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FLOWS OF VISCOUS LIQUID AT VARIOUS REYNOLDS NUMBERS

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Abstract. *Based on existing experimental data, the flows occurring in the range* $\mathrm{Re}'_{кр} < \mathrm{Re} < \mathrm{Re}_{кр}$ *of Reynolds numbers Re are analyzed. The question is investigated: are these flows Poiseuille flows. The results of numerical calculations and their comparison with experimental data (for pressure) are graphically shown.*
Keywords: *Laminar, rod, turbulent flows; lower and transition critical Reynolds numbers; energy dissipation, flow disturbance force.*

In the dynamics of a viscous incompressible fluid, laminar flows and turbulent flows are distinguished. The former occur at low, the latter at high Reynolds numbers. In this paper, we will consider the motion of a fluid in a "flat" pipe (flows between parallel solid walls) and its motion in a pipe of circular cross-section at different Re Reynolds numbers.

The existence of a critical number Re has been experimentally established, which has the following property, described below (we will call this number the lower critical Reynolds number and denote it by $\mathrm{Re}'_{кр}$). It defines the stability boundary with respect to disturbances of finite intensity. Under the condition $\mathrm{Re} < \mathrm{Re}'_{кр}$ no undamped non-stationary motion can exist in the pipe at all. If turbulence occurs in any section, then under the specified condition the turbulent region, carried downstream, narrows until it disappears. Under $\mathrm{Re} < \mathrm{Re}'_{кр}$ on the contrary, it will expand over time, capturing an ever larger section of the flow. If disturbances of the flow continuously occur at the entrance to the pipe, then at a value of the number $\mathrm{Re}'_{кр}$ less than the lower critical value $\mathrm{Re} < \mathrm{Re}'_{кр}$, they will certainly fade at some distance from the entrance, no matter how strong they are. Under the condition $\mathrm{Re} < \mathrm{Re}'_{кр}$ the flow, on the contrary, can become turbulent throughout the entire length of the pipe, and for this, the weaker disturbances are sufficient, the greater Re [1] .

Repeatedly confirmed experiments have established that in the case of a liquid flow in a pipe of circular cross-section, undamped turbulence is observed already at a value of $\mathbf{Re} \approx 1800$ $(\mathbf{Re}'_{\kappa p} \leq 1800)$, in the case of its flow between parallel planes (flow in a "flat" pipe) - starting from a value of $\mathbf{Re} \approx 1000$ $(\mathbf{Re}'_{\kappa p} \leq 1000)$. With very careful elimination of disturbances at the entrance to the pipe, it is possible to maintain a flow that does not turn into turbulence up to very large value Re (in fact, it was possible to observe it up to a value of $\mathbf{Re} \approx 10^5$) [1, 2] .

Let us conduct a detailed analysis of the experimental results, considering the flow in a flat pipe. Let us assume that a viscous incompressible liquid in the presence of a pressure gradient moves in the direction of the axis Ox_1 between two stationary parallel plates (solid walls) located in the planes $x_2 = \pm h$. Such a flow can be considered as a flow through a cylindrical pipe of rectangular cross-section, if we imagine that the liquid is also enclosed between two imaginary planes $x_3 = \pm 0.5$, located at a distance of 1 from each other, along which it slides freely, without experiencing frictional resistance (i.e. the pipe is a rectangular parallelepiped) [3]. Suppose that of the three components of the velocity vector v_1, v_2, v_3 only $v_1 \neq 0$, and v_2 and v_3 and and are identically equal to zero, the continuity equation is satisfied identically [1]. The Navier-Stokes equations are significantly simplified and give a parabola for the velocity v_1 in an arbitrary cross-section of the pipe $x_1 = c$ $(0 < c < L, c = const)$ parabola

$$v_1 = -\frac{h^2}{2\mu} \cdot \frac{dp}{dx_1} \left(1 - \frac{x_2^2}{h^2}\right), \tag{1}$$

where the pressure p depends only on the value of x_1, $\frac{dp}{dx_1} = const$, μ – is the dynamic viscosity coefficient, L – length of the pipe, and the velocity v_1 depends only on the coordinate x [2]. Formula (1) is obtained under the assumption that the pipe is infinitely long. Such an idealization of a real flow implies a finite, but sufficiently long, pipe and the position of the section $x_1 = c$, in which formula (1) already takes place, sufficiently distant from the pipe entrance .

From the facts obtained in the experiments and listed above, it follows that there is a critical number Re (let us denote it $\mathbf{Re}_{\kappa p}$),at which the flow actually passes into a turbulent regime and which may not coincide with $\mathbf{Re}'_{\kappa p}$ (let us call it $\mathbf{Re}_{\kappa p}$ *the transition critical* Reynolds number). Indeed, $\mathbf{Re}'_{\kappa p} \leq 10^3$ and $\mathbf{Re}'_{\kappa p} \leq 1.8 \cdot 10^3$ and for a flat pipe and a pipe of circular cross-section, respectively, and $\mathbf{Re}_{\kappa p} \approx 10^5$ with very careful elimination of disturbances at the pipe entrance and sufficiently smooth solid walls. This means that there are flows determined by inequalities $\mathbf{Re}'_{\kappa p} < \mathbf{Re} < \mathbf{Re}_{\kappa p}$, which are not turbulent. Such flows are called laminar in [1] (however, it is indicated that in the interval between values $\mathbf{Re}'_{\kappa p}$ and $\mathbf{Re}_{\kappa p}$ laminar flow is metastable) and they are also called "stretched laminar" in [2]. Let us ask ourselves the question: are these latter flows described by formula (1)? To answer it, we will carry out the following chain of reasoning, taking into account the fact that the motion of a viscous fluid is considered, and the presence of viscosity (internal friction) leads to dissipation (scattering) of energy [1, 2]. Recall that a detailed analysis of the experimental results is carried out for flows in a flat pipe.

To carry out our reasoning, we will obtain the necessary relationships under the assumption that the analyzed flows are described by formula (1). We find

$$v_{cp} = -\frac{1}{2Lh}\cdot\frac{h^2}{2\mu}\cdot\frac{dp}{dx_1}\int_{-h}^{h}\left(1-\frac{x_2^2}{h^2}\right)dx_2\int_0^L dx_1\int_{-0.5}^{0.5}dx_3 = -\frac{h^2}{3\mu}\cdot\frac{dp}{dx_1};\quad \mathrm{Re} = \frac{2\rho h}{\mu}\cdot v_{cp} = -\frac{2\rho h^3}{3\mu^2}\cdot\frac{dp}{dx_1}. \quad 2)$$

It is also known that if we find the time derivative of the total kinetic energy of a liquid and then integrate it over a certain volume, V, we get

$$\frac{\partial}{\partial t}\int\frac{\rho v^2}{2}dV = -\oint\left[\rho\bar{v}\left(\frac{v^2}{2}+\frac{p}{\rho}\right)-(\bar{v},\bar{\sigma}')\right]d\bar{f} - \int\sigma'_{ik}\frac{\partial v_i}{\partial x_k}dV, \tag{3}$$

where ρ – density of liquid, σ'_{ik} – viscous stress tensor, $\oint[-"-]d\bar{f}$ – integral over the surface of the volume V, the second term on the right-hand side (taken with the opposite sign) represents the decrease in kinetic energy per unit time due to dissipation. If we extend the integration over the entire volume of the liquid, then the integral over the surface disappears (it vanishes due to the condition that the velocity on the wall is zero). From this follows the formula for energy dissipation in an incompressible liquid

$$E'_{кин} = -\frac{\mu}{2}\int\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)^2 dV. \tag{4}$$

Dissipation leads to a decrease in kinetic energy, i.e. there must be $E'_{кин} < 0$. In the formula (4) $E_{кин}$ – total kinetic energy of an incompressible fluid in its entire volume V, $E'_{кин}$ – the derivative of this energy with respect to time, the integration is performed over the entire volume V [1]. The first term in square brackets (the right side of (3)) is the energy flow associated with the simple transfer of the mass of the liquid during its movement, coinciding with the energy flow in an ideal liquid. The second term $(\bar{v},\bar{\sigma}')$ is the energy flow associated with internal friction processes. Since we are interested in the influence of only viscous forces on the nature of the flow, we leave only the second term for consideration and consider the integral over the surface for the flow (1) (in $(\bar{v},\bar{\sigma}')$ only the term of the form remains $v_1 \cdot v'_{1x_2}$).

The integral over the lateral surface is equal to $I_1 = \int_{-0.5}^{0.5}dx_3\int_\Gamma v_1\cdot v'_{1x_2}dl$, where Γ – a closed contour that is a rectangle in a plane $x_3 = 0$ with sides lying on the lateral faces of the parallelepiped. Passing the contour in the chosen direction, taking into account that on a solid wall $v_1 = 0$ and the integrand does not depend on x_1, let's find out

$$I_1 = 1\cdot\int_\Gamma v_1\cdot v'_{1x_2}dl = \int_{-h}^{h}v_1v'_{1x_2}dx_2\bigg|_{x_1=L} + \int_{h}^{-h}v_1v'_{1x_2}dx_2\bigg|_{x_1=0} = \int_{-h}^{h}v_1v'_{1x_2}dx_2\bigg|_{x_1=c} + \int_{h}^{-h}v_1v'_{1x_2}dx_2\bigg|_{x_1=c} = 0\ .$$

The upper and lower faces are considered in the same way as the lateral faces. Let us turn to $E'_{кин}$.

Having found the derivative v'_{1x_2} and integrating over volume, we obtain

$$E'_{кин} = -\frac{4Lh^3}{3\mu}\cdot\left(\frac{dp}{dx_1}\right)^2. \qquad (5)$$

In order to be able to compare disturbances and to give a more definite meaning to the words «weak» and «strong» disturbances, we introduce the term «disturbance force» F. Let us return to the question: are the flows defined by the inequalities $\mathrm{Re}'_{кр} < \mathrm{Re} < \mathrm{Re}_{кр}$, described by formula (1), and assume that the answer is positive. Let us take two values $\mathrm{Re}_1, \mathrm{Re}_2$ $(\mathrm{Re}_1 < \mathrm{Re}_2)$ from the interval $(\mathrm{Re}'_{кр}, \mathrm{Re}_{кр})$. More precisely, let ρ, h, μ, L and be fixed $\mathrm{Re}_{кр}$ – the number Re, at which, in a given pipe, when a given fluid moves, the flow actually passes into a turbulent regime ($\mathrm{Re}_{кр}$ – the transition critical number). In addition, let $\mathrm{Re}_1 = \mathrm{Re}'_{кр} + 0.1\cdot(\mathrm{Re}_{кр} - \mathrm{Re}'_{кр})$, and $\mathrm{Re}_2 = \mathrm{Re}'_{кр} + 0.9\cdot(\mathrm{Re}_{кр} - \mathrm{Re}'_{кр})$. Let us begin our consideration with the number Re_1 and will disturb the flow occurring at this number Re, gradually increasing the disturbance force until we reach a force that will make the flow turbulent throughout the pipe . It follows from the above that such a force exists. This means that there is a maximum value $F^{(1)}_{\max}$ of the disturbance force at which our flow $(\mathrm{Re} = \mathrm{Re}_1)$ is not yet turbulent, but an insignificant increase in this value leads to turbulence. The flow at number $\mathrm{Re} = \mathrm{Re}_1$, we will call flow 1.

Let the flow occur at number $\mathrm{Re} = \mathrm{Re}_2$ (we will call it flow 2). We will now disturb this flow, gradually increasing the force of disturbance until we reach such a force that will make the flow turbulent throughout the entire length of the pipe (such a force exists). There is a minimum value $F^{(2)}_{\min}$ of the force of disturbance at which the flow $(\mathrm{Re} = \mathrm{Re}_2)$ is turbulent, but a slight decrease in this value leads to the disappearance of turbulence.

It follows from the above that $F^{(2)}_{\min} < F^{(1)}_{\max}$. On the other hand, $\mathrm{Re}_2 > \mathrm{Re}_1$. The latter means that the dissipation of the fluid energy at Re_2 is greater (in absolute value) than at Re_1 (see. (2) and (5)). The dissipation (scattering) of the mechanical energy occurs due to that part of the work of the internal forces that is determined by viscosity [2]. It is the presence of viscosity that leads to the dissipation of the energy of turbulent pulsations, and the greater the dissipation (in absolute value), the sooner these pulsations will disappear. But it is greater at number Re_2 and, therefore, the pulsations should disappear sooner in flow 2, especially since in this flow they are weaker (since $F^{(2)}_{\min} < F^{(1)}_{\max}$), but, as we can see, flow 2 is turbulent, while flow 1 is not. We have come to the conclusion that weaker pulsations are not suppressed by greater dissipation, while stronger pulsations are suppressed by lesser (in absolute value) dissipation. This conclusion contradicts the experimental data, and we came to it by assuming that the answer to the question posed above is positive. Thus, the flow under the condition $\mathrm{Re}'_{кр} < \mathrm{Re} < \mathrm{Re}_{кр}$ is not a flow (1). Such a flow is called a rod flow. A strict definition of a rod flow is given in [4].

Thus, we consider the motion of a viscous incompressible fluid in a «flat» pipe (flows between parallel solid walls) at different Reynolds numbers Re.

Let us call the condition $\mathbf{Re < Re'_{кр}}$ *condition* 1, and the condition $\mathbf{Re'_{кр} < Re < Re_{кр}}$ – condition 2, and deal with the question of the transverse velocity profile v_2. The existence of a rod flow, which occurs when condition 2 is satisfied and which is not described by the formula

$$v_1 = -\frac{h^2}{2\mu}\cdot\frac{dp}{dx_1}\left(1-\frac{x_2^2}{h^2}\right). \tag{1}$$

In (1) the pressure p depends only on the value of x_1, $\frac{dp}{dx_1} = const$, μ – the viscosity coefficient, v_1, v_2, v_3 – the components of the velocity vector, $v_1 \neq 0$, and v_2 and v_3 are identically equal to zero, $2h$ – is the distance between the solid walls of the «flat» pipe located in the planes $x_2 = \pm h$.

To solve the question of the existence of a rod flow, the «integral approach» was sufficient. Above, we considered the dissipation of energy $E'_{кин}$, expressed by the integral equality (see (4)), in which the integration was performed over the entire volume of the liquid. It follows that the viscous forces existing in a rod flow and taken over the entire volume of the liquid, i.e. in the «integral sense» («integral approach»), are less than they would be if the flow at a number Re, satisfying condition 2 were a flow (1). In the question of the transverse velocity profile in a rod flow, a more subtle differentiated approach will be required.

Thus, the flow under condition 2 is not a flow (1). The assumption that the components v_2 and v_3 and are identically equal to zero leads to formula (1). Since v_3 Since cannot be different from zero, we come to the conclusion: the assumption that the transverse component of the velocity (hereinafter the velocity component is called the velocity) v_2 is identically equal to zero is confirmed by experiment for flows occurring under condition 1, and contradicts it in the case of condition 2 (we assume that the fluid motion is described by the Navier-Stokes equations). Consequently, the transition of the number Re (as it increases) through the value $\mathbf{Re'_{кр}}$ leads to the appearance of a non-zero transverse velocity and, therefore, to the appearance of a non-zero partial derivative $\frac{\partial v_2}{\partial x_2}$, since $v_2\big|_{x_2=\pm h} = 0$. But then, by virtue of the continuity equation $\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0$, and $\frac{\partial v_1}{\partial x_1} \neq 0$.

The boundary conditions $v_2\big|_{x_2=\pm h} = 0$ express the fact that the liquid cannot penetrate behind the solid surface. Note that in the case of the motion of an ideal liquid, the boundary conditions to the corresponding equations (Euler's equations) require that only the velocity vanish on the solid walls of the pipe v_2. In the case of the motion of a viscous liquid, these conditions, already to the Navier-Stokes equations, require that both the velocity v_2, and the velocity be zero v_1. This additional requirement is related to the fact (see [1]) that between the surface of a solid and a viscous liquid there always exist forces of molecular adhesion (a property of viscosity), leading to the fact that the layer of liquid adjacent to the solid wall is completely retained, as if adhering to it (the adhesion condition). Note also that the solutions of the Euler equations cannot satisfy the extra (compared to the case of an ideal liquid) boundary condition of the velocity vanishing v_1. Mathematically, this is due to the lower (first) order of these equations in coordinate derivatives than the order (second) of the Navier-Stokes equations.

Due to the above, the layers of liquid adjacent to the layer adjacent to the solid wall are also delayed (not completely), due to viscosity (internal friction). This leads to the braking of these layers (a decrease in the velocity v_1 in the direction x_1 near the solid wall). It follows that (in a rod flow) near the solid walls, the partial derivative $\frac{\partial v_1}{\partial x_1} < 0$ (it was noted above that it is not equal to zero). But then, due to the same continuity equation, $\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0$, we obtain (near the solid walls) $\frac{\partial v_2}{\partial x_2} > 0$. Since the coordinate axis $\boldsymbol{Ox_2}$ (point O – the origin of the coordinate system) is directed from the wall $x_2 = -h$ (left wall) to the wall $x_2 = h$ (right wall) and, obviously, $v_2\big|_{x_2=0} = 0$, then at $-h < x_2 < 0$ the velocity $v >$, and at $0 < x_2 < h$ v_2 is less than zero. This implies the profile of the transverse velocity v_2, which is a graph of some continuous function $f(x_2)$ (a function of one variable x_2 for fixed values of the variables $x_1 = c_1, x_3 = c_3$, where $0 < c_1 < L, -0.5 < c_3 < 0.5$). The values of function $f(x_2)$ while changing x_2 from value $-h$ to value h: first increase from zero ($(f(-h) = 0)$ to some maximum value $f_{\max} = f(-b) > 0$ (on this section $\frac{\partial v_2}{\partial x_2} > 0$), then decrease from $f_{\max}$ to $f_{\min} = f(b) < 0$, while passing through the zero value (on this section $\frac{\partial v_2}{\partial x_2} < 0$), then again increase $f_{\min}$ to zero $(f(h) = 0)$ ($\frac{\partial v_2}{\partial x_2} > 0$). Points $x_2 = \pm b$ – extremum function points $f(x_2)$. To clarify the velocity profile v_2 and finding its dependance on the number Re letls define, if it is possible more precisely finding the points $x_2 = \pm b$ in the interval $(-h, h)$ at different Re. Let us turn to (1) (it is true under the condition 1). Firstly note that the number Re, in case of flow (1), is defined by equality (see. equality (2))

$$\mathrm{Re} = -\frac{2\rho h^3}{3\mu^2} \cdot \frac{dp}{dx_1}. \tag{2}$$

Now, in order to establish the general form of the *viscous stress tensor,* σ'_{ik}, we will use the considerations described in [1]. Without repeating them and assuming the fluid to be incompressible (in this case the tensor takes a simple form), we will immediately write the tensor σ'_{ik} in an incompressible fluid (see [1]):

$$\sigma'_{ik} = \mu\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) \tag{6}$$

If the the flow occurs (1), then in (6) only velocity exists v_1 (the rest is equal to zero). This velocity depends only on the axis x_2. Therefore, from all derivatives of the formula (6) non-zero will only be $\frac{\partial v_1}{\partial x_2}$,which stop to be private derivative and then is deisgnated as $\frac{dv_1}{dx_2}$. So, the viscous stress (the viscous force acting on a unit area), which we will denote τ, in case of flow (1), it will take the form (see. as well [5]) $\tau = \mu \frac{dv_1}{dx_2}$. Taking into account (1) we get the equalities $\frac{dv_1}{dx_2} = \frac{1}{\mu} \cdot \frac{dp}{dx_1} x_2$, $\tau = \tau(x_2) = \frac{dp}{dx_1} x_2 \left(\frac{dp}{dx_1} = const \right)$. Since the axis x_2 changes the sign ($-h < x_2 < h$), and we will be interested in the absolute value of viscous stress, then below we will refer to the equality

$$|\tau(x_2)| = \left| \frac{dp}{dx_1} x_2 \right|. \tag{7}$$

In the further discussions we thing that $\rho = \rho_0$, $\mu = \mu_0$, $h = h_0$ are the fixed values of parameters of the flow and it means (see. (2)) value Re can be increased only during increasing $\left| \frac{dp}{dx_1} \right|$. Пусть Re^* $(\mathrm{Re}^* > \mathrm{Re}'_{кр}$, $\mathrm{Re}^* \approx \mathrm{Re}'_{кр})$ is the value Re, which is more $\mathrm{Re}'_{кр}$ and at which the flow is already core. We note that under the condition $\mathrm{Re} < \mathrm{Re}'_{кр}$ we have $v_2 = 0$. With Re increasing (within $\mathrm{Re} < \mathrm{Re}'_{кр}$) module $|\tau(x_2)|$ increases (at all x_2: $-h < x_2 < h$, despite the value $x_2 = 0$), moreover $|\tau|$ takes its greatest values near solid walls (see (2) and (7)). When $\mathrm{Re} = \mathrm{Re}^*$ v_2 is not equal to zero. So, on the one hand, $|\tau|$, by increasing, reached values $|\tau(x_2)|^*$, enough for existing $v_2 \neq 0$ and $\frac{\partial v}{\partial x} \neq$. On the other hand, the viscous stress existing in the rod flow in the "integral sense" is less than it would be if the flow were at a number $\mathrm{Re} = \mathrm{Re}^*$ the flow(1) . From the last two seemingly contradictory sentences it follows that $|\tau(x_2)|^*$ has values sufficient in magnitude for the existence of a core flow only in a sufficiently small part (more precisely, in two parts) of the entire volume of liquid, in which $\frac{\partial v_2}{\partial x_2} > 0$ $(-h < x_2 < -b$ and $b < x_2 < h)$. In the rest, the greater part of the entire volume (where $\frac{\partial v_2}{\partial x_2} < 0$), the value $|\tau(x_2)|^*$ is small and the viscous stress in it cannot suppress sufficiently strong disturbances (we will call this part the flow core).

Experiments show that with very careful elimination of disturbances at the entrance to the pipe and its sufficiently smooth (internal) walls, the flow in it does not go into a turbulent regime until $\mathrm{Re} \approx 10^5$ [1,2]. In these experiments, the causes of flow disturbances are the roughness of the pipe walls and the unevenness of its edge at the inlet (i.e., both occur at $x_2 = \pm h$) . For the flow to transition to a turbulent regime, the weaker the disturbances, the greater Re [1]. From the above considerations and the facts of this paragraph it is easy to conclude that with increasing Re (if condition 2 is met) the flow rod will expand. This means that the point $x_2 = -b$ will move to the left wall, and the point $x_2 = b$ – to the right. This is exactly what the experimental data confirms.

Below are the graphic results of calculations performed by the numerical method of work [6] (there are also the indicated results there). The graphs fully confirm the above reasoning (the coordinates designated in the article x_1, x_2, x, y are indicated at the picture, and velocity components v_1, v_2 – u, v).

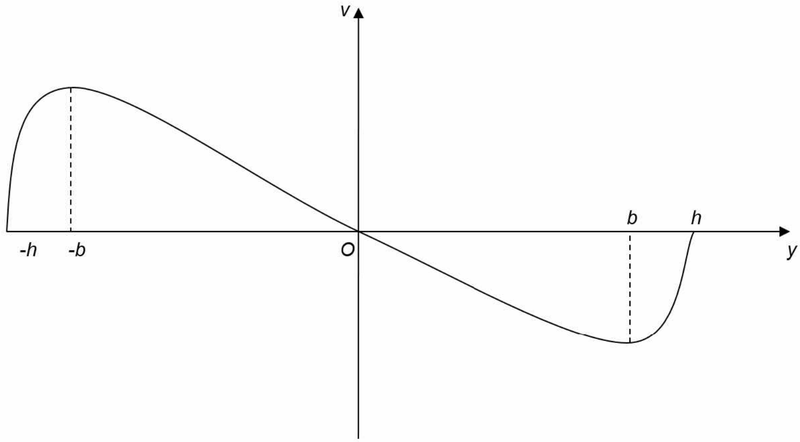


Figure 1. *Distribution of transverse velocities across the pipe vidth*

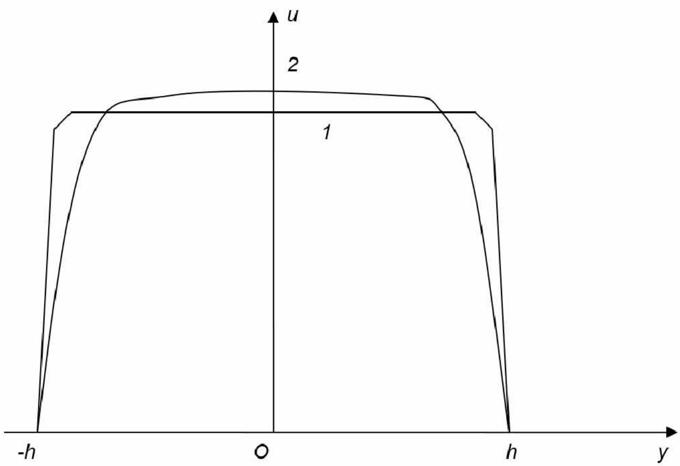
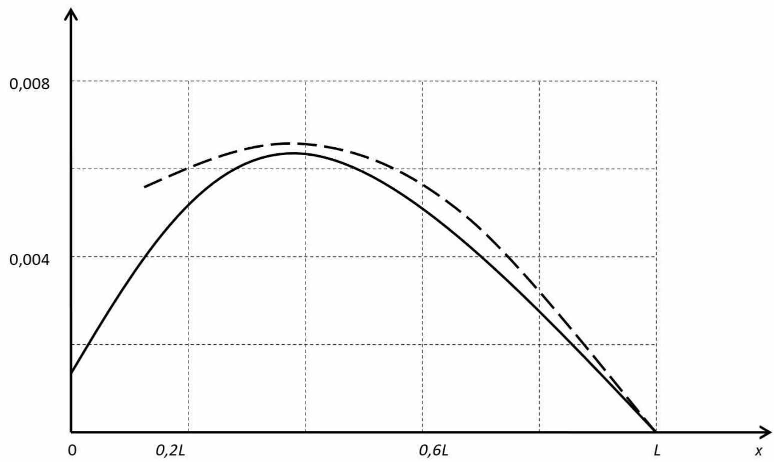


Figure 2. *Distribution of longitudinal velocity across the width of the pipe: curve 1 – section at the pipe inlet, curve 2 – at a distance from the inlet*

Fig. 1 shows the distribution of the transverse velocity component u_2 in the direction of the coordinate x_2 in the pipe section $x_1 = c$ $(0 < c < L)$. At $x_2 = \pm h$ velocity $u_2 = 0$ (adhesion conditions), when $x_2 = 0$ velocity $u_2 = 0$ (symmetry condition), when $x_2 = \pm b$ velocity u_2 reaches the biggest value (at absolute value).

Fig. 2 shows the distribution of the longitudinal velocity component. u_1 in the direction of the coordinate x_2 in two sections of the pipe $x_1 = c_1$ (curve *1*) and $x_1 = c_2$ (curve *2*), where $0 < {}_1 < c_2 < L$. The figure shows that when moving downstream (in the direction of the coordinate x_1) the boundary layer expands and the flow core narrows.

The region of the flow in which $\frac{\partial u_2}{\partial x_2} < 0(-b < x_2 < b)$, is called the core of the flow, and the flow itself, where the transverse velocity has the profile shown in Figure 1, is called the core flow. The regions of the flow in which $\frac{\partial u_2}{\partial x_2} > 0(-h < x_2 < -b, b < x_2 < h)$, are called the flow shell (the names encountered earlier are shown in the figure).



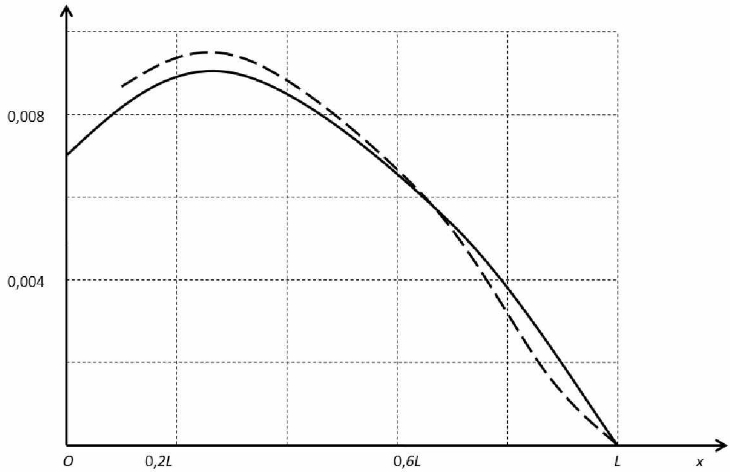


Figure 3. *Distribution of pressure along the channel length at Re=10,000 and Re=30,000*

Fig. 3 shows the pressure distributions along the pipe length obtained as a result of experimental measurements and calculations using the numerical method indicated above: the upper figure corresponds to the value Re=10,000, the lower figure – Re=30,000; the dashed line is the experiment, the solid line is the calculation.

The graphs represent the relative (dimensionless) excess pressure in relation to atmospheric pressure. Calculations and comparison of their results with experimental data were carried out in a wide range of Reynolds numbers ($\mathbf{8000 \le Re \le 30000}$).

Comparison of numerical calculations with experimental data showed that the calculated values are fully confirmed by the experimental data. The proposed numerical method can be used to calculate rod flows in the range of Reynolds numbers $10^3 < \mathrm{Re} < 10^5$, i.e. flows that take place in reality [2]. These flows, which are no longer Poiseuille and have not yet become turbulent, occupy an intermediate position between the two. As noted in [7], the corresponding differential problem in its classical formulation can be overdetermined. The difficulties associated with this situation are overcome by applying the mathematical model presented in [6] to the calculation of rod flows. The issues of solvability of the equations of rod flow and the Navier-Stokes equations are considered in [4] and [8]–[10].

References

1. *Landau L.D., Lifshits E.M. Hydrodynamics. – Moscow: Nauka, 1988.*
2. *Loitsyanskii L.G. Mechanics of Liquid and Gas. – Moscow: Nauka, 1987.*
3. *Milne-Thomson L.M. Theoretical Hydrodynamics. – Moscow: Nauka, 1964.*
4. *Kayanovich S.S. Boundary Value Problem for Rod Flow in a Channel. // Bulletin of the NAS of Belarus. Series of Phys. and Mathematical Sciences. No. 4. 2016. Pp. 55–66.*
5. *Kochin N.E., Kibel I.A., Roze N.V. Theoretical Hydromechanics, Part 2. – Moscow: State Publ. of Phys. and Mathematical Literature, 1963.*
6. *Kayanovich S.S. Rod flow of a viscous fluid // Bulletin of the NAS of Belarus. Series of physical and technical sciences. No. 3. 2013. Pp. 32-35.*
7. *Kayanovich S.S. Navier – Stokes equations and paradoxes of a viscous incompressible fluid // Proceedings of BSTU. Issue II. Series V. Phys. and Mathematics. – 1995. Pp. 49-55.*
8. *Kayanovich S.S. Solvability of the differential model of rod flow. // Bulletin of the NAS of Belarus. Series of physical and mathematical sciences. No. 1. 2015. Pp. 52–59.*
9. *Kayanovich S.S. Mathematical issues of two-dimensional hydrodynamics. – Minsk: Bestprint, 2024.*
10. *Kayanovich Siarhei. On the Solvability of a differential-difference Problem for the Navier – Stokes Equations. // International Conference "Scientific research of the SCO countries: synergy and integration". December 4, 2024. Beijing, PRC. Part 3. P. 145-151*