processes on the output of the antenna system of receiving channels [1, 2]

$$\widehat{\mathbf{\Phi}} = \frac{1}{ML}\sum_{m=1}^{M}\sum_{\ell=-L}^{L}\mathbf{u}_{m,\ell}\mathbf{u}_{m,\ell}^{+}, \tag{6}$$

where the " + " sign in uppercase indicates the Hermit conjugation operation (complex conjugation and transposition of a matrix or vector).

Considering the small sample size we perform regularization [2-4] of the sample correlation matrix

$$\mathbf{\Phi}_{\text{p}} = \widehat{\mathbf{\Phi}} + \mu\mathbf{E}, \tag{7}$$

where μ is the regularization parameter; $\mathbf{E}$ is the unit matrix.

The parameter can be selected as $\mu \approx \frac{1}{I} tr(\widehat{\mathbf{\Phi}})$, where $tr(\bullet)$ is the operator for calculating the trace (the sum of elements on the main diagonal) of the matrix.

Let's calculate the vectors weight coefficients of processing

$$\boldsymbol{\omega} = \mathbf{\Phi}_{\text{p}}^{-1}\mathbf{f} \tag{8}$$

and form discrete compensated accepted implementations

$$\dot{Z}_{n,m} = \boldsymbol{\omega}^{+}\mathbf{u}_{m,n},\ \mathbf{u}_{m,n} = \begin{pmatrix} \dot{U}_{i,m}(n\cdot\Delta t) \\ ... \\ \dot{U}_{I,\text{m}}(n\cdot\Delta t) \end{pmatrix}. \tag{9}$$

Further processing are the measurement and filtering of the pseudo range or using the classic system with two correlates, which is implemented in a "belated" in relation to real time.

The modeling technique

The algorithm is well suited for implementation in an SDR receiver and is simple for mathematical modeling. When performing mathematical modeling, the source of the navigation signal, the consumer equipment, and the mirror reflection point were assumed to be stationary.

The simulation sequence is shown below.

1. By setting the coordinates of the mirror point (x_{mi}, y_{mi}, z_{mi}) and the direction of arrival of the navigation signal (θ_s, φ_s), calculate Δd and $(\theta_{mi}, \varphi_{mi})$. It's necessary to set the delay time of the navigation signal.

2. Reproduced the $m = \overline{1, M}$ independent discrete implementations $\dot{Y}_{i,m}(n \cdot \Delta t)$ of input and $\dot{U}_{i,m}(n \cdot \Delta t)$ output signals of matched filters in each of the channels of the receiving system according to (1), (2). The amplitude of the navigation signal in the absence of multipath propagation are selected in accordance with signal-to-noise ratio for the GPS receiver (10...15 dB). a pair of Fourier transforms were used to calculate the output signal of the matched filter.

3. The average maximum index was determined according to (3), (4).

4. Processing was implemented in accordance with (5)-(8).

5. Using the adapted channel signals $\dot{Z}_{n,m}$, determined the integer value of the maximum index $n_m^{\max} = \arg\max_n |\dot{Z}_{n.m}|$ and its refined (fractional) value $\left\{n_m^{\max}\right\}$ in each repetition period . When specifying the maximum position, the signal envelope at the output of the matched filter in the vicinity of the maximum is approximated by a parabola $|Z(n)| = a \cdot n^2 + b \cdot n + c$, whose coefficients are the following

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} (n_m^{\max} - 1)^2 & (n_m^{\max})^2 & 1 \\ (n_m^{\max})^2 & n_m^{\max} & 1 \\ (n_m^{\max} + 1)^2 & n_m^{\max} + 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} |\dot{Z}_{n_m^{\max} - 1, m}| \\ |\dot{Z}_{n_m^{\max}, m}| \\ |\dot{Z}_{n_m^{\max} + 1, m}| \end{pmatrix}, \tag{10}$$

and

$$\left\{n_m^{\max}\right\} = -\frac{b}{2a}. \tag{11}$$

To find the error of estimation of pseudo range

$$\delta t_m^a = c \cdot \left(\left\{n_m^{\max}\right\} \Delta t - (t_0 + t_F) \right), \tag{12}$$

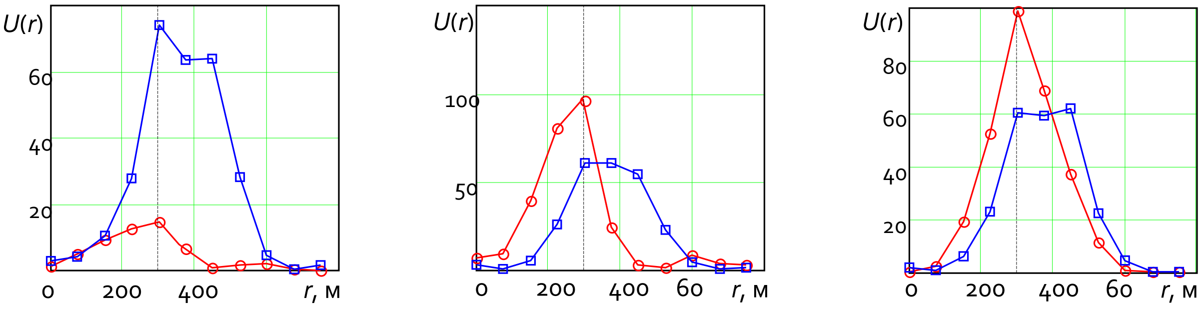
where t_F is the delay constant in the matched filter; c is the speed of light.

6. Get statistical characteristics (mathematical expectation, variance, sample probability density) of errors δt^a and δt^0 for a set of implementations .

Some results of the simulation

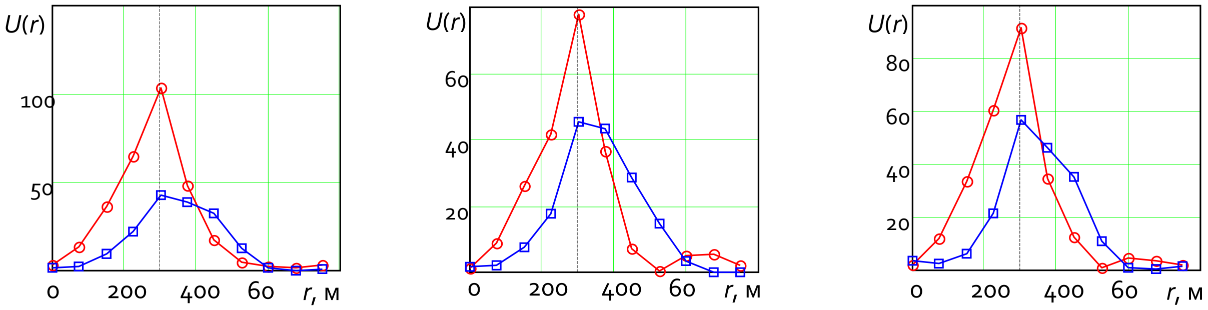
The simulation was performed for the civilian signal of the GPS system at the distance of the reflection point at Δd =150 m. The signal-to-noise ratio at the output of the matched filter for the direct signal was 13 dB. The consumer equipment antenna system consisted of a three-element array of patch antennas located at the vertices of an equilateral triangle with a side length of 0.1 m. The signal-to-noise ratio at the output of the matched filter for the direct signal at the outputs of the receiving channels was 13 dB. The formation of a sample correlation matrix was carried out for M =10...100 periods of repetition of the navigation signal. The sampling rate was 4 MHz, parameter L =2.

Fig. 4 for M =10; 50 and 100 and $|K|$=0.8 shows the results of processing in the absence of (μ=0) regularization of the sample correlation matrix in accordance with (7). Figures 5 ($|K|$=0.5) and 6 ($|K|$=0.8) for M =10 show three typical signal implementations at the output of the processing system in the presence (round markers) and absence (square matrices) of multipath compensation and regularization (7). the dotted line shows the range (rounded by an integer number of sampling intervals) at which the signal peak should be located in the absence of interference.



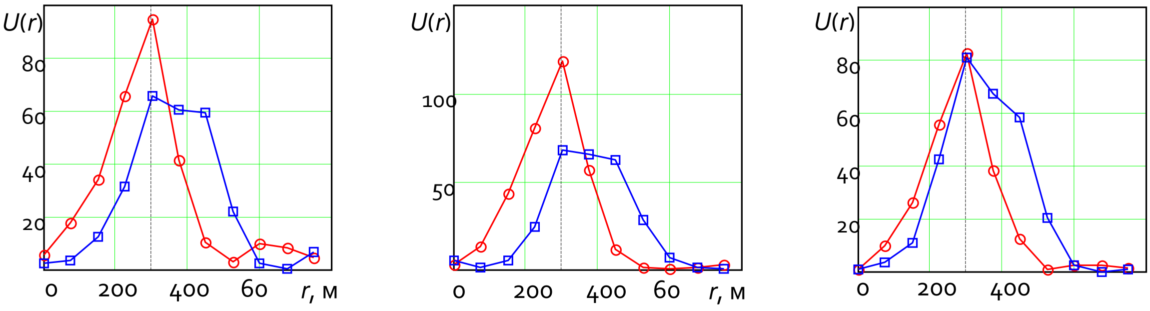
a) *M*=10 b) *M*=50 c) *M*=100

Fig. 4. Implementation of signals at the output of the processing system at $|K|$=0,8 and no regularization



a) b) c)

Fig. 5. Implementation of signals at the output of the processing system at $|K|$=0.5 and the presence of regularization



a) b) c)

Fig. 6. Implementation of signals at the output of the processing system at $|K|$=0.8 and the presence of regularization

As follows from the results shown in fig. 4-6, the proposed algorithm provides a high degree of multipath compensation: the peak of the signal after compensation in all cases is located in a small neighborhood from the true value. Operation (7) of regularization of the sample correlation matrix is fundamental: in its absence, due to a sharp increase in the norm of weight coefficients for an unclassified sample [3, 4], the algorithm's characteristics deteriorate to the point of complete loss of performance, or a significant increase in the sample size is required, up to 10 times, with a corresponding complication of the equipment and possible deterioration of characteristics when the consumer's equipment moves. In this regard, the proposed algorithm is simpler than [5], where the cyclostationarity of the correlation matrix at the output of matched filters of receiving channels is also used as one of the processing ideas, but a complex projection algorithm is used to compensate for external noise interference and spoofing.

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