Towards the Theory of Semantic Space

Valerian Ivashenko
Department of Intelligent Information Technologies
Belarusion State University of Informatics and Radioelectronics
Minsk, Republic of Belarus
ivashenko@bsuir.by

***Abstract*—The paper considers models for investigating the structure, topology and metric features of a semantic space using unified knowledge representation.**

The classes of finite structures corresponding to ontological structures and sets of classical and non-classical kinds are considered, and the enumerability properties of these classes are investigated.

The notion of operational-information space as a model for investigating the interrelation of operational semantics of ontological structures of large and small step is proposed.

Quantitative features and invariants of ontological structures oriented to the solution of knowledge management problems are considered.

***Keywords*—Semantic Space, Neg-entropy, Operational-information space, Enumerable sets, Natural numbers, Ackermann coding, Generalized formal language, Enumerable self-founded Hereditarily finite sets, Countable non-identically-equal Hereditarily finite sets, Multigraph, Hypergraph, Metagraph, Orgraph, Unoriented graph, Quasi-metric, Orgraph invariant, Homomorphism, Isomorphism, Homeomorhpism, Oriented sets, Graph wave-front, Dynamic graph system, Receptor, Effector, Transmitter, Resonator, Graph dimension, Fully-connected orgraph period, Rado graph, Universal model, Stable structure, Operational semantics, Denotational Semantics, Infinite structures, Generalized Kleene closure**

I. Introduction

There are different approaches to the study of topological, metrical and other properties of signs in texts leading to the consideration of corresponding semantic (or meaning (sense)) spaces [56].

Space is convenient because it is connected with some ordinal or metric scale which allows to reduce the cost of solving such cognitive tasks as searching (synthesis) or checking (analysis) the presence of an element (including for the purpose of eliminating redundancy) in a set organized as a space.

Knowledge integration based on unification is necessary both to eliminate redundancy and to compute semantic metrics. For this purpose, the developed model of unified knowledge representation [1], [5] can be adopted.

II. Approaches to the construction of a meaning space

The history of the development of the concept of "meaning space" and the corresponding models are described in the works [2], [11], [32], [56].

As stated in [56], the main approaches to the construction and research of the organization of meaning space include:

- exterior studying the physical nature [30], [33], [48] of processes including thinking processes [29],
- (quantitative) interior using quantitative and soft models, including probabilistic description of processes [11], [34], [35], [42], based on the practice of using words of language [20], [53],
- (qualitative) interior investigating the structure of represented knowledge and its dynamics [12], using formal semiotic models [51].

In some cases, it is possible to combine these elements of these approaches.

The following models and methods are used to construct and investigate the semantic space:

- mathematical models of spaces [37]–[41], [43], [44],
- formal and generalized formal languages [45], [56],
- methods of probability theory [11], [36], [54],
- methods of formal concepts analysis [58], [59],
- other models [3], [4], [45], [46], [49].

Further in the paper we consider the main classes of structures, their attributes and corresponding types of subspaces of the semantic space using unified knowledge representation [5], [12].

III. Unified representation and classification of fully representable finite knowledge structures

At the level of syntax, using syntactic links, it is possible to represent only finite knowledge structures in a unified (explicit) way.

Let us consider the principles of unified representation of knowledge [5], [12] with a structure that is one of finite structures of different kinds. Let us compare a certain class of structures to each kind of finite knowledge structures.

Note that finite structures can be divided into two main types: oriented finite structures and unoriented finite structures [21].

The simplest unoriented finite structures are hereditarily finite sets [63]. The class of hereditarily finite sets can be expressed as follows:

$$\emptyset^{(+^{*}_{1})} = H_{\aleph_0}$$

where

$$\binom{0}{A} \overset{def}{=} 2^{\emptyset} = \{\emptyset\}$$

$$\binom{1}{A} \overset{def}{=} \left(\bigcup_{x \in A} 2^{\{x\}}\right) / \binom{0}{A}$$

$$\binom{\iota + 1}{A} \overset{def}{=} \left(\binom{\iota}{A} \breve{\cup} \binom{1}{A}\right) / \binom{\iota}{A}$$

$$A \breve{\cup} B \overset{def}{=} \bigcup_{\langle P, Q \rangle \in A \times B} \{P \cup Q\}$$

$$2^{\left(\emptyset + \sum_{x \in A} \{x\}\right)} \overset{def}{=} \bigcup_{\iota \in \mathbb{N} \cup \{0\}} \binom{\iota}{A}$$

$$A^{\left(+_k^1\right)} \overset{def}{=} \tau_k \left(\rho_k \left(\langle A, A \rangle\right) \cup \sigma_k \left(\langle A, A \rangle\right)\right)$$

$$A^{\left(+_k^{\iota+1}\right)} \overset{def}{=} \tau_k \left(\rho_k \left(\left\langle A, A^{\left(+_k^{\iota}\right)} \right\rangle\right) \cup \sigma_k \left(\left\langle A, A^{\left(+_k^{\iota}\right)} \right\rangle\right)\right)$$

$$A^{\left(+_k^{*}\right)} \overset{def}{=} \bigcup_{\iota \in \mathbb{N} / \{0\}} A^{\left(+_k^{\iota}\right)}$$

$$\tau_1 \left(A\right) \overset{def}{=} A$$

$$\rho_1 \left(\langle A, B \rangle\right) \overset{def}{=} \emptyset$$

$$\sigma_1 \left(\langle A, B \rangle\right) \overset{def}{=} 2^{\emptyset + \sum_{x \in (A \cup B)} \{x\}}$$

According to Ackermann coding [62], all hereditarily finite sets can be a mutually uniquely matched to natural numbers and thus enumerated [27]:

$$f\left(S\right) = 0 + \sum_{x \in S} 2^{f(x)}$$

A generalization of the class of hereditarily finite sets is the class of generalized hereditarily finite sets.

$$A^{\left(+_1^{*}\right)}$$

Generalized hereditarily finite sets can be embedded in (classical non-generalized) hereditarily finite sets:

$$\emptyset \sim 2^{\emptyset}$$

$$a_k \sim 2^{\{\emptyset\}_k}$$

$$g\left(\emptyset\right) = \{\emptyset\}$$

$$g\left(a_k\right) = \left\{\{\emptyset\}_k, \emptyset\right\}$$

$$g\left(X\right) = \{g\left(x\right) | x \in X\}$$

or alternatively:

$$\emptyset \sim d\left(1\right) = \{\emptyset\}$$

$$a_k \sim d\left(2 * k + 1\right)$$

$$d\left(k\right) = \bigcup_{i=1}^{\lfloor \log_2 k \rfloor} \left\{ d\left(\left\lfloor \frac{k}{2^i} \right\rfloor mod 2\right) \right\}$$

$$d\left(0\right) = \emptyset$$

$$d\left(1\right) = \{\emptyset\}$$

$$d\left(2\right) = \{\{\emptyset\}\}$$

$$d\left(3\right) = \{\{\emptyset\}, \emptyset\}$$

$$d\left(4\right) = \{\{\{\emptyset\}\}\}$$

$$d\left(5\right) = \{\{\{\emptyset\}\}, \emptyset\}$$

$$d\left(6\right) = \{\{\{\emptyset\}\}, \{\emptyset\}\}$$

$$d\left(7\right) = \{\{\{\emptyset\}\}, \{\emptyset\}, \emptyset\}$$

$$d\left(8\right) = \{\{\{\emptyset\}, \emptyset\}\}$$

$$d\left(9\right) = \{\{\{\emptyset\}, \emptyset\}, \emptyset\}$$

$$d\left(10\right) = \{\{\{\emptyset\}, \emptyset\}, \{\emptyset\}\}$$

$$d\left(11\right) = \{\{\{\emptyset\}, \emptyset\}, \{\emptyset\}, \emptyset\}$$

$$d\left(12\right) = \{\{\{\emptyset\}, \emptyset\}, \{\{\emptyset\}\}\}$$

$$d\left(13\right) = \{\{\{\emptyset\}, \emptyset\}, \{\{\emptyset\}\}, \emptyset\}$$

$$d\left(14\right) = \{\{\{\emptyset\}, \emptyset\}, \{\{\emptyset\}\}, \{\emptyset\}\}$$

$$d\left(15\right) = \{\{\{\emptyset\}, \emptyset\}, \{\{\emptyset\}\}, \{\emptyset\}, \emptyset\}$$

$$d\left(16\right) = \{\{\{\{\emptyset\}\}\}\}$$

$$\ldots$$

$$g\left(\emptyset\right) = \{\emptyset\}$$

$$g\left(a_k\right) = d\left(2 * k + 1\right)$$

$$g\left(X\right) = \{g\left(x\right) | x \in X\}$$

In this way we obtain an ordering of generalized hereditarily finite sets (as example) in accordance with the Ackermann numbering and embedding in hereditarily finite (unoriented) sets.

As for oriented structures (oriented, "ordered" sets), if we take the von Neumann-Bernays-Gödel axiomatics [63] as a basis then with some "traditional" approach (representation of oriented pairs according to K. Kuratowski) [24] an empty string [10], [13], [14], an empty an oriented set [22] cannot be represented as unfounded sets in a theory with the von Neumann-Bernays-Gödel axiomatics [26], [63].

Accepted:

$$x = \langle x \rangle$$

in this case, the oriented pair of K. Kuratovsky:

$$\langle x, y \rangle = \{\{x\}, \{x, y\}\}$$

also

$$\langle x_1, x_2, x_3 \rangle = \langle \langle x_1, x_2 \rangle, x_3 \rangle$$

$$\langle x_1, x_2, x_3, x_4 \rangle = \langle \langle x_1, x_2, x_3 \rangle, x_4 \rangle$$

$$\langle x_1, x_2, x_3, x_4, x_5 \rangle = \langle \langle x_1, x_2, x_3, x_4 \rangle, x_5 \rangle$$

$$\langle x_1, x_2, x_3, x_4, x_5, x_6 \rangle = \langle \langle x_1, x_2, x_3, x_4, x_5 \rangle, x_6 \rangle$$

$$\langle x_1, x_2, ..., x_i, ..., x_{n-1}, x_n \rangle = \langle \langle x_1, x_2, ..., x_i, ..., x_{n-1} \rangle, x_n \rangle$$

$$A^n = \{\langle x_1, x_2, ..., x_i, ..., x_{n-1}, x_n \rangle | x_i \in A\}$$

The consequence of this is that strings are conditionally dimensional, that is, the length of a string is not its function, and therefore cannot be calculated uniquely from a string; an empty string cannot be represented by a set in the von Neumann-Bernays-Gödel theory:

$$\langle x, x \rangle = \{\{x\}\} = \langle \{x\} \rangle$$

$$\langle x, x \rangle = \langle \{x\} \rangle$$

$$2 = length\,(\langle x, x \rangle) \neq length\,(x) = 1$$

$$n = length\,(\langle x_1, x_2, ..., x_i, ..., x_{n-1}, x_n \rangle) \neq$$

$$length\,(\langle x_1, x_2, ..., x_i, ..., x_{n-1}, x_n \rangle) = 2$$

During understanding the string length function, if we move from a function (as in the formulas above) to a higher-order function with respect to the set of elements of an oriented set this does not solve the problem:

$$2 = length\,(\{\langle x, x \rangle, \{x\}\})\,(\langle x, x \rangle) \neq$$

$$length\,(\{\langle x, x \rangle, \{x\}\})\,(\langle \{x\} \rangle) = 1$$

Another consequence of this is that the Cartesian power can exhibit the following non-obvious and non-intuitive properties:

$$\exists A \left(A = A^1 \supset A^2 \supset A^3 \supset ... \supset A^i \supset ... \right)$$

Inability to represent the empty string ε when representing strings as oriented sets.

Let be:

$$E = \{\varepsilon\}$$

Required:

$$E^n = E$$

We have:

$$E^2 = \{\langle \varepsilon, \varepsilon \rangle\} = \{\{\{\varepsilon\}\}\}$$

$$\varepsilon = \{\{\varepsilon\}\}$$

$$E^3 = \{\langle \varepsilon, \varepsilon, \varepsilon \rangle\} = \{\langle \langle \varepsilon, \varepsilon \rangle, \varepsilon \rangle\} = \{\{\{\langle \varepsilon, \varepsilon \rangle\}, \{\langle \varepsilon, \varepsilon \rangle, \varepsilon\}\}\}$$

$$E^3 = \{\{\{\{\{\varepsilon\}\}\}, \{\{\{\varepsilon\}\}, \varepsilon\}\}\}$$

$$\varepsilon = \{\{\{\{\varepsilon\}\}\}, \{\{\{\varepsilon\}\}, \varepsilon\}\}$$

The latter violates the axiom of regularity (foundation), otherwise:

$$E^2 \neq E$$

$$E^3 \neq E$$

The use of non-founded sets is evidence of a transition to non-classical mathematical models.

There are approaches to representing strings in von Neumann-Bernays-Gödel set theory by equivalence classes of groupoids (which is complex) over oriented sets or functions (requires the construction of a set of ordinal numbers). In the first case, the representation grows exponentially, and in the second case, it is necessary to use oriented pairs [23], [25] (the number of characters for a string of length n grows no faster than $1+14*n+p\,(n)$, where $p\,(n) = 1 + n * (n+3)/2$ – number of characters to represent ordinal numbers). These approaches do not require a transition to non-classical mathematical models. However, a string of one element is not this element.

Let's consider another approach to representing strings and oriented sets, which does not require, overcomes the identified difficulties within the framework of classical mathematical models and uses pairs not according to K. Kuratowski, which cannot counter-intuitively have cardinality (length) equal to one.

Let us define the concept of disposing of a set.

$$1^S \overset{def}{=} S$$

$$(\iota + 1)^S \overset{def}{=} \left\{ \iota^T \middle| T \subseteq S \right\}$$

Example.

$$2^{\{x\}} = \left\{ 1^{\{x\}}, 1^{\emptyset} \right\} = \{\{\chi\}, \emptyset\}$$

$$3^{\{x\}} = \left\{ 2^{\{x\}}, 2^{\emptyset} \right\} = \left\{ \{\{\chi\}, \emptyset\}, \left\{ 1^{\emptyset} \right\} \right\} = \{\{\{\chi\}, \emptyset\}, \{\emptyset\}\}$$

$$4^{\{x\}} = \left\{ 3^{\{x\}}, 3^{\emptyset} \right\} = \{\{\{\{\chi\}, \emptyset\}, \{\emptyset\}\}, \{\{\emptyset\}\}\}$$

$$5^{\{x\}} = \left\{ 4^{\{x\}}, 4^{\emptyset} \right\} = \{\{\{\{\{\chi\}, \emptyset\}, \{\emptyset\}\}, \{\{\emptyset\}\}\}, \{\{\{\emptyset\}\}\}\}$$

$$4^{\emptyset} = \left\{ 3^{\emptyset} \right\} = \left\{ \left\{ 2^{\emptyset} \right\} \right\} = \left\{ \left\{ \left\{ 1^{\emptyset} \right\} \right\} \right\} = \{\{\{\emptyset\}\}\}$$

Also, let us define the concept of an individual set.

$$\{x\}_1 \overset{def}{=} \{x\}$$

$$\{x\}_{\iota+1} \overset{def}{=} \{\{x\}_\iota\}$$

note that:

$$(\iota + 1)^{\emptyset} = \{\emptyset\}_\iota$$

Finite oriented set:

$$\bigcup_{i=1}^{k} \left\{ (k - i + 1)^{\{a_i\}} \right\}_i$$

The number of characters to represent it is no more than $1 + n * (5 * n + 1)\,/2 + q\,(n)$ where $q\,(n) = 2 * n + 1$ is the number of characters per representation of individual sets of the empty set.

Examples:

$$\langle\rangle = \emptyset$$

$$\langle \chi \rangle = \{\{\chi\}\}$$

A pair like a Wiener pair [23]:

$$\langle \chi, \gamma \rangle = \{\{\{\chi\}, \emptyset\}, \{\{\gamma\}\}\}$$

Other examples:

$$\langle \chi, \gamma, \zeta \rangle = \{\{\{\{\chi\}, \emptyset\}, \{\emptyset\}\}, \{\{\{\gamma\}, \emptyset\}\}, \{\{\{\zeta\}\}\}\}$$

Also let it be true:

$$A^n = \{\langle x_1, x_2, ..., x_i, ..., x_{n-1}, x_n \rangle | x_i \in A\}$$

From the above this follows:

$$\exists A \left(A^1 \neq A\right)$$

Based on the introduced concepts, we can give a definition of the Kleene closure:

$$A^* \overset{def}{=} A^{*_A}$$

where

$$A^{*_\Sigma} \overset{def}{=} A \cup \left(\bigcup_{\iota \in \mathbb{N} \cup \{0\}} A^\iota \right)^{\oplus \Sigma}$$

$$\Lambda^{\oplus \Sigma} = \bigcup_{\chi \in \Lambda} \left\{ \bigoplus_{\iota=1}^{|\chi|} \kappa\left(\langle \chi_\iota, \Sigma \rangle\right) \right\}$$

$$\kappa\left(\langle \gamma, \Sigma \rangle\right) \in \{\gamma | \tau\left(\langle \gamma, \Sigma \rangle\right)\} \cup \{\langle \gamma \rangle | \neg \tau\left(\langle \gamma, \Sigma \rangle\right)\}$$

$$\tau\left(\langle \gamma, \Sigma \rangle\right) = \Big((|\gamma| \in \mathbb{N} \cup \{0\}) \wedge \Big(\exists B \left(\gamma \in B^{|\gamma|} / \Sigma \right) \Big) \Big) \wedge \left(\bigcup_{\iota=1}^{\gamma} \gamma_\iota \subseteq \Sigma \right)$$

Kleene closure properties:

$$|\emptyset^*| = |\{\langle\rangle\}| = 1$$

$$\left|\{\emptyset\}^{*_\emptyset}\right| = \left|\{\langle\rangle\}^{*_\emptyset}\right| = 1$$

if $0 < |A| \leq |\mathbb{N}|$ then $|A^*| = |\mathbb{N}|$.

Extensiveness property of the Kleene closure:

$$A \subseteq A^*$$

Monotonic property of the Kleene closure:

$$A^* \subseteq (A \cup \Delta)^{*_A}$$

Idempotency property of the Kleene closure:

$$(A^*)^{*_A} = A^*$$

The formal language through the introduced Kleene closure is defined:

$$\Lambda \subseteq A^* / \left(A / A^1\right)$$

Generalized strings differ from strings in that their elements can be not only elements of the alphabet but also strings and other generalized strings.

Let us define a generalized Kleene closure (the set of all generalized strings with respect to the alphabet A, extending the alphabet A).

$$A^{(*^*)} \overset{def}{=} \bigcup_{\iota \in \mathbb{N} / \{0\}} A^{(*^\iota)}$$

where

$$A^{(*^1)} \overset{def}{=} A^*$$

$$A^{(*^{\iota+1})} \overset{def}{=} \left(A^{(*^\iota)} \cup A\right)^*$$

Properties of the generalized Kleene closure: if $0 \leq |A| \leq |\mathbb{N}|$, then $\left|A^{(*^*)}\right| = |\mathbb{N}|$.

Extensibility property of the generalized Kleene closure:

$$A \subseteq A^* \subseteq A^{(*^*)}$$

Monotonicity property of the generalized Kleene closure:

$$A^{(*^*)} \subseteq (A \cup \Delta)^{(*^*)}$$

Idempotency property of the generalized Kleene closure:

$$\left(A^{(*^*)}\right)^{(*^*)} = A^{(*^*)}$$

A generalized formal language is defined:

$$\Lambda \subseteq A^{(*^*)} / \left(A / A^1\right)$$

The following is true:

$$\emptyset^{(*^*)} \subseteq \emptyset^{(+^*_1)}$$

Generalized formal languages are included in generalized hereditarily finite (unoriented) sets:

$$A^{(*^*)} \subseteq A^{(+^*_1)}$$

By embedding generalized strings (generalized hereditarily finite sets) into hereditarily finite sets, they can be ordered according to Ackermann coding.

Besides, all generalized strings can be mutually unambiguously mapped to strings of a formal language whose alphabet (extended alphabet) differs from the alphabet of generalized strings by two additional symbols: the start symbol and the end symbol of the string. Accordingly, given a linear ordering of the symbols of the extended alphabet, all generalized strings can be lexicographically ordered as corresponding strings of a formal language on the extended alphabet.

It is possible to consider unoriented sets that are not oriented sets such sets we will call ultra-unoriented sets.

Together with the corresponding generalized oriented sets, generalized ultra-unoriented hereditarily finite sets can be constructed as the class $A^{(+^*_2)}$ according to the expressions:

$$\tau_2(B) \overset{def}{=} B$$

$$\rho_2\left(\langle A, B \rangle\right) = (A \cup B)^*$$

$$\sigma_2(\langle A,B\rangle)=\pi_2\left(2^{\emptyset+\sum_{x\in A\cup B}\{x\}}\right)$$

$$\pi_2(B)=\{h(x)\,|x\in B\}$$

$$h(\emptyset)=\emptyset$$

$$\left\{h(a)\middle|(a\in A)\bigwedge(h(a)=a)\right\}=A$$

$$h(X)=\emptyset\cup\bigcup_{x\in X}\left\{2^{\{h(x)\}}\right\}$$

where h defines a mutually unambiguous mapping of generalized unoriented hereditarily finite sets into generalized ultra-unoriented hereditarily finite sets.

In this case:

$$A^{(+^*_2)}\subseteq A^{(+^*_1)}$$

Together with generalized unoriented finite sets, generalized ultra-unoriented finite sets can be mapped mutually unambiguously to inherited finite sets.

The classical mathematical model works with justified (grounded) sets, but some non-classical models correspond to other structures. While the structure of hereditarily finite sets is acyclic and involves various hierarchies, other (e.g., cholarchical [65] structures consist of elements where each element is a part and is a whole (composed of parts). Some such structures can be visualized as periodic or cyclic structures. Another example of structures that may not fit within the framework of classical mathematical models are non-trivially automorphic structures, since such models adhere to the abstraction of identity,. However, despite this, some of these "non-classical" structures can (under certain conditions) be represented by classical mathematical models. First of all, among such structures we will be interested in enumerable structures, i.e., such structures that can be enumerated.

Let us consider the representation of $A^{(+^*_3)}$ in generalized hereditarily finite sets of the class of countably non-identically-equal generalized hereditarily finite sets on the alphabet $\dot{A}$, which can be given according to the expressions:

$$\tau_3(B)\overset{def}{=}B$$

$$\rho_3(\langle A,B\rangle)=C\times(A\cup B)^*$$

$$\sigma_3(\langle A,B\rangle)=D\times\pi_3\left(2^{\emptyset+\sum_{x\in A\cup B}\{x\}}\right)$$

$$\pi_3(B)=B$$

$$a_{2k+1}=\begin{cases}\dot{a}_k\left|k\le\left|\dot{A}\right|\right.\\ d(2*k+1)\left|k>\left|\dot{A}\right|\right.\end{cases}$$

$$a_{2k}=d(2*k)$$

$$C\cup D=E$$

$$E=\left\{a_{2k}\middle|(a_{2k}\in\dot{A})\bigwedge(k\in\mathbb{N})\right\}$$

$$E\subseteq A$$

If

$$C\cap D=\emptyset$$

then the representations of oriented sets in $A^{(+^*_3)}$ will not intersect with the representations of unoriented sets in $A^{(+^*_3)}$.

Fulfilled:

$$A^{(+^*_3)}\subseteq A^{(+^*_1)}$$

Together with generalized finite sets, representations of countable non-identically-equal generalized hereditarily finite sets can be mapped one-to-one onto hereditarily finite sets.

The representation of $A^{(+^*_4)}$ is analogous to $A^{(+^*_2)}$.

The representation of classes of unorient mixed countably non-identically-equal generalized hereditarily finite sets can be given according to the expressions:

$$\tau_5(A)=2^{\left(\emptyset+\sum_{x\in A}\{x\}\right)}$$

$$\rho_5=\rho_3$$

$$\sigma_5=\sigma_3$$

$$\pi_5=\pi_3$$

The representation of classes of orient mixed countably non-identically-equal generalized hereditarily finite sets can be given according to the expressions:

$$\tau_6(A)=A^{\left(*^1\right)}$$

$$\rho_6=\rho_3$$

$$\sigma_6=\sigma_3$$

$$\pi_6=\pi_3$$

Consider the representation of $A^{(+^*_6)}$ in generalized hereditarily finite sets of the class of countably non-identically-equal generalized hereditarily finite sets on the alphabet $\dot{A}$ with codes $\ddot{A}$, which can be given according to expressions:

$$\tau_7(B)\overset{def}{=}B$$

$$\rho_7(\langle A,B\rangle)=C\times(A\cup B)^*$$

$$\sigma_7(\langle A,B\rangle)=D\times\pi_7\left(2^{\emptyset+\sum_{x\in A\cup B}\{x\}}\right)$$

$$\pi_7(B)=B$$

$$a_{2k+1}=\begin{cases}\dot{a}_k\left|k\le\left|\dot{A}\right|\right.\\ d(2*k+1)\left|k>\left|\dot{A}\right|\right.\end{cases}$$

$$a_{2k}=d(2*k)$$

$$C\cup D=E$$

$$E=\left\{a_{2k}\middle|(a_{2k}\in\dot{A})\bigwedge(k\in\mathbb{N})\right\}$$

$$E \subseteq A$$

where

$$\ddot{A} \cap \mathbb{N} \neq \emptyset$$

If a number in $\ddot{A}$ is the Ackermann coding of the corresponding set in the representation of enumerably self-founded generalized hereditarily finite sets by hereditarily finite sets, then such a set is self-founded, i.e. this set is considered as an element of the alphabet at the same position in $\dot{A}$ as its code $\ddot{A}$.

To ensure the enumerability of self-founded generalized hereditarily finite sets, it is required to ensure that they are all finitely mutually-founded, that is, that they are not infinitely mutually-founded. In this case, the extensional closure will be reduced to a finite structure and there will be an algorithm for comparing these structures provided they are reduced to canonical form.

In general, it is not possible at this stage to canonize the representation of a structure that is a union of structures of all sets of a given class. To canonize the representation of such structures requires a separate investigation of the conditions under which this may be possible.

The embedding of structures in hereditarily finite sets and natural numbers (Fig. 1), using Ackermann coding, gives a structure isomorphic to the Rado graph [62] which is universal for any graph, i.e. it allows us to isomorphically enclose any graph and its supergraphs. Stability is one of the important properties of Rado Graph has. There are known studies of universal uncountable structures and corresponding theories including studies of the property of their stability [60]. As for uncountable structures [31] associated with operational semantics [16], [55] one direction of research is to study an approach based on the use of decision procedures without the presence of enumeration procedures as well as the use of Büchi automata [61] and their hierarchy.

Using the considered classes of structures, one can represent arbitrary finite graph, pseudograph, multigraph, metagraph, hypergraph structures, including abstract simplicial complexes [28], their combinations and others.

The applied value of the considered classes is the possibility of algorithmic construction of canonical forms (representations) of knowledge structures.

IV. Similarity, proximity, other attribute and invariants of structures of meaning space and corresponding models

The analysis of structural properties implies consideration of topological relations and relations of similarity (similarity and analogies) and difference. These relations can be algorithmically realized within the framework of the knowledge specification model in accordance with the knowledge integration model. The knowledge specification model, by considering finite structures, allows for the decidability of the corresponding analysis algorithms.

As for the similarity relations, they can be formed from property detection relations or non-detection relations. A property detection relation itself can be a similarity relation when the property itself is revealed in its specification (i.e., the relation is reflexive); as a rule, such relations are transitive. If a property is not revealed in its specification, but is revealed in other structures, then all these structures can be united into a class of (pairwise) similar structures.

In the first case, such relations can be reduced to the identification of full or partial embeddings, and morphisms: isomorphic, homomorphic or homeomorphic embedding.

The composition of two binary similarity relations is a binary similarity relation.

The union of two ij-similarity relations is a ij-similarity relation.

The intersection of two ij-similarity relations is a ij-similarity relation.

The difference of a ij-similarity relation and a ij-difference relation is a ij-similarity relation.

The difference of two ij-similarity relations, the first of which is a subset of the second, is a ij-difference relation.

The difference of a ij-difference relation and a second relation is a ij-difference relation.

The difference of two ij-difference relations is a ij-difference relation.

The union of two ij-difference relations is a ij-difference relation.

The intersection of two ij-difference relations is a ij-difference relation.

In the second case, similarity can be formed as an equivalence relation or a partial order relation on the set of structures for which no enumerated embeddings are identified (in the first case).

Topological properties [47] can be investigated through the consideration of operations on structures that correspond to topological closure.

Among the topological similarities we can distinguish: similarity on the inclusion of closures, similarity on the inclusion of derived sets, similarity on the inclusion of touch points and others.

Among topological similarities we can distinguish: similarity by equivalence of closures, similarity by equivalence of derived sets, similarity by equivalence of touch points and others.

In addition to search operations for extensional closures, we can identify closure operations on the class of automorphic elements of the structure.

It is also possible to identify search operations by a single (simple) pattern (reverse homomorphism) in exten-

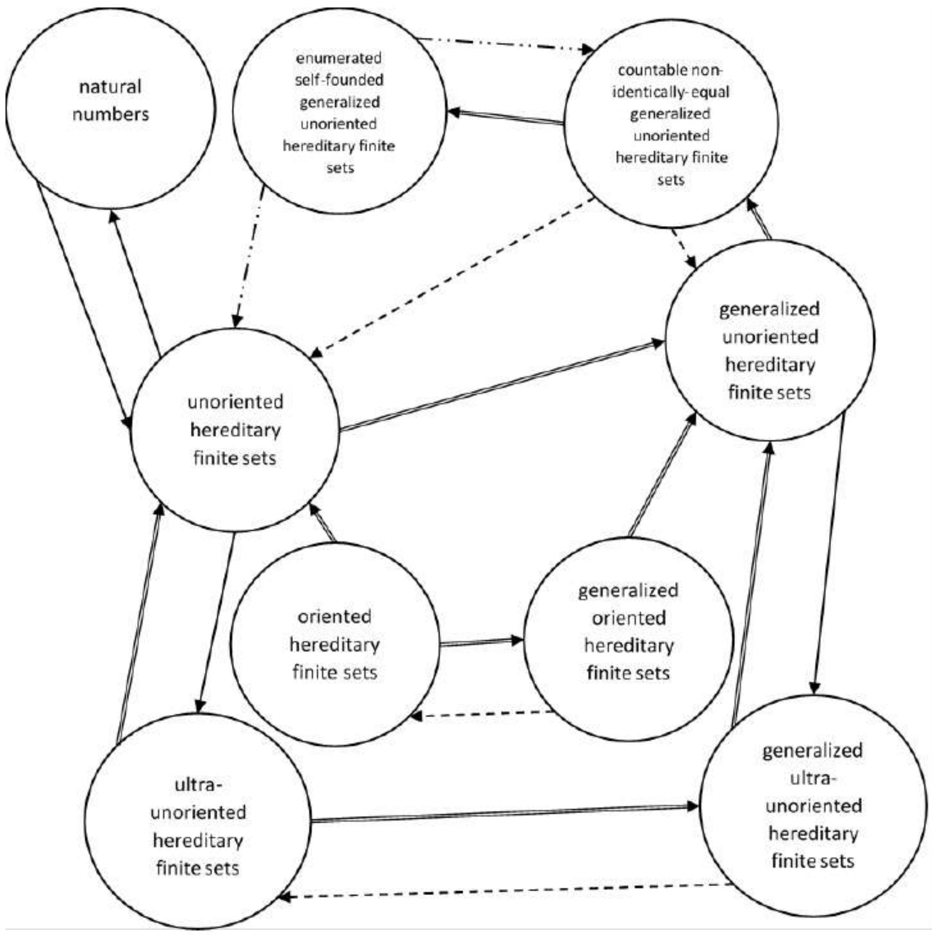

Figure 1. General finite structures classes

sional closures covering all elements of some distance-defined neighborhood of a concept, and all elements incident to the concept must be all elements of the automorphism class in this neighborhood. Among such neighborhoods we can choose a maximal neighborhood. The corresponding neighborhood can be considered as a closed set if it is not included in any other such neighborhood. In this case, the search operation on the corresponding pattern can also be considered as a closure operation.

The number of mappings of a pseudograph to the ordinal scale distinguishable with precision up to the order of vertices does not exceed $|V|^{|V|}$. $|V|^{|V|}$.

The number of mappings of the pseudograph to the metric scale distinguishable with order-of-magnitude accuracy of pairwise distances does not exceed $\left(\left|V^2\right| - |V|\right)^{\left(\left|V^2\right| - |V|\right)}$ (k-ary generalized distances – $\left(\left|V^k\right| - |V|\right)^{\left(\left|V^k\right| - |V|\right)}$).

For finite pseudographs and other representations of G for which the smallest A is found:

$$G \in A^{(+^*_7)}$$

a quantitative attribute model can be based on the following class of quantitative features:

$$\mathbb{N}r_+{}^{A^{(+^*_7)}}$$

where

$$\mathbb{R} \subseteq \mathbb{N}r$$

All closures defined on finite structures are finite. Finite structures, including finite (extensional) closures, have the following (global) characteristics: length, width, graph dimension, neg-entropy, sets (sets) of local characteristics and others.

Global characteristics can be considered as invariants [17], [68], on the basis of which characteristics similarity or proximity measures (metrics or pseudo-metrics) can be calculated.

Local (conditional) numerical characteristics include: centrality, metrics [10] and others.

Another example of numerical features are measures of scalar and coscalar product for introorthogonal sets considered in taxonomy management problems [].

The features for a finite set can be computed as a generalized or stepped average:

$$\sqrt[p]{\frac{\sum_{x\in S} x^p}{|S|}}$$

For a finite set of objects (an element of the product of metric spaces), metrics based on the (stepped) mean can be considered

$$d\left(\langle P,Q\rangle\right)=\sqrt[p]{\sum_{x\in P}\sum_{y\in Q} d\left(\langle x,y\rangle\right)^p}$$

The transition from an unbounded metric to a metric as a bounded monotone (fuzzy) measure, can be realized according to:

$$\lim_{z\to d(\langle x,y\rangle)}\frac{z}{1+z}$$

or any other semi-additive monotonically increasing bounded function from the point $(0;0)$:

$$f\left(x+y\right)\leq f\left(x\right)+f\left(y\right)$$

For a metric on linear representations by generalized strings, we can use either the edit distance between them or the minimum of the edit distances between the results of the action of a subgroup of the symmetric group of permutations of linear representations by generalized strings.

Let us consider an algorithm for finding the extensional closure of a concept:

1 The distensible set of the extensional closure is empty.
2 Include a concept in the current front.
3 Construct a new next front by going deep down the extensional.
4 Subtract the current front from the next front.
5 Add the elements of the current edge to the distensional set of the extensional closure.
6 If the next edge is empty return the extensional closure.
7 Make the next edge the current edge, go to step 3.

Consider an algorithm for computing the metric (quasi-metric) on the union of (finite) extensional closures of concepts:

1 Find the extensional closure of the first concept.
2 Find the extensional closure of the second concept.
3 If the intersection of the found closures is empty then return the metric equal to infinity.
4 If one concept lies in the extensional closure of another, then find the distance from the other to the first and return the metric equal to the smallest of the found ones.
5 Descend from each concept deep into the extensional closure to the intersection of the extensional closures and remember the lengths of descent as A and B.
6 Stepwise descent in the intersection of closures to continue until the moment of meeting, calculate the corresponding lengths A' and B'.
7 Return the metric $A+A'+(B+B')*1$.

Each static structure (with its own denotational semantics [19]) and its characteristics can be related (see Fig.6, Fig.7) to the dynamic structure of a (formal) information processing model based on that static structure and its operations (possibly within $A^{(+\frac{*}{?})}$) [57]. Such dynamic structures and their corresponding operations are described by a big-step operational semantics. The connection between small-step [16], [52] and big-step operational semantics [6] can be revealed through the connections between the operational-information space and the (formal) information processing model.

Operational information space can be described as follows. A set of data instances V, a set of channels P, a set of operators K, a set of (commutative) operations O, a set of configurations C, a sequence of configurations R

$$R\subseteq C\times C$$

$$O\subseteq 2^{V^2\times V}\cup 2^{V\times V^2}$$

$$C\subseteq 2^{K\cup P\cup(P\times(V\cup K))\cup(K\times(O\cup P))}$$

If a new parameter (with a value) is added to a configuration operation, it is possible to jump to the updated configuration that contains it.

If a new operation is added to the configuration parameters, it is possible to switch to the updated configuration that contains it.

It is possible to switch to a configuration that does not contain an operation.

It is possible to jump to a configuration that does not contain a parameter that is not used by an operation.

Consider the flows of an open (acyclic (Fig.2)) or closed path (cyclic (Fig.3)) for the corresponding open or closed structures and assume the following requirements for its flow c_{ij} [57].

Each edge is mapped to a flow (energy) c_{ij}. To each vertex s there is a flow $c_s=\sum_{j=1}^{n}c_{sj}$. In addition to the forward flow, the reverse flow $c_{ji}^{-1}, c_s^{-1}=\sum_{j=1}^{n}c_{sj}^{-1}$ is also computed. Their difference is equal to: $d_{ij}=c_{ij}-c_{ji}^{-1}$, $d_s=c_s-c_s^{-1}$. The direct (local) amplitude is calculated as follows $p_{ij}=\frac{d_{ij}}{c_i+c_i^{-1}}+\frac{1}{\sum_{j=1}^{V}a_{ij}}$.

$$\sum_{j=1}^{N}c_{ij}=\sum_{j=1}^{N}c_{ji}$$

$$c_{ij}=\frac{\sum_{j=1}^{N}c_{ij}}{\sum_{j=1}^{N}a_{ij}}*a_{ij};c_{ij}*\sum_{j=1}^{N}a_{ij}=a_{ij}*\sum_{j=1}^{N}c_{ij}$$

Also in matrix form we have:

$$A^T*C=(A*1)\bullet C$$

For the structure in Fig.3 and its forward flow we have:

$$\begin{cases} c_{11} = c_{12} \\ c_{56} = c_{57} \\ c_{89} = c_{814} \\ c_{1112} = c_{1113} \end{cases}$$

Let us find the minimum solution in natural numbers.

As a result of fulfillment of these requirements we obtain the following table of results (see Fig.3, Fig.4, Fig.8, Fig.5).

Table I
Table of dynamic structure characteristics

Edge number	*Flow difference*	*Forward amplitude*	*Backward amplitude*
0	3	19/32	35/35=1
1	-3	13/32	29/29=1
2	3	35/35=1	35/35=1
3	-3	29/29=1	29/29=1
4	3	35/35=1	19/34
5	-1	15/29	15/34
6	-2	14/29	30/30=1
7	2	34/34=1	34/34=1
8	-2	30/30=1	30/30=1
9	0	16/34=8/17	16/30=8/15
10	2	18/34=9/17	18/35
11	-2	30/30=1	14/30=7/15
12	-2	30/30=1	30/30=1
13	1	17/30	33/33=1
14	-3	13/30	29/29=1
15	1	33/33=1	17/35
16	-3	29/29=1	29/29=1
17	3	35/35=1	35/35=1
18	-3	29/29=1	13/32
19	3	35/35=1	19/32

An analogous result can be obtained for an open (non-closed) structure (Fig. 2).

Each strongly connected structure has an (own) period T, which is the GCD of all periods (lengths of simple cycles) in this structure, and has a partition by levels of wave fronts corresponding to this period. We will call the number of these levels the length of the structure $L = T$. The length L of an acyclic structure is the maximum length of the shortest route for two connected vertices. Each (acyclic) structure has a mapping W of the set of numbers of moments of time [9] to the set of subsets of vertices by levels of wavefronts at given moments of time, the number of which does not exceed the length and diameter of the structure. Each wavefront has an energy $E(t) = \sum_{s \in W(t)} c_s$. The wavefront energy can be different from 1. The amplitude at the top of the wavefront $p_s^t = \frac{c_s}{E(t)}$ is in the interval $[0;1]$. The average amplitude is inversely proportional to the number of front elements $\frac{E(t)}{|W(t)|}$.

The amount of information of the wavefront at the moment t is expressed.

$$-\sum_{s \in W(t)} \left(\frac{\left|U_s^{(t)}\right|}{p_s^t} * \ln \frac{\left|U_s^{(t)}\right|}{p_s^t} \right)$$

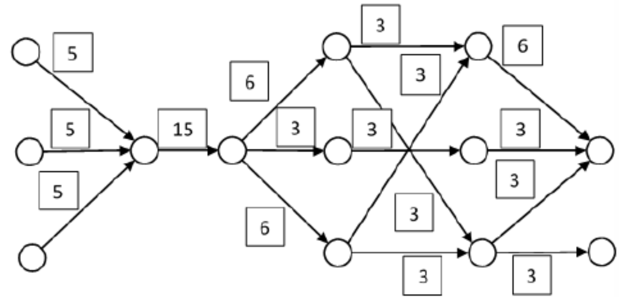

Figure 2. Acyclic orgraph weighted structure.

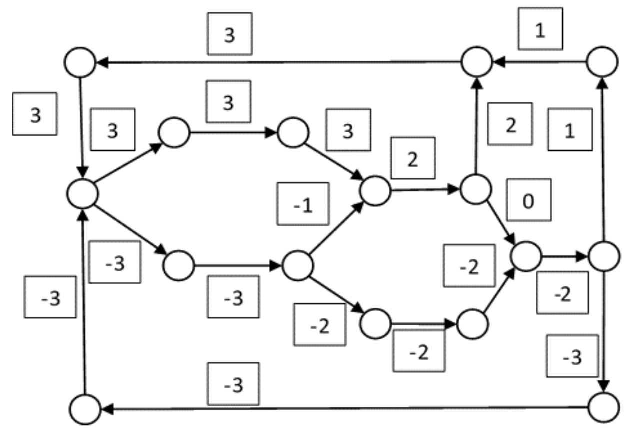

Figure 3. Strong connected flow difference weighted orgraph.

Set of undistinguishable vertices of the wavefront

$$\{s\} \subseteq U_s^{(t)} \subseteq W(t)$$

The average (arithmetic) amount of information of the structure:

$$-\frac{1}{T} * \sum_{t=1}^{T} \sum_{s \in W(t)} \left(\frac{\left|U_s^{(t)}\right|}{p_s^t} * \ln \frac{\left|I_{(t)}\right| * \left|U_s^{(t)}\right|}{T * p_s^t} \right)$$

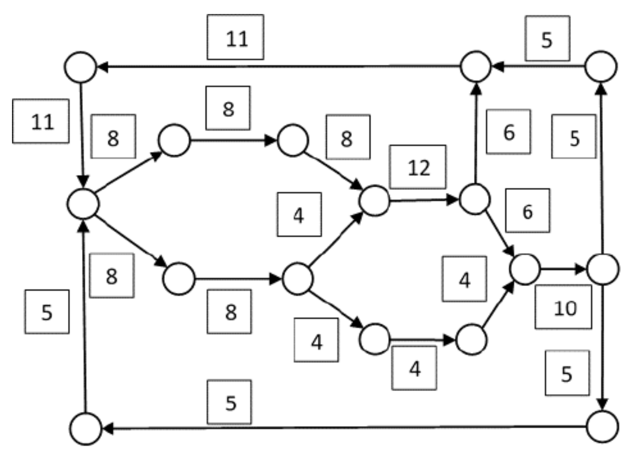

Figure 4. Strong connected orgraph forward flow.

$$A = \begin{matrix} 0 & 3 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Figure 5. Strong connected orgraph flow difference weights matrix.

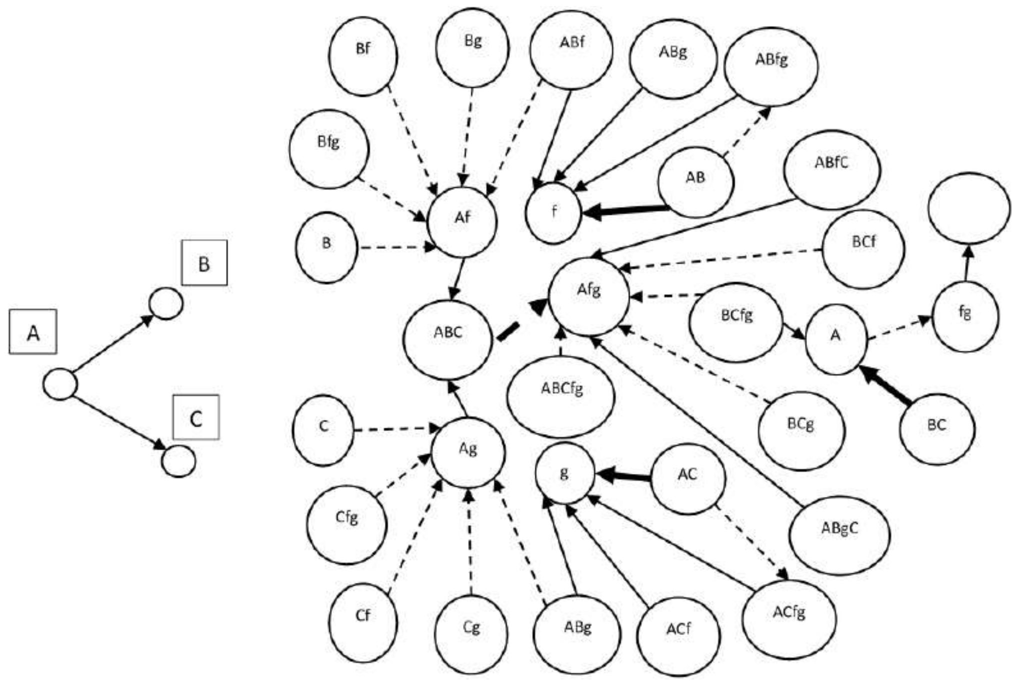

Figure 6. Asymmetric ontological structure with corresponding dynamic structure.

A set of indistinguishable moments in time

$$\{t\} \subseteq I_{(t)} \subseteq Dom\,(W)$$

The information in a strongly connected structure will be called real (elliptic), and in an acyclic structure will be called imaginary (hyperbolic).

Two kinds of structures are considered: a (finite) acyclic graph and a strongly connected pseudograph. An arbitrary (finite) pseudograph structure can be decomposed into its connected components. An arbitrary (finite) connected pseudograph structure can have different kinds of substructures and, in particular, can be broken down into the two kinds of structures discussed earlier: strongly connected components (subpseudographs), acyclic graphs (subgraphs). However, within a structure, these substructures can have different relationships and fulfill different roles [66]. Let us describe different types of substructures according to their roles (relations) performed (available) in the structure [66].

Resonators are maximal strongly connected subpseu-

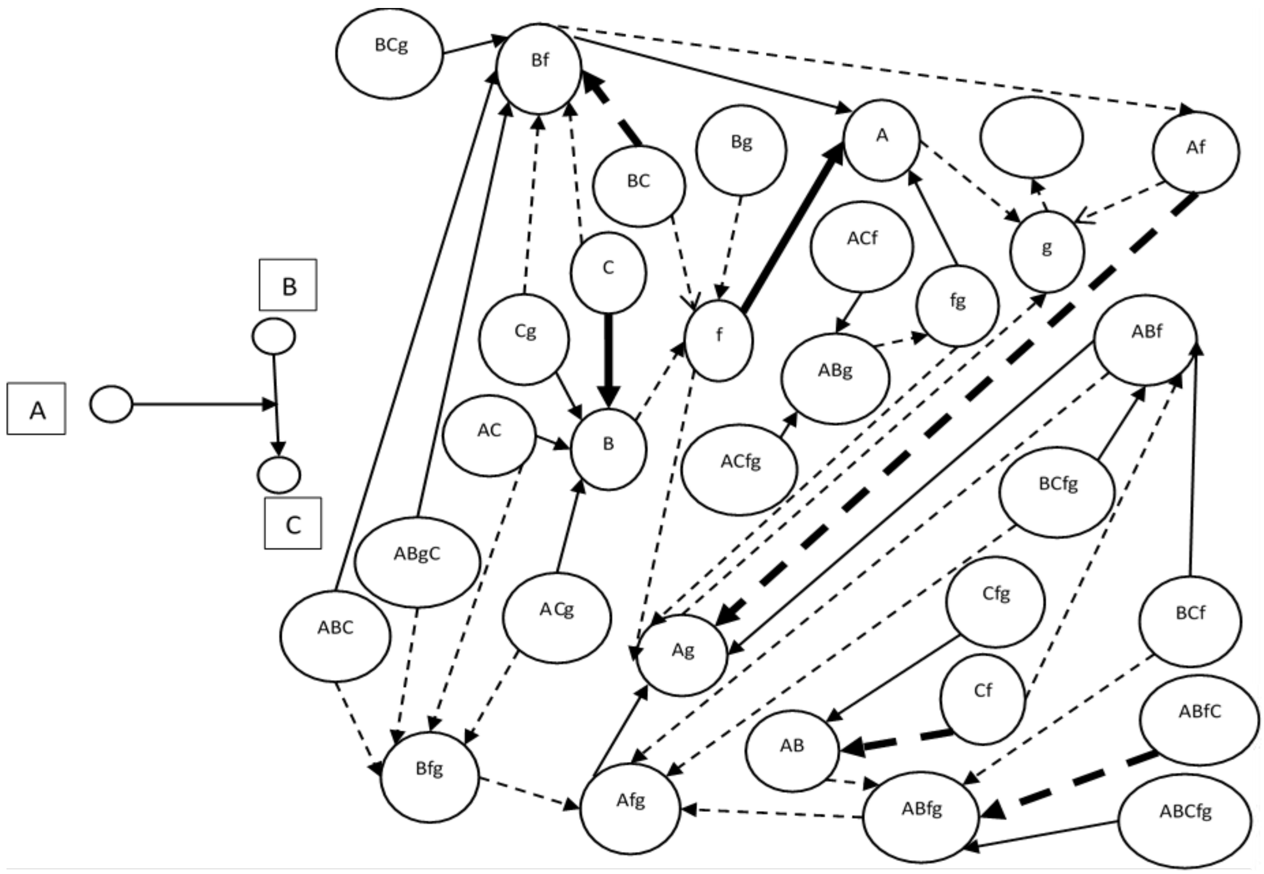

Figure 7. Asymmetric ontological structure with corresponding dynamic structure.

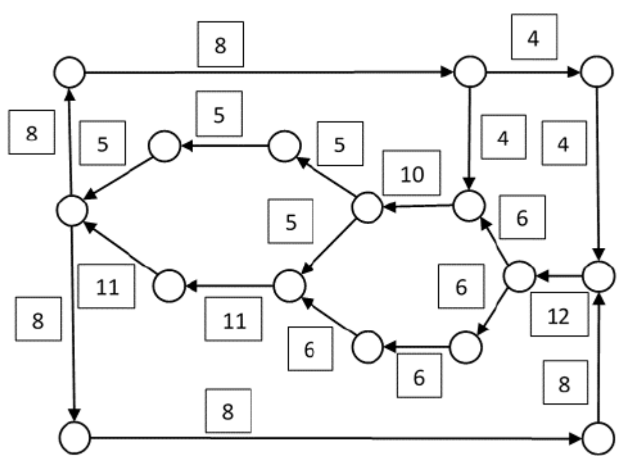

Figure 8. Strong connected orgraph backward flow.

dographs.

Sensors (receptors) are acyclic subgraphs whose elements are not reachable from any resonator.

Dispensers (effectors) are acyclic subgraphs whose elements are not reachable from any resonator.

Transmitters are acyclic subgraphs whose elements are reachable from at least one resonator and from whose elements at least one other (different) resonator is reachable.

Transmitters and transmitters are consumers.

Sensors and transmitters are suppliers.

We can also consider generating resonators (not reachable from other resonators) and consuming resonators (reachable from other resonators).

Among resonators, we can distinguish unimodal (harmonic) resonators and multimodal (non-harmonic) resonators (multimodal waveform). All multimodal resonators are consuming resonators. Each mode is phase shifted by less than a period.

A resonator-free subgraph can be extracted from any structure, which is the set of all vertices and edges reachable from the receptor elements.

The remaining edges together with their initial and final vertices form the resonator subpseudograph.

A receptor element is called a sensor element if it has no supplier.

A dispenser element is called a dispenser element if it has no consumer.

If there are no suppliers (sensors or transmitters) whose elements are suppliers to the generating resonator, then all its elements are sensor elements.

If there are no consumers (dispensers or transmitters) whose elements are consumers of the resonator, then all its elements are dispenser elements.

The method for determining the capacitive characteristics of structures used in problem solving is summarized in the following principles.

Each resonator has a period, which is the partial GCD T of its own period and periods of all (its) suppliers, from which it consumes, to the GCD of this GCD (T) and all its divisors k, for which the convolution of the phases $k * n$ with the phases (taking into account their shifts) of all modes (waveforms) of the signal is equal to $k\ T - k * ((T - k)!)$.

For each consumer (dispenser or transmitter) element, the period is calculated similarly, except that instead of the GCD of the period itself and the periods of all (its) suppliers from which it consumes, the GCD of the periods of all its suppliers is taken.

For each supplier element, the potential period, which is the period of the consumer in the inverted pseudograph (inverse ratio pseudo-graph), can be similarly determined.

For sensor elements, it may be assumed, unless otherwise accepted, that their period is equal to their potential period.

For each sensor element (supplier), a period can be calculated that is equal to the LCM of the periods of all its consumers (taking into account the phase shift).

The entropy of a set of resonators is calculated on a period equal to the GCD of periods of all resonators of this set.

If resonators are present, then the (maximum possible) entropy of the entire pseudograph is valid and can be computed as the greatest entropy of the greatest entropy of the greatest entropies of the smallest sets of resonators cutting the set (paths) of the set of smallest sets of paths connecting all sensor elements to all dispenser elements.

If all sensor elements and dispenser elements are resonator elements, then the (minimum required) entropy of the entire pseudograph is valid and can be computed as the smallest entropy of the smallest entropies of the smallest sets of resonators cutting the set (paths) of the set of smallest sets of paths connecting all sensor elements to all dispenser elements.

If all sensor elements and dispenser elements are not resonator elements, then the (minimum necessary) entropy of the whole pseudograph is invalid (imaginary) and is computed on the period equal to the LCM of all periods (elements) of the subpseudographs of the pseudograph and the maximum of the lengths of the (simple) paths from the sensor element to the dispenser element.

The entropy of an unbound pseudograph can be calculated as the average (minimum, maximum, etc.) of the entropies of its components on a period equal to the LCM of periods (on which calculations for) its components were performed.

Conclusions

The classification of enumerable finite structures and their representations is proposed, relations between classes of this classification are considered. The classification is oriented on unification of knowledge representation with finite structure and algorithmization of solutions of problems of investigation of topological and metric properties of structures of meaning space in order to exclude redundant fragments of knowledge at representation in meaning space.

Approaches are considered and concepts for investigation of structural-topological and metric properties of structures of the sense space for the purpose of optimization of structures of knowledge bases are proposed. General quantitative evaluations of mappings of finite structures with accuracy up to order preservation to ordinal and metric scales in the study of corresponding properties of these structures are given. An approach to the classification of quantitative features of finite structures is proposed. An algorithm for metric computation on the union of extensional closures of sense space concepts is proposed.

A model and a method for computing entropy (as one of the invariants) for finite dynamic structures of information processing models (in accordance with the models of graph dynamical system and generalized finite automaton [64]) based on analytical calculation of transition probabilities on the state graph in accordance with its structure are proposed.

The features of structures of semantic space that can be used as invariants in order to reduce the time to identify redundant fragments in the semantic space are considered.

A model of operational-information space is proposed, which corresponds to the model of model-parametric space [67], is oriented to solving the problems of knowledge management [15], [50], [56] in information processing and the study of the relationship between the attributes of structures with operational semantics expressed by the operational semantics of small and large step.

An approach to the consideration of infinite structures through (limit) sequences of finite structures converging to them is proposed in accordance with the classification of finite structures, the model of knowledge specification [5], the algebraic system of extensible sets and the metamodel of semantic space [56].

References

[1] V.V. Golenkov. Otkrytyi proekt, napravlennyi na sozdanie tekhnologii komponentnogo proektirovaniya intellektual'nykh sistem [An open project aimed at creating a technology for the

component design of intelligent systems], Otkrytye semanticheskie tekhnologii proektirovaniya intellektual'nykh system [Open semantic technologies for intelligent systems], 2013, pp. 55—78.

[2] V.V. Golenkov, N.A. Gulyakina, D.V. Shunkevich. Otkrytaya tekhnologiya ontologicheskogo proektirovaniya, proizvodstva i ekspluatatsii semanticheski sovmestimykh gibridnykh intellektual'nykh komp'yuternykh system [Open technology for ontological design, production and operation of semantically compatible hybrid intelligent computer systems], Minsk, Bestprint, 2021. 690 p.

[3] Z. Pawlak. Rough sets. International Journal of Parallel Programming, 1982, vol. 11, no. 5, pp. 341—356.

[4] D. Dubois, H. Prade. Rough fuzzy sets and fuzzy rough sets International Journal of General Systems, 1990, vol. 17, no. (2-3), pp. 191-–209.

[5] V.P. Ivashenko. Modeli i algoritmy integratsii znanii na osnove odnorodnykh semanticheskikh setei (disc. na soiskanie stepeni kand. tekhn. nauk: 05.13.17) [Models and algorithms for knowledge integration based on homogeneous semantic networks (thesis for the degree of Candidate of Technical Sciences: 05.13.17)] , Minsk, BGUIR, 2014, 152 p.

[6] V.P. Ivashenko Operatsionnaya semantika mnogoagentnykh sistem obrabotki znanii [Operational semantics of multi-agent knowledge processing systems], Information Technologies and Systems, 2020, Minsk, BGUIR, pp. 78–79.

[7] V.P. Ivashenko. Modeli i algoritmy integratsii znanii na osnove odnorodnykh semanticheskikh setei [Models and algorithms for knowledge integration based on homogeneous semantic networks], Otkrytye semanticheskie tekhnologii proektirovaniya intellektual'nykh system [Open semantic technologies for intelligent systems], 2015, Minsk, BGUIR, pp. 111—132.

[8] V.P. Ivashenko Tekhnologiya razrabotki programmnykh komponentov intellektual'nykh sistem na osnove integratsionnoi platformy [Technology for the development of software components of intelligent systems based on an integration platform], Information Technologies and Systems, 2021, Minsk, BGUIR, pp. 84–85.

[9] V.P. Ivashenko. Ontologicheskaya model' prostranstvenno-vremennykh otnoshenii sobytii i yavlenii v protsessakh obrabotki znanii [Ontological model of the space-time relations of the event and the phenomenon in the processes of knowledge processing] 2017, vol. 5, no. 107, Minsk, BSUIR, pp. 13–17.

[10] V.P. Ivashenko. String processing model for knowledge-driven systems. Minsk, Doklady BGUIR, 2020, vol. 18, no. 6, pp. 33–40.

[11] Yu. Manin, M. Marcolli. Semantic spaces. Published, Location, 2016. 32 p. (arXiv)

[12] V.V. Golenkov, O.E. Eliseeva, V.P. Ivashenko. Predstavlenie i obrabotka znanii v parallel'nykh grafodinamicheskikh assotsiativnykh mashinak [Representation and processing of knowledge in parallel graphodynamic associative machines], Minsk, BGUIR, 2001. 412 p.

[13] G. Rozenberg, A. Salomaa. Handbook of Formal Languages, Volume 1: Word, Language, Grammar. Verlag, Berlin, Heidelberg, Springer, 1997. 873 p.

[14] B. Smith. Metody i algoritmy vychislenii na strokakh [Computing Patterns in Strings], Moscow, OOO "I.D. Williams", 2006. 496 p.

[15] D.A. Pospelov. Situatsionnoe upravlenie: teoriya i praktika [Situational management: theory and practice], Moscow, Nauka, 1986. 288 p.

[16] A. Blass. A game semantics for linear logic. Annals of Pure and Applied Logic, 1992, vol. 56, pp. 183–220.

[17] J.H. Conway. On Numbers and Games. AK Peters/CRC Press, 2000. 242 p.

[18] G. Plotkin. Call-by-name, call-by-value and the lambda-calculus. Theoretical Computer Science, 1975, vol. 1, no. 2. pp. 125–159.

[19] D. Scott, Ch. Strachey. Toward a mathematical semantics for computer languages. Proceedings of the Symposium on Computers and Automata, Microwave Research Institute Symposia Series, Polytechnic Institute of Brooklyn Press, New York, 1971, vol. 21, pp. 19—46.

[20] F. Daneš. On Prague school functionalism in linguistics. Functionalism in Linguistics, 1987, pp. 3–38.

[21] A. R. D. Mathias. Unordered pairs in the set theory of Bourbaki 1949, Archiv der Mathematik, 2010, vol. 94, pp. 1–10.

[22] W. Quine. On ordered pairs. Journal of Symbolic Logic, 1945, vol. 10, no. 3, pp. 95—96.

[23] J. Heijenoort. From Frege to Gödel. A source book in mathematical logic, 1879–1931. Cambridge, Mass., Harvard University Press, 1967, 664 p.

[24] C. Kuratowski. Sur la notion de l'ordre dans la Théorie des Ensembles. Fundamenta Mathematicae, 1921, vol. 2, no. 1,pp. 161–171.

[25] A.P. Morse. A Theory of Sets. Academic Press, 1965, 130 p.

[26] A. Levy. Basic Set Theory, Springer-Verlag, 1979, 391 p.

[27] K.J. Devlin. Fundamentals of contemporary set theory. Universitext. Springer-Verlag, 1979, 182 p.

[28] J.M. Lee. Introduction to Topological Manifolds, Springer, 2011, 452 p.

[29] G. W. F. Hegel. Nauka logiki [Science of Logic]: v 3-h t. T.1, Moscow, "Mysl'", 1970. 501 p.

[30] R.T.W. Arthur. Leibniz on Time, Space, and Relativity, Oxford University Press, 2022, 432 p.

[31] D. Hilbert. On the infinite. In Philosophy of Mathematics (1984), Selected Readings, Cambridge, Cambridge University Press, 2012, pp. 183–201.

[32] D. Bohm and B.J. Hiley. The Undivided Universe: An Ontological Interpretation of Quantum Theory, London, Routledge, 1993. xii + 397 p.

[33] D. Bohm. Wholeness and the Implicate Order, London, Routledge, 2002. 284 p.

[34] V.V. Nalimov, Zh.A. Drogalina. Real'nost' nereal'nogo. Veroyatnostnaya model' bessoznatel'nogo [The reality of the unreal. Probabilistic model of the unconscious], Moscow, Mir idei, AO Akron, 1995. 432 p.

[35] V.V. Nalimov. Spontannost' soznaniya. Veroyatnostnaya teoriya smyslov i smyslovaya arkhitektonika lichnosti [Spontaneity of consciousness. Probabilistic theory of meanings and semantic architectonics of personality], Moscow, Prometei, 1989. 287 p.

[36] V.V. Nalimov. Veroyatnostnaya model' yazyka. O sootnoshenii estestvennykh i iskusstvennykh yazykov [Probabilistic model of language. On the relationship between natural and artificial languages] Moscow, Nauka, 1979. 304 p.

[37] S. Roman, Advanced Linear Algebra, Graduate Texts in Mathematics, vol. 135 (2nd ed.), Berlin, New York, 2005. 488 p.

[38] M.A. Armstrong, Basic Topology [1979], Undergraduate Texts in Mathematics, Springer, 1983. 263 p.

[39] L. Collatz. Functional Analysis and Numerical Mathematics, New York, San Francisco, London, Academic Press, 1966. xx 473 p.

[40] J. Heinonen. Lectures on analysis on metric spaces, New York, Springer, 2001. X+141 p.

[41] H.H. Schaefer, M.P. Wolff. Topological Vector Spaces, New York, Springer New York Imprint Springer, 1999. 361 p.

[42] I. Mel'˘cuk. Language: from Meaning to Text. Ed. by D. Beck. Moscow & Boston, 2016.

[43] J. Harris. Algebraic Geometry: A First Course, New York, Springer, 1992. 330 p.

[44] A. Kostrikin, Yu. Manin. Linear Algebra and Geometry, Gordon and Breach Science Publishers, 1997. 320 p.

[45] V.V. Martynov. V tsentre soznaniya cheloveka [At the center of human consciousness], Minsk, BGU, 2009. 272 p.

[46] V.V. Martynov. Yazyk v prostranstve i vremeni: k probleme glottogeneza slavyan (2-e izd.) [Language in space and time: to the problem of glottogenesis of the Slavs (2nd ed.)], Moscow, Editorial URSS, 2004. 106 p.

[47] C. Marijuan. Finite topologies and digraphs. Proyecciones (Antofagasta), 2010, vol. 29, pp. 291–307.

[48] R. Penrose. The Road to Reality, Oxford, Oxford University Press, 2004. 1094 p.

[49] A.K. Guts. Khronogeometriya. Aksiomaticheskaya teoriya otnositel'nosti [Chronogeometry. Axiomatic theory of relativity], Omsk, OOO "UniPak", 2008. 340 p.

[50] M. Bansal, D. Burkett, G. de Melo, Gerard, D. Klein, Dan. Structured Learning for Taxonomy Induction with Belief Propagation. 52nd Annual Meeting of the Association for Computational

Linguistics, ACL 2014 – Proceedings of the Conference, 2015, vol. 1, pp. 1041–1051.
[51] Yu.I. Zhuravlyov. Ob algebraicheskom podkhode k resheniyu zadach raspoznavaniya ili klassifikatsii [On an algebraic approach to solving problems of recognition or classification]. Problemy kibernetiki, 1978, vol. 33. pp. 5–68.
[52] V.V. Golenkov, D.V. Shunkevich. Agentno-orientirovannye modeli, metodika i sredstva razrabotki sovmestimykh reshatelei zadach intellektual'nykh sistem [Agent-based models, methodology and tools for developing compatible problem solvers for intelligent systems]. Programmnye produkty i sistemy, 2020, vol. 33, no. 3. pp. 404–412.
[53] J.A. Fodor. The Language Of Thought. Crowell Press, 1975. 214 p.
[54] A.N. Kolmogorov. Problems of Probability Theory. Theory of Probability & Its Applications, 1994, vol. 38, no. 2, pp. 177–178.
[55] A.N. Kolmogorov, V.A. Uspenskij K opredeleniyu algoritma [To the definition of the algorithm] UMN, 1958, vol. 13. no. 4.
[56] Ivashenko, V. General-purpose semantic representation language and semantic space Otkrytye semanticheskie tekhnologii proektirovaniya intellektual'nykh system [Open semantic technologies for intelligent systems] Minsk, BGUIR, 2022. vol. 6. pp. 41–64.
[57] V. Ivashenko, Structures and Measures in Knowledge Processing Models / V. Ivashenko // Pattern Recognition and Information Processing. Artificial Intelliverse: Expanding Horizons : Proceedings of the 16th International Conference, Minsk, October 17–19, 2023 / ed.: A. Nedzved. A. Belotserkovsky / Belarusian State University. – Minsk, 2023. – P. 16–21.
[58] J. Poelmans, D. Ignatov, S. Kuznetsov, G. Dedene Fuzzy and rough formal concept analysis: A survey. International Journal of General Systems, 2014, vol. 43. 10.1080/03081079.2013.862377.
[59] B. Ganter, R. Wille, Formal Concept Analysis: Mathematical Foundations, translated by C. Franzke, Springer-Verlag, Berlin, 1998. ISBN 3-540-62771-5
[60] S. Shelah, Classification theory and the number of nonisomorphic models, Studies in Logic and the Foundations of Mathematics (2nd ed.), Elsevier, 1990 [1978], vol. IX, no. 1.19, p.49. ISBN 978-0-444-70260-9
[61] B. Khoussainov, A. Nerode. Automata Theory and its Applications, Springer Science & Business Media, 6 December 2012.
[62] W. Ackermann, "Die Widerspruchsfreiheit der allgemeinen Mengenlehre". Mathematische Annalen, 1937 , vol. 114, pp.305–315.
[63] A. Kanamori, "Bernays and Set Theory", Bulletin of Symbolic Logic, 2009, vol. 15, no. 1, pp.43—-69.
[64] H.S. Mortveit, C.M. Reidys, An Introduction to Sequential Dynamical Systems. Universitext, 2007.
[65] A. Koestler. The Ghost in the Machine. London: Hutchinson. 1, 1990 reprint edition [1967], Penguin Group.
[66] V.P. Ivashenko Informatsionnye kharakteristiki ontologicheskikh struktur [Information characteristics of ontological structures], Information Technologies and Systems, 2023, Minsk, BGUIR, pp. 55–56.
[67] Yu.R. Val'kman, V.I. Gritsenko, A.Yu. Rykhal'skii. Model'no-parametricheskoe prostranstvo: teoriya i primenenie [Model-parametric space: theory and application], 2012, Kiev, Naukova Dumka. 192 p.
[68] B.A. Kulik. Logika estestvennykh rassuzhdenii. [Logic of natural reasoning.], 2001, SPb, Nevskii Dialekt. 128 p.

К ТЕОРИИ СМЫСЛОВОГО ПРОСТРАНСТВА

Ивашенко В. П.

Статья рассматривает модели для исследования структуры, топологии и метрических признаков смыслового пространства, использующего унифицированное представление знаний.

Рассмотрены классы конечных структур, соответствующие онтологическим структурам и множествам классического и неклассического вида, исследованы свойства перечислимости этих классов.

Предложено понятия операционно-информационного пространствм, как модели для исследования взаимосвязи операционной семантики онтологических структур большого и малого шага.

Рассмотрены количественные признаки и инварианты онтологических структур, ориентированные на решение задач управления знаниями.

Received 03.04.2024