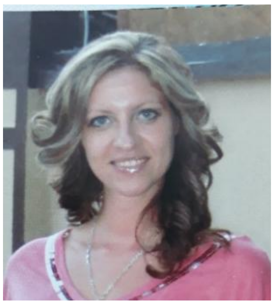
УДК 621.865.8:629.055

# MOTION CONTROL OF MOBILE ROBOT WITH DIFFERENTIAL DRIVE AND TWO CASTOR WHEELS


***Yang Shixin***
*PhD student of the Department of «Electric drive and automation of industrial installations and technological complexes» of BNTU*
*yang_shi_xin@163.com*

***A.A. Velchenko***
*Associate Professor of the Department of «Electric drive and automation of industrial installations and technological complexes» of BNTU, PhD of Technical Sciences, Associate Professor*
*anna.velchenko@gmail.com*

***S.A. Pauliukavets***
*Head of the Department of «Electric drive and automation of industrial installations and technological complexes» of BNTU, PhD of Technical Sciences, Associate Professor*
*sap@bntu.by*
*s.pauliukavets@gmail.com*

***S.A. Pauliukavets***
*PhD in Technical Sciences, Associate Professor, Head of the of the «Electric Drive and Automation of Industrial Installations and Technological Complexes» Department of the Belarusian National Technical University. The area of scientific interests is related to the development of methods and algorithms for controlling mobile robots.*

***A.A. Velchenko***
*PhD in Technical Sciences, Associate Professor of the «Electric Drive and Automation of Industrial Installations and Technological Complexes» Department of the Belarusian National Technical University. The area of scientific interests is related to the development and research of neural network controllers for electric drives.*

***Yang Shixin***
*PhD student at the Belarusian National Technical University. The area of scientific interests is related to the study of control algorithms for wheeled mobile robots.*

**Abstract.** This article presents a mathematical model of a motion control system for a nonholonomic mobile robot, an independent suspension with two independent drive wheels and two castor wheels. A model that takes into account the position of a wheeled robot relative to a given path, using transfer design function devices, which allows the use of methods of both classical and modern control theories for the synthesis of control devices.
**Keywords**: mobile robot, mathematical model, castor wheel, kinematics, control system, motion trajectory.

**Introduction.** The differential mobile robot is simple in structure, low in cost, relatively easy to analyze and establish the kinematics model, and currently has many applications in the market [1], so this paper chooses the differential mobile robot as the research object. The system input of the differential robot is only the speed of the left and right driving wheels, and the forward, backward, turning and other actions are realized through the speed difference. Usually, in order to improve the balance of the robot, a universal wheel is installed in the front, which only plays a supporting role and does not provide power [2]. The establishment of the robot motion model is a process from bottom to top. The motion of the robot is realized through the action of the wheels, but the wheels also constrain the motion of the robot. Multiple wheels are connected together through a mechanical structure to form a robot chassis. Therefore, the constraints of each wheel are combined to generate motion constraints on the robot chassis. At this time [3], it is necessary to analyze the kinematics model of the robot.

To improve the design of robots and reduce costs, it is necessary to research and use new, stronger and lighter materials, develop fundamentally new designs, and optimize robot control algorithms. Optimizing a robot control algorithm or developing a new algorithm involves studying the functioning of the robot using a mathematical model. When developing mathematical models, special attention should be paid to modeling the external conditions in which the robot is located, i.e. modeling the load on its drive motors. Many mathematical models have been developed that describe the electrical and mechanical parts of electric drives of mobile robots and other mechanisms [4–6]. This paper sets the task of controlling the movement of a mobile robot based on its mathematical model. When developing a mathematical model, it is necessary to take into account the external conditions in which the robot is located and moves.

**Difference robot coordinate transformation.** There are multiple coordinate systems in the robot system, including the world coordinate system ($X_W O_W Y_W$), robot coordinate system ($X_R O_R Y_R$), Sensor coordinate system ($X_S O_S Y_S$), As shown in Figure 1. The environment in which the robot moves is called the world coordinate system, which is fixed and used to describe the global information of the robot. The origin of the robot coordinate system is the midpoint of the two driving wheels, and the instantaneous orientation of the robot is the positive direction of the coordinate axis. When the robot calculates the turning radius, it also calculates the distance from the instantaneous center of rotation to the midpoint of the driving wheels. The robot coordinate system is used to describe the robot's own information, and it is also the conversion medium from the sensor coordinate system to the world coordinate system. The information collected by the sensor is based on the sensor coordinate system. For example, the data information detected by the lidar is based on the lidar coordinate system.

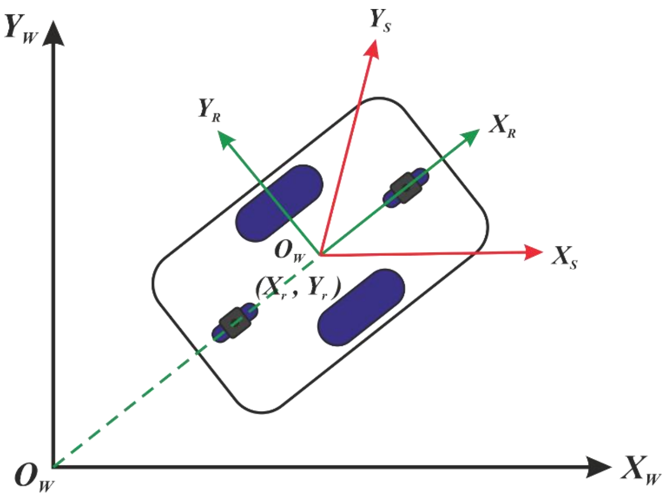



*Figure 1.* Schematic diagram of chassis movement of differential mobile robot

The origin position and orientation of the robot coordinate system and the sensor coordinate system change with the movement of the robot, so the robot posture and sensor data information cannot be directly applied, but should be changed to the world coordinate system. The transformation from the robot coordinate system to the world coordinate system is related to the pose of the robot at the current moment. The sensor is fixed on the robot through a mechanical structure, so the transformation relationship from the sensor coordinate system to the robot coordinate system is determined by the installation position of the sensor.

The origin of the robot coordinate system $X_R O_R Y_R$ is $O_R$. The pose in the world coordinate system $X_W O_W Y_W$ is $P_{WR}(x_r, y_r, \theta_r)$. The conversion matrix from the robot coordinate system $X_R O_R Y_R$ to the world coordinate system $X_W O_W Y_W$ is:

$$T_{WR} = \begin{bmatrix} \cos(\theta_r) & -\sin(\theta_r) & x_r \\ \sin(\theta_r) & \cos(\theta_r) & y_r \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \quad (1)$$

Of which:

$$R = \begin{bmatrix} \cos(\theta_r) & -\sin(\theta_r) \\ \sin(\theta_r) & \cos(\theta_r) \end{bmatrix} \quad (2)$$

$$t = \begin{bmatrix} x_r \\ y_r \end{bmatrix} \quad (3)$$

The pose of the origin of the sensor coordinate system in the robot coordinate system is $P_{RS} = (x_{rs}, y_{rs}, \theta_{rs})$. The coordinates in the world coordinate system are as follows:

$$P_{WS} = P_{RS} T_{WR} = (x_{rs}, y_{rs}, \theta_{rs}) \cdot \begin{bmatrix} \cos(\theta_r) & -\sin(\theta_r) & x_r \\ \sin(\theta_r) & \cos(\theta_r) & y_r \\ 0 & 0 & 1 \end{bmatrix} = (x_{ws}, y_{ws}, \theta_{ws}) \quad (4)$$

From this, the conversion matrix of the sensor coordinate system $X_S O_S Y_S$ to the world coordinate system $X_W O_W Y_W$ can be obtained:

$$T_{WS} = \begin{bmatrix} \cos(\theta_{ws}) & -\sin(\theta_{ws}) & x_{ws} \\ \sin(\theta_{ws}) & \cos(\theta_{ws}) & y_{ws} \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

**Robot kinematics model.** The pose of the robot in the environment is described as $(x, y, \theta)$. There are three degrees of freedom. The input of the system is only the speed of the left and right wheels $(v_L, v_R)$. So the differential mobile robot is an underactuated system, and its motion is coupled. Figure 2 is a schematic diagram of the chassis motion of the differential mobile robot studied in this paper.

What is known in the above figure is the movement speed of the left and right wheels $v_L, v_R$. It can be calculated by the photoelectric encoder. The distance between the centers of the two driving wheels is 2d, which has been determined during installation and can be obtained by measurement. Other information of the robot can be deduced from the known information through the establishment of the kinematics model. Assume that the linear and angular velocities of the robot chassis are $v, \omega$. The angular velocity of the left and right wheels is $\omega_L, \omega_R$. The instantaneous turning radius of the robot when moving is r. The movement of the differential mobile robot at any moment can be regarded as a circular arc movement. The entire robot system is a rigid body connected uniformly. When the robot moves, the angular velocity of the left and right wheels is equal to the overall angular velocity of the robot.

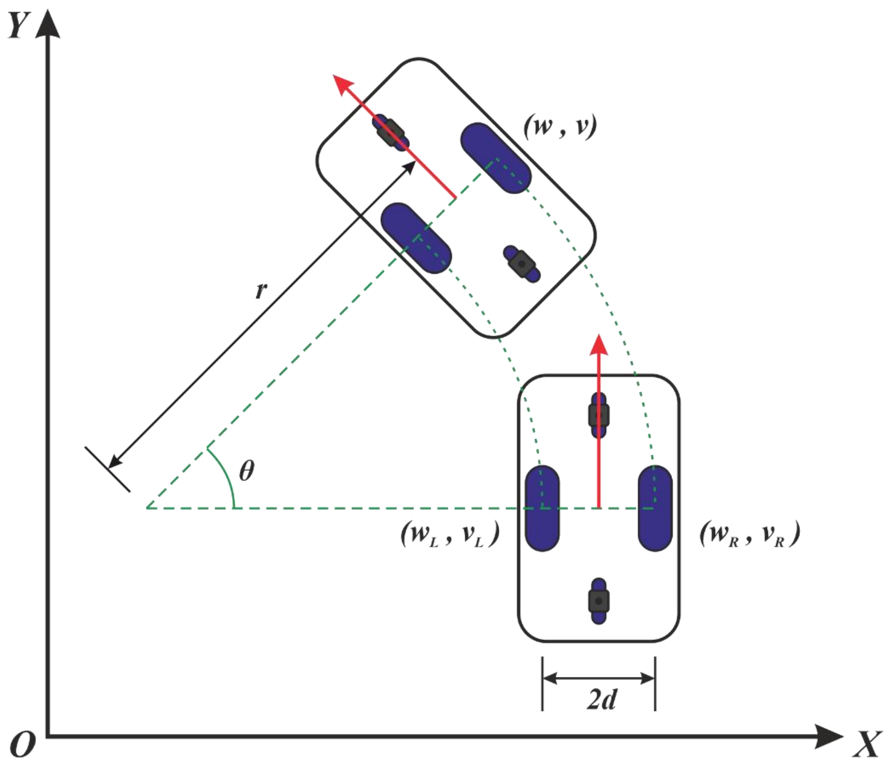



*Figure 2.* Schematic diagram of differential chassis movement

$$\omega = \omega_L = \omega_R \tag{6}$$

From the relationship between angular velocity and linear velocity, we can get:

$$\frac{v_L}{r-d} = \frac{v_R}{r+d}$$

$$v_L(r+d) = v_R(r-d) \tag{7}$$

$$r(v_R - v_L) = (v_R + v_L)d$$

So the formula for the instantaneous radius of rotation is as follows:

$$r = \frac{(v_R + v_L)d}{v_R - v_L} \tag{8}$$

Calculated from the angular velocity formula:

$$\omega = \frac{v_R}{r+d} \tag{9}$$

$$r + d = \frac{(v_R + v_L)d}{v_R - v_L} + d = 2\frac{v_R d}{v_R - v_L} \tag{10}$$

Substitute formula (10) into formula (9) to get the angular velocity formula:

$$\omega = \frac{v_R - v_L}{2d} \tag{11}$$

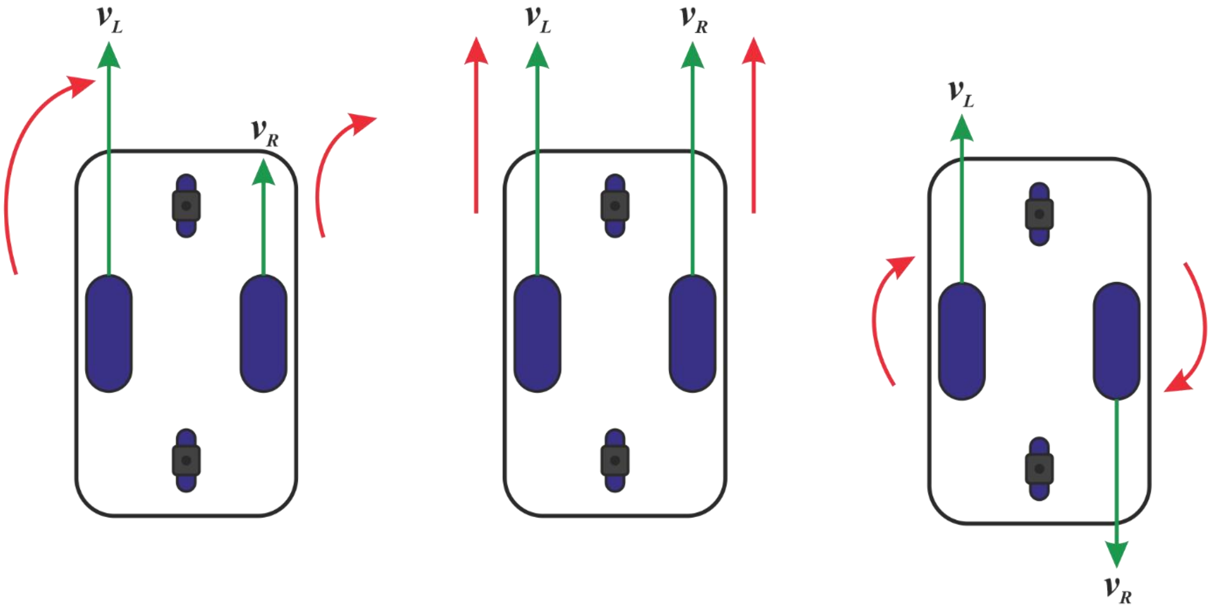
Linear speed of differential robot chassis:

$$v = \omega * r = \frac{v_R - v_L}{2d}\frac{(v_R + v_L)d}{v_R - v_L} = \frac{v_R + v_L}{2} \tag{12}$$

The kinematics model is expressed in matrix form as follows:

$$\begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{-1}{2d} & \frac{1}{2d} \end{bmatrix} \begin{bmatrix} v_L \\ v_R \end{bmatrix} \tag{13}$$

The differential drive robot determines three different motion states according to the speed difference between the left and right wheels, as shown in Figure 3.




(a) turn right (b) linear motion (c) rotate in place

*Figure 3.* Three motion states of the differential robot

When $v_L > v_R$, make a circular motion and turn right.
When $v_L < v_R$, true life.
When $v_L = v_R$, make a straight line motion.
When $v_L =- v_R$, rotate in situ with the center point of the left and right wheels.

**Conclusion.** The control of a nonholonomic mobile robot with a differential drive and two castor wheels along a reference curve is presented, where the controller consists of two parts: feedforward control and feedback control. The former uses robot inverse kinematics to calculate feedforward inputs based on a reference curve, while the latter removes the effects of noise, disturbances, and initial state errors. The presented example proves that a mobile robot can follow a desired reference path according to a given speed profile with satisfactory accuracy. Thus, the presented model is useful for a preliminary study of the designed motion control laws of a mobile robot.

**References**

[1] Berman S., Schechtman E., Edan Y. Evaluation of Automatic Guided Vehicle Systems // Robotics and Computer-Integrated Manufacturing. 2008. doi: 10.1016/j.rcim.2008.02.009.

[2] Siegwart R., Nourbakhsh I. R., Scaramuzza D. Introduction to autonomous mobile robots. Publishing house of Massachusetts Institute of Technology, 2004. 321 p.

[3] Xiang Yuyun. Research on indoor robot navigation control method based on SLAM [D]. Northwest A&F University,2021.

[4] R. X. Xu. Path planning of mobile robot based on multi-sensor information fusion[J]. EURASIP Journal on Wireless Communications and Networking, 2019,2019(1): 1-8.

[5] Zeng Wente, Su Jianbo. A network robot system based on distributed intelligence [J]. Robotics,2009.

[6] Tian Ye, Chen Hongwei, Wang Fasheng, et al. A Review of SLAM Algorithms for Indoor Mobile Robots J. Computer Science, 2021,48(09):223-234.

**Авторский вклад**

Все авторы внесли равноценный вклад в написании статьи.

# УПРАВЛЕНИЕ ДВИЖЕНИЕМ МОБИЛЬНОГО РОБОТА С ДИФФЕРЕНЦИАЛЬНЫМ ПРИВОДОМ И ДВУМЯ ПОВОРОТНЫМИ КОЛЕСАМИ

***Ян Шисинь***
*Аспирант кафедры «Электропривод и автоматизация промышленных установок и технологических комплексов» БНТУ*

***А.А. Вельченко***
*Доцент кафедры «Электропривод и автоматизация промышленных установок и технологических комплексов» БНТУ, кандидат технических наук, доцент*

***С.А. Павлюковец***
*Заведующий кафедры «Электропривод и автоматизация промышленных установок и технологических комплексов» БНТУ, кандидат технических наук, доцент*

**Аннотация.** В данной статье представлена математическая модель системы управления движением неголономным мобильным роботом, имеющего шасси с двумя независимыми ведущими колесами и двумя кастор колесами. Модель, учитывающая положение колесного мобильного робота относительно заданного пути, построена с использованием аппарата передаточных функций, что позволяет применять методы как классической, так и современной теории управления для синтеза устройств управления.

**Ключевые слова:** мобильный робот, математическая модель, поворотное колесо, кинематика, система управления, траектория движения.