Approaches to the Study of Semantic Space and Integrated Logical Inference Models Using Similarity, Difference and Other Measures

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***Abstract*—Approaches to the analysis of proximity, similarities, and differences in knowledge structures are considered for the purpose of extracting meaning from data and knowledge. Approaches and models for representing knowledge with operational semantics for component-based design of intelligent systems are discussed. An ontology of generalized measures and relations of difference and similarity, including similarity relations, has been developed. A class of pre-order scales and measures on structures with operational semantics has been proposed, along with an investigation of their algebraic properties. The concept of operational-informational space has been clarified. A model of the semantics of fuzzy temporal logic is proposed for specifying knowledge structures with operational semantics based on previously suggested parameterized fuzzy logic. For the ontological structures corresponding to the obtained results and the three levels of semantic analysis, principles of their integration are formulated as metric subspaces of the semantic space.**

***Keywords*—Semantic space, Generalized measure, Similarity measure, Similarity relation, difference relation, Dynamic system entropy, operational information space, fuzzy temporal logic, CTL*, Reo Coordination Language, Semantic metric space, Logical inference model integration, preorder scale, modian, quasisimilarity, quasieuclid relation, qausireflexive relation, quasitransitive relation, quasiirreflexive relation, semantic unification**

I. Inroduction

For the new generation of intelligent systems [11], [52] it is important to strive for convergence of knowledge processing processes. This also applies to the processes of logical inference, including classical and non-classical deductive, inductive, abductive logical inference [4], etc. One of the primary mechanisms in analysis tasks that precede data and knowledge processing is inductive logical inference, which is based on difference and similarity relations [14]. In the presence of knowledge non-factors [12], these relations are characterized by means of non-classical logics [4], [14], [40] based on the notion of measure [10], [15]. Measures [16], as well as attributes, are closely related to such knowledge attribute as scaling [13]. Measures are also used in the study of spatial properties and relations and the definition of metrics [19], [20], [33], including semantic metrics [16], [17].

II. Review

Similarity and difference relations appear in the sources in one of the following contexts: as reflexive relations and irreflexive relations (ij-reflexive and ij-irreflexive relations) [21], as relations used in inductive inference methods and mechanisms [4], in fuzzy logic [14], as relations of formation of concepts and ontological structures in knowledge processing processes [18], [22]. The name of relations of “similarity” and “difference” in the second case is not quite good, more suitable names would be “relations of conceptual confirmation” or “relations of conceptual refutation”.

In case of presence of non-factors characteristic for the stages of analysis of incoming data (texts) for the purpose of analysis of their sense [23], [24], relations can be characterized by means of attributes taking values on the ordinal or numerical (metric) scale [16]. In this case, the analysis can take place at several levels [24] (essentially-systematic, relationally-phenomenological, intensionally-logical), which correspond to their own ordinal and metric scales. Such attributes and scales can serve as the basis for models of topological, metric, pseudometric (etc.) semantic spaces for ontological structures. Organization of ontological structures into semantic spaces with the help of scales and attributes can ensure minimization of duplication of meanings and optimization of mechanisms of search and knowledge processing in knowledge bases, including logical inference.

One of the main properties of the similarity relation is reflexivity; a similarity relation can be reflexive, quasireflexive, left and right quasireflexive, weakly left and right quasireflexive [36].

Among the properties of similarity relations that serve to distinguish the corresponding subclasses of similarity relations, the following can be identified:

symmetricity, transitivity, quasi-transitivity, left Euclidean, right Euclidean, Euclidean, and others.

One of the basic properties of the difference relation is irreflexivity; the similarity relation can be irreflexive, quasi-irreflexive, left and right quasi-irreflexive, weakly left and right quasi-irreflexive.

Among the properties of difference relations that serve to distinguish the corresponding subclasses of similarity relations, the following symmetry, antitransitivity and others can be identified.

The main classes of measures that can be used both to characterize similarity relations and for subsequent logical inference are: monotonicity, subadditivity, additivity, superadditivity, submodularity, supermodularity, modularity.

For efficient logical inference in distributed multi-agent systems and transition from observed facts and phenomena to rules and regularities, it is necessary to move from static structures to dynamic ones, from denotational semantics to operational semantics. In this case, efficiency can be ensured by parallel reduction of the original task into subtasks that are distributed among agents. In this connection, it is necessary to take into account not only the semantics of classical logic models, but also the operational semantics of logic models oriented to dynamic subject areas [28], and the game semantics of logic models [51] oriented to subject areas considering agents' interaction.

For the purpose of integrating different types of operational semantics ("big" and "small step"), a model of operational-information space [25] was proposed.

Reo Coordination Language (RCL) [26] is an example of such a model and language, which allows to design integrable reusable components based on a unified semantic framework [29] with different semantics descriptions ranging from category-theoretic to logical and semantics of probabilistic automata and automata with constraints [27]. However, the relation of its semantics to the semantics of "big step" is poorly described; from the point of view of denotational semantics, RCL is also insufficiently unified.

When analyzing, including within the semantic space, models of knowledge processing and representation, it is necessary to identify various attributes and measures. One of the key measures is entropy. As for the analysis of graph structures and their corresponding matrices, several definitions of entropy are known: von Neumann entropy [30], entropy of spectral values [32] and others. One of the most general notions of entropy is the Hinchin entropy [31]. The concept of entropy is closely related to the notions of convergence and divergence [35], which are closely related to the relations of similarity and difference [33], [34].

When taking into account non-factors in logical inference processes, including inductive, abductive and deductive, it is required to integrate models using fuzzy measures, relational semantics of temporal and spatio-temporal logics [42], [43]. It is also important to take into account the similarities and differences between the logic models themselves and the knowledge structures in the problems they are used to solve.

As a result of the integration of the above approaches, their ontological models, and convergence of the corresponding problem solving processes, a certain model is formed that can be organized as a semantic space, i.e., have spatial features [46]–[50] when knowledge is scaled [13].

To model and study the sense space [44], the paper uses models and methods of discrete mathematics, graph theory, and mathematical logic.

The semiotic conceptual approach [13] based on notions (concepts) is also used. Each concept has a name (term), a denotation (referent) and a significat.

A concept is understood as a sign that has an extension and an intension by analogy to the formal concepts of the analysis of formal concepts [57].

The basic thesis: the meaning of a concept is completely determined by its signifier (extensional closure).

Structural and metric features [9], [25] that are resistant to ontology structure changes are used in modeling the semantic space. The model of unified knowledge representation [6], [21], whose texts are understood as associative, symmetric, pseudo-orgraphic generalized formal languages [25], is used for representation.

Definition 1. An associative generalized formal language is Λ if and only if, in its texts, there is a generalized string of Δ symbols of length n in addition to T symbols:

$$\exists T \exists \Delta \exists n \left(\emptyset \subset \left((\Delta^n \cup T)^* \cap \Lambda\right) / (T^*)\right). \qquad (1)$$

Definition 2. A symmetric generalized formal language is Λ if and only if for any Δ^n:

$$((\emptyset \subset \Delta^n \cap \Lambda) \to (\Delta^n \subseteq \Lambda)). \qquad (2)$$

Definition 3. A (Pseudo(or))graph generalized formal language Λ , if and only if there exists A for any T, Δ, n:

$$\begin{gathered} \Lambda \subseteq \left(A^2 \cup A\right)^*; \\ \left(\left(\emptyset \subset \left(\left(\Delta^2 \cup T\right)^n \cap \Lambda\right) / (T^n)\right) \to (\Delta \subseteq T)\right). \end{gathered} \qquad (3)$$

The approach to modeling the semantic space [6], [44] can be referred to the theoretical-synthetic (interior) approaches based on unification. Unification assumes that for a set of notations $N = C \cup I$ (signs $S \subseteq C$, concepts C and names (terms) I), there is a set of values (meanings) E, between which correspondences are defined.

Correspondence between symbols and their meanings:

$$v_N \subseteq N \times E. \qquad (4)$$

Similarity relation t_N, tolerance of meanings (values):

$$t_N \overset{def}{=} v_N \circ \left({v_N}^{-1}\right). \qquad (5)$$

The projection of t_N onto the set of concepts C is the equivalence relation of meanings:

$$\sim_C \overset{def}{=} t_N \cap (C \times C)\,. \tag{6}$$

The equivalence relation $\sim_C$ forms equivalence classes $C_{\sim_C}$, between which and the signs of S there is a bijection $C_{\sim_C}| \leftrightarrow |S$, with (denotation function) $d_N \overset{def}{=} v_N \circ (C \times E)$ is an injection: $S| \leftrightarrow E$.

The unification is to go by the similarity relation $t_N \circ (\sim_C \cap (C \times S))$ from the N signs to the S signs.

The semantic normalization of [45] in the languages of the knowledge representation model used assumes that primary meanings $F \subseteq 2^{N \cup F}$ are selected among the meanings $E \subseteq 2^{N \cup F}$.

III. Generalization of the notion of measure

A. *Kinds of generalized measures*

Let us introduce the notion of a generalized measure. Let us first consider the notion of an ordered commutative monoid.

Definition 4: An ordered commutative monoid:

$$\langle M, \{\trianglelefteq, \boxplus\}\rangle \tag{7}$$

satisfies the properties:

$$\begin{array}{c} \{0_M\} \times \{0_M\} \subseteq \trianglelefteq \subseteq M \times M; \\ (a \in M) \to (a \trianglelefteq a)\,; \\ ((a \trianglelefteq b) \wedge (b \trianglelefteq a)) \to (a = b)\,; \\ ((a \trianglelefteq b) \wedge (b \trianglelefteq c)) \to (a \trianglelefteq c)\,; \\ (a \boxplus 0_M) = a; \\ (a \boxplus b) = (b \boxplus a)\,; \\ ((a \boxplus b) \boxplus c) = (a \boxplus (b \boxplus c))\,; \\ (a \in M) \to ((a \trianglelefteq 0_M) \vee (0_M \trianglelefteq a))\,; \\ (a \trianglelefteq b) = ((a \boxplus c) \trianglelefteq (b \boxplus c))\,. \end{array} \tag{8}$$

Definition 5. An ordered commutative monoid bounded from above is an ordered commutative monoid satisfying the additional property:

$$M \times \{\top_M\} \subseteq \trianglelefteq \subseteq M \times M. \tag{9}$$

Definition 6. An ordered commutative monoid bounded from below is an ordered commutative monoid satisfying the additional property:

$$\{\bot_M\} \times M \subseteq \trianglelefteq \subseteq M \times M. \tag{10}$$

Definition 7. A linearly ordered commutative monoid is an ordered commutative monoid satisfying the additional property:

$$((a \in M) \wedge (b \in M)) \to ((a \trianglelefteq b) \vee (b \trianglelefteq a))\,. \tag{11}$$

Definition 8. An ordered commutative group is an ordered commutative monoid satisfying the additional property:

$$\exists b\,(a \boxplus b) = 0_M. \tag{12}$$

Definition 9. A forward well-ordered commutative monoid is an ordered commutative monoid satisfying the additional property:

$$\begin{array}{c} (A \subseteq M) \to \\ (\exists \bar{a}(((\emptyset \subset A \times \{\bar{a}\}) \wedge (A \times \{\bar{a}\} \subseteq \trianglelefteq)) \wedge \\ ((A \times \{a\} \subseteq \trianglelefteq) \to (\{\bar{a}\} \times \{a\} \subseteq \trianglelefteq)))). \end{array} \tag{13}$$

Definition 10. A backward well-ordered commutative monoid is an ordered commutative monoid satisfying the additional property:

$$\begin{array}{c} (A \subseteq M) \to \\ (\exists \underline{a}(((\emptyset \subset A \times \{\underline{a}\}) \wedge (A \times \{\underline{a}\} \subseteq \trianglelefteq)) \wedge \\ ((\{a\} \times A \subseteq \trianglelefteq) \to (\{a\} \times \{\underline{a}\} \subseteq \trianglelefteq)))). \end{array} \tag{14}$$

Definition 11. Generalized measure is defined as

$$\mu \in M^X, \tag{15}$$

where M is the support of an ordered commutative monoid.

Definition 12. A generalized measure bounded from below is a generalized measure satisfying the additional property:

$$\bot_M \trianglelefteq \mu\,(\chi)\,. \tag{16}$$

Definition 13. A generalized measure bounded from above is a generalized measure satisfying the additional property:

$$\mu\,(\chi) \trianglelefteq \top_M. \tag{17}$$

Definition 14. A bounded measure is a generalized measure bounded from above and below. A special case of a bounded measure is a finite-bounded measure.

Definition 15. A basis measure is a generalized measure bounded from below satisfying the additional property:

$$\exists x\,(\mu\,(x) = \bot_M = 0_M)\,. \tag{18}$$

Definition 16. Regular (normalized) measure is a (basis) generalized measure bounded from above satisfying the additional property:

$$\exists x\,(\mu\,(x) = \top_M = 1_M)\,. \tag{19}$$

Definition 17. A preorder-monotone measure is a generalized measure such that there is a preorder (a reflexive transitive binary relation) $\preceq$ for which:

$$(a \preceq b) \to (\mu\,(a) \trianglelefteq \mu\,(b))\,. \tag{20}$$

Given a scale $\langle\langle\langle X, \Sigma\rangle, \langle Y, \{\rho\}\rangle\rangle, \sigma\rangle$, where $\sigma \in Y^X$, and $\rho \subseteq Y \times Y$ is the preorder relation (reflexive and transitive similarity relation) characterized by the predicate-measure $\pi \in M^Y$:

$$\pi = (\rho \times \{1_M\}) \cup (((Y \times Y)\,/\rho) \times \{0_M\})\,. \tag{21}$$

Such given scale will be called a pre-order scale.

Definition 18. Filtering ϕ is:

$$\varphi\,(\langle\sigma, \rho, \chi\rangle) \overset{def}{=} \left\{\left((\sigma \circ \rho) \circ \left(\sigma^{-1}\right)\right)(\chi)\right\}. \tag{22}$$

Definition 19. Nominal (λ-nominal) measure satisfies:

$$\psi_\lambda\left(\langle\sigma,\rho,\chi\rangle\right)\overset{def}{=}\left|X/\varphi\left(\langle\sigma,\rho,\chi\rangle\right)\right|- \\ \lambda*\left|\varphi\left(\langle\sigma,\rho,\chi\rangle\right)\right|. \tag{23}$$

When λ equals 0 the measure is called the mode measure, when λ equals 1 it is called the meda measure.

Definition 20. Medium (λ-medium) measure:

$$\mu\left(\chi\right)\overset{def}{=}\left|\left|\varphi\left(\left\langle\sigma,\left\{\pi^{-1}\left(1_M\right)\right\},\chi\right\rangle\right)\right|- \\ \left|\varphi\left(\left\langle\sigma,\left\{\pi^{-1}\left(1_M\right)\right\}^{-1},\chi\right\rangle\right)\right|\right|+ \\ \left|\psi_\lambda\left(\left\langle\sigma,\left\{\pi^{-1}\left(1_M\right)\right\}^{-1}\cup\left\{\pi^{-1}\left(1_M\right)\right\},\chi\right\rangle\right)\right|. \tag{24}$$

When λ equals 0, the measure is called a modian measure, when λ equals 1, it is called a median measure.

Definition 21. The centroid (τ-centroid) of δ of measure μ is:

$$\delta\left(\mu\right)\overset{def}{=}\lim_{\tau\to\varepsilon}\sqrt[\tau]{\sum\nolimits_{\chi\in X}\left(\mu\left(\chi\right)\right)^\tau}. \tag{25}$$

Let a sigma-algebra be given.

$$\left\langle\Sigma,\left\{\cap,\cup,^{-}\right\}\right\rangle \tag{26}$$

A measurable space (of sets) on X is also given:

$$\left\langle X,\left\langle\Sigma,\left\{\cap,\cup,^{-}\right\}\right\rangle\right\rangle, \tag{27}$$

where $\Sigma\subseteq 2^X$.

As an ordered commutative monoid we will further consider as an ordered commutative monoid by default:

$$\left\langle\mathbb{R},\left\{\leq,+\right\}\right\rangle. \tag{28}$$

Definition 22. A monotone measure (of sets) is a generalized measure on a measurable space $\left\langle X,\left\langle\Sigma,\left\{\cap,\cup,^{-}\right\}\right\rangle\right\rangle$ that satisfies the additional property:

$$\left(A\subseteq B\right)\to\left(\mu\left(A\right)\trianglelefteq\mu\left(B\right)\right). \tag{29}$$

Definition 23. A monotone Sugeno measure is a monotone measure (of sets) satisfying the additional property:

$$\mu\left(\lim_{k\to\omega}\bigcup\nolimits_{i=1}^{k}A_i\right)=\lim_{k\to\omega}\left(\mu\left(\bigcup\nolimits_{i=1}^{k}A_i\right)\right). \tag{30}$$

Definition 24. A subadditive measure is a monotone measure (of sets) satisfying the additional property:

$$\mu\left(\left(A/B\right)\cup\left(B/A\right)\right)\trianglelefteq\mu\left(A/B\right)\boxplus\mu\left(B/A\right). \tag{31}$$

Definition 25. Superadditive measure is a monotone measure (of sets) satisfying the additional property

$$\mu\left(A/B\right)\boxplus\mu\left(B/A\right)\trianglelefteq\mu\left(\left(A/B\right)\cup\left(B/A\right)\right). \tag{32}$$

Definition 26. An additive measure is a monotone measure (of sets) satisfying the additional property:

$$\mu\left(A/B\right)\boxplus\mu\left(B/A\right)=\mu\left(\left(A/B\right)\cup\left(B/A\right)\right). \tag{33}$$

Definition 27. A submodular measure is a monotone measure (of sets) satisfying the additional property:

$$\mu\left(\left(A/B\right)\cup\left(B/A\right)\right)\boxplus\mu\left(\left(A/B\right)\cap\left(B/A\right)\right) \\ \trianglelefteq\boxplus\mu\left(A/B\right)\mu\left(B/A\right). \tag{34}$$

Definition 28. A supermodular measure is a monotone measure (of sets) satisfying the additional property:

$$\mu\left(A/B\right)\boxplus\mu\left(B/A\right) \\ \trianglelefteq\mu\left(\left(A/B\right)\cup\left(B/A\right)\right)\boxplus\mu\left(\left(A/B\right)\cap\left(B/A\right)\right). \tag{35}$$

Definition 29. A modular measure is a monotone measure (of sets) satisfying the additional property:

$$\mu\left(A/B\right)\mu\boxplus\left(B/A\right)= \\ \mu\left(\left(A/B\right)\cup\left(B/A\right)\right)\boxplus\mu\left(\left(A/B\right)\cap\left(B/A\right)\right). \tag{36}$$

Definition 30. A necessity measure is a monotone measure (of sets) on a backward well-ordered monoid satisfying the following additional properties:

$$\mu\left(\left(A/B\right)\cap\left(B/A\right)\right)=\min\left(\left\{\mu\left(A/B\right)\right\}\cup\left\{\mu\left(B/A\right)\right\}\right) \\ \mu\left(\bigcap\nolimits_i A_i\right)=\inf\left(\left\{\mu\left(A_i\right)|i\right\}\right). \tag{37}$$

Definition 31. A possibility measure is a monotone measure (of sets) on a forward well-ordered monoid satisfying the additional properties:

$$\mu\left(\left(A/B\right)\cup\left(B/A\right)\right)=\max\left(\left\{\mu\left(A/B\right)\right\}\cup\left\{\mu\left(B/A\right)\right\}\right) \\ \mu\left(\bigcup\nolimits_i A_i\right)=\sup\left(\left\{\mu\left(A_i\right)|i\right\}\right). \tag{38}$$

Definition 32. A symmetric measure is a monotone measure (of sets) satisfying the additional property:

$$\left(\left|A/B\right|=\left|B/A\right|\right)\to\left(\mu\left(A/B\right)=\mu\left(B/A\right)\right). \tag{39}$$

B. *Measures and metrics*

Special kinds of measures are: pre-metric, quasi-metric, pseudometric, metric, ultrametric, metametric, etc.

Theorem 1. Let $\lambda\geq 1$ be a pseudometric space with pseudometric π and a metric space with metric μ on the set X. Then $\langle X,\rho\rangle$ is a metric space where

$$\rho\left(\langle\chi,\gamma\rangle\right)=\left(\left(\mu(\langle\chi,\gamma\rangle)^\lambda\right)+\left(\pi(\langle\chi,\gamma\rangle)^\lambda\right)\right)^{\left(\lambda^{-1}\right)} \tag{40}$$

or

$$\rho\left(\langle\chi,\gamma\rangle\right)=\max\left(\left\{\mu\left(\langle\chi,\gamma\rangle\right)\right\}\cup\left\{\pi\left(\langle\chi,\gamma\rangle\right)\right\}\right). \tag{41}$$

IV. Relations of similarity and difference

V. Kinds of similarity and difference relations

Based on the introduced measures, let us consider types (characteristic functions) of similarity and dissimilarity relations. We will consider some types of n-ary similarity relations, but mainly we will consider types of binary similarity relations.

The 1-2-similarity relation is a special case of the ij-similarity relation.

Definition 33. A right 1-2-similarity relation is an n-ary relation satisfying:

$$\left(\mu\left(\langle x_1,x_2,x_3,...x_n\rangle\right)=1\right)\to \\ \left(\exists c\left(\left(\mu\left(\langle c,x_2,x_3...x_n\rangle\right)=1\right)\wedge\left(\mu\left(\langle c,x_1,x_n...,x_3\rangle\right)=1\right)\right)\right). \tag{42}$$

Definition 34. A left 1-2-similarity relation is an n-ary relation satisfying:

$$\begin{gathered}(\mu(\langle x_1, x_2, x_3, ...x_n\rangle) = 1) \to \\ (\exists c((\mu(\langle x_1, c, x_3...x_n\rangle) = 1) \wedge (\mu(\langle x_2, c, x_n..., x_3\rangle) = 1))).\end{gathered} \tag{43}$$

Definition 35. A 1-2-similarity relation is a right and left 1-2-similarity relation.

Definition 36. A total right 1-2-similarity relation is an n-ary relation satisfying:

$$\begin{gathered}(\langle x_1, x_2, x_3, ...x_n\rangle \in X^n) \to \\ (\exists c((\mu(\langle c, x_2, x_3...x_n\rangle) = 1) \wedge (\mu(\langle c, x_1, x_n..., x_3\rangle) = 1))).\end{gathered} \tag{44}$$

Definition 37. A total left 1-2-similarity relation is an n-ary relation satisfying:

$$\begin{gathered}(\langle x_1, x_2, x_3, ...x_n\rangle \in X^n) \to \\ (\exists c((\mu(\langle x_1, c, x_3...x_n\rangle) = 1) \wedge (\mu(\langle x_2, c, x_n..., x_3\rangle) = 1))).\end{gathered} \tag{45}$$

Definition 38. A total 1-2-similarity relation is an total right and total left 1-2-similarity n-ary relation.

Obviously, a relation of (left\right) total 1-2 similarity is a relation of (left\right) 1-2 similarity.

Example. The ternary 1-2-similarity relation is satisfied:

$$\begin{gathered}(\mu(\langle x, y, z\rangle) = 1) \to \\ (\exists c((\mu(\langle c, y, z\rangle) = 1) \wedge (\mu(\langle c, x, z\rangle) = 1))).\end{gathered} \tag{46}$$

The 1-2-reflexive relation is a special case of the ij-reflexive relation.

Definition 39. A 1-2-reflexive relation is an n-ary relation satisfying:

$$(\langle x_1, x_2, x_3, ...x_n\rangle \in X^n) \to (\mu(\langle x_1, x_1, x_3...x_n\rangle) = 1). \tag{47}$$

Obviously, any 1-2-reflexive relation is a 1-2-similarity relation.

Definition 40. A right 1-2-quasi-reflexive relation is an n-ary relation satisfying:

$$(\mu(\langle x_1, x_2, x_3...x_n\rangle) = 1) \to (\mu(\langle x_1, x_1, x_3...x_n\rangle) = 1). \tag{48}$$

Definition 41. A left 1-2-quasi-reflexive relation is an n-ary relation satisfying:

$$(\mu(\langle x_1, x_2, x_3...x_n\rangle) = 1) \to (\mu(\langle x_2, x_2, x_3...x_n\rangle) = 1). \tag{49}$$

Definition 42. A 1-2-quasireflexive relation is an n-ary right and left 1-2-quasireflexive relation.

Obviously, any 1-2-reflexive relation is 1-2-quasi-reflexive.

Definition 43. A right similarity (binary similarity relation) is a binary relation satisfying:

$$(\mu(\langle a, b\rangle) = 1) \to (\exists c((\mu(\langle c, a\rangle) = 1) \wedge (\mu(\langle c, b\rangle) = 1))). \tag{50}$$

Definition 44. A left similarity (binary similarity relation) is a binary relation satisfying:

$$(\mu(\langle a, b\rangle) = 1) \to (\exists c((\mu(\langle a, c\rangle) = 1) \wedge (\mu(\langle b, c\rangle) = 1))). \tag{51}$$

Definition 45. A similarity (binary similarity relation) is a right and left similarity binary relation.

Definition 46. A total right similarity is a binary relation satisfying:

$$(\langle a, b\rangle \in X \times X) \to (\exists c((\mu(\langle c, a\rangle) = 1) \wedge (\mu(\langle c, b\rangle) = 1))). \tag{52}$$

Definition 47. A total left similarity is a binary relation satisfying:

$$(\langle a, b\rangle \in X \times X) \to (\exists c((\mu(\langle a, c\rangle) = 1) \wedge (\mu(\langle b, c\rangle) = 1))). \tag{53}$$

Definition 48. Total similarity is a total right and total left similarity binary relation.

Definition 49. Right quasisimilarity is a binary relation satisfying:

$$(\mu(\langle a, b\rangle) = 0) \to (\exists c((\mu(\langle c, a\rangle) = 1) \wedge (\mu(\langle c, b\rangle) = 1))). \tag{54}$$

Definition 50. Left quasisimilarity is a binary relation satisfying:

$$(\mu(\langle a, b\rangle) = 0) \to (\exists c((\mu(\langle a, c\rangle) = 1) \wedge (\mu(\langle b, c\rangle) = 1))). \tag{55}$$

Definition 51. A quasi-similarity is a right and left quasi-similarity binary relation.

A reflexive binary similarity relation will be called a stable similarity relation, otherwise a binary similarity relation will be called an unstable similarity relation. similarity relations are defined below.

Quasireflexivity of binary relations, the left and right quasireflexivity of binary relations is defined in the standard way as a special case of 1-2-quasireflexivity.

Other useful properties of similarity and binary relations are defined below.

Definition 52. Right antiquasireflexivity of a binary relation:

$$((\mu(\langle a, b\rangle) = 0) \wedge (\neg(a = b))) \to (\mu(\langle a, a\rangle) = 1). \tag{56}$$

Definition 53. Left antiquasireflexivity of a binary relation:

$$((\mu(\langle a, b\rangle) = 0) \wedge (\neg(a = b))) \to (\mu(\langle b, b\rangle) = 1). \tag{57}$$

Definition 54. Antiquasireflexivity of a binary relation:

$$((\mu(\langle a, b\rangle) = 0) \wedge (\neg(a = b))) \to (\mu(\langle b, b\rangle) = 1). \tag{58}$$

Definition 55. Weak right quasireflexivity of a binary relation:

$$\begin{gathered}((\mu(\langle a, b\rangle) = 0) \wedge (\neg(a = b))) \to \\ ((\mu(\langle a, a\rangle) = 1) \wedge (\mu(\langle b, b\rangle) = 1)).\end{gathered} \tag{59}$$

Definition 56. Weak left quasireflexivity of a binary relation:

$$((\mu(\langle a, b\rangle) = 1) \wedge (\mu(\langle b, a\rangle) = 0)) \to (\mu(\langle b, b\rangle) = 1). \tag{60}$$

Definition 57. Weak quasireflexivity of a binary relation:

$$\begin{gathered}((\mu(\langle a, b\rangle) = 1) \wedge (\mu(\langle b, a\rangle) = 0)) \to \\ ((\mu(\langle a, a\rangle) = 1) \wedge (\mu(\langle b, b\rangle) = 1)).\end{gathered} \tag{61}$$

The symmetric stable similarity relation will be called the convergence relation.

The transitive stable similarity relation will be called the resemblance relation.

Definition 58. Weak transitivity of a binary relation:

$$\begin{gathered}(((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle b,c\rangle)=1))\wedge\\((\mu(\langle c,b\rangle)=0)\wedge(\mu(\langle b,a\rangle)=0)))\rightarrow\\(\mu(\langle a,b\rangle)=1)\,.\end{gathered}\tag{62}$$

Definition 59. Antiantitransitivity of a binary relation:

$$((\mu(\langle a,b\rangle)=0)\wedge(\mu(\langle b,c\rangle)=0))\rightarrow(\mu(\langle a,c\rangle)=1).\tag{63}$$

Definition 60. Quasitransitivity of a binary relation:

$$\begin{gathered}(((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle b,c\rangle)=1))\wedge\\((\mu(\langle c,b\rangle)=0)\wedge(\mu(\langle b,a\rangle)=0)))\rightarrow\\((\mu(\langle a,c\rangle)=1)\wedge(\mu(\langle c,a\rangle)=0))\,.\end{gathered}\tag{64}$$

Definition 61. Euclidean binary relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=1)\wedge(\mu(\langle c,a\rangle)=1))\vee\\((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle a,c\rangle)=1)))\rightarrow\\(\mu(\langle b,c\rangle)=1)\,.\end{gathered}\tag{65}$$

Definition 62. Left Euclidean binary relation:

$$((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle a,c\rangle)=1))\rightarrow(\mu(\langle b,c\rangle)=1)\,.\tag{66}$$

Definition 63. Right Euclidean binary relation:

$$((\mu(\langle b,a\rangle)=1)\wedge(\mu(\langle c,a\rangle)=1))\rightarrow(\mu(\langle b,c\rangle)=1)\,.\tag{67}$$

Definition 64. Sub-Euclidean binary relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=1)\wedge(\mu(\langle c,a\rangle)=1))\wedge\\((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle a,c\rangle)=1)))\rightarrow(\mu(\langle b,c\rangle)=1)\,.\end{gathered}\tag{68}$$

Definition 65. Quasi-Euclidean binary relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=1)\wedge(\mu(\langle c,a\rangle)=1))\wedge\\((\mu(\langle a,b\rangle)=0)\wedge(\mu(\langle a,c\rangle)=0)))\rightarrow\\(\mu(\langle b,c\rangle)=1)\\(((\mu(\langle b,a\rangle)=0)\wedge(\mu(\langle c,a\rangle)=0))\wedge\\((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle a,c\rangle)=1)))\rightarrow\\(\mu(\langle b,c\rangle)=1)\,.\end{gathered}\tag{69}$$

Definition 66. Left quasi-Euclidean binary relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=1)\wedge(\mu(\langle c,a\rangle)=1))\wedge\\((\mu(\langle a,b\rangle)=0)\wedge(\mu(\langle a,c\rangle)=0)))\rightarrow\\(\mu(\langle b,c\rangle)=1)\,.\end{gathered}\tag{70}$$

Definition 67. Right quasi-Euclidean binary relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=0)\wedge(\mu(\langle c,a\rangle)=0))\wedge\\((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle a,c\rangle)=1)))\rightarrow\\(\mu(\langle b,c\rangle)=1)\,.\end{gathered}\tag{71}$$

Definition 68. Left antidivergence of a binary relation:

$$((\mu(\langle a,c\rangle)=0)\wedge(\mu(\langle b,c\rangle)=0))\rightarrow(\mu(\langle a,b\rangle)=1)\,.\tag{72}$$

Definition 69. Right antidivergence of a binary relation:

$$((\mu(\langle c,a\rangle)=0)\wedge(\mu(\langle c,b\rangle)=0))\rightarrow(\mu(\langle a,b\rangle)=1)\,.\tag{73}$$

Definition 70. Antidivergence of a binary relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=0)\wedge(\mu(\langle c,a\rangle)=0))\vee\\((\mu(\langle a,b\rangle)=0)\wedge(\mu(\langle a,c\rangle)=0)))\rightarrow\\(\mu(\langle b,c\rangle)=1)\,.\end{gathered}\tag{74}$$

Definition 71. Quasi-difference (binary quasi-difference relation):

$$\begin{gathered}(\mu(\langle a,b\rangle)=1)\rightarrow\\(\exists\alpha(\exists c((\alpha\in\{-1,1\})\wedge((\mu(\langle a,c\rangle)+\alpha=\mu(\langle b,c\rangle))\vee\\(\mu(\langle c,a\rangle)+\alpha=\mu(\langle c,b\rangle))))))).\end{gathered}\tag{75}$$

Definition 72. Total quasi-difference (total binary quasi-difference relation):

$$\begin{gathered}(\langle a,b\rangle\in X\times X)\rightarrow\\(\exists\alpha(\exists c((\alpha\in\{-1,1\})\wedge((\mu(\langle a,c\rangle)+\alpha=\mu(\langle b,c\rangle))\vee\\(\mu(\langle c,a\rangle)+\alpha=\mu(\langle c,b\rangle))))))).\end{gathered}\tag{76}$$

Definition 73. Right quasi-difference:

$$\begin{gathered}(\mu(\langle a,b\rangle)=1)\rightarrow\\(\exists c((\mu(\langle a,c\rangle)=1+\mu(\langle b,c\rangle))\vee\\(\mu(\langle b,c\rangle)=1+\mu(\langle a,c\rangle)))).\end{gathered}\tag{77}$$

Definition 74. Total right quasi-difference:

$$\begin{gathered}(\langle a,b\rangle\in X\times X)\rightarrow\\(\exists c((\mu(\langle a,c\rangle)=1+\mu(\langle b,c\rangle))\vee\\(\mu(\langle b,c\rangle)=1+\mu(\langle a,c\rangle)))).\end{gathered}\tag{78}$$

Definition 75. Left quasi-difference:

$$\begin{gathered}(\mu(\langle a,b\rangle)=1)\rightarrow\\(\exists c((\mu(\langle c,a\rangle)=1+\mu(\langle c,b\rangle))\vee\\(\mu(\langle c,b\rangle)=1+\mu(\langle c,a\rangle)))).\end{gathered}\tag{79}$$

Definition 76. Total left quasi-difference:

$$\begin{gathered}(\langle a,b\rangle\in X\times X)\rightarrow\\(\exists c((\mu(\langle c,a\rangle)=1+\mu(\langle c,b\rangle))\vee\\(\mu(\langle c,b\rangle)=1+\mu(\langle c,a\rangle)))).\end{gathered}\tag{80}$$

Definition 77. Left and right quasi-difference:

$$\begin{gathered}(\mu(\langle a,b\rangle)=1)\rightarrow\\(\exists c((\mu(\langle a,c\rangle)=1+\mu(\langle b,c\rangle))\vee\\(\mu(\langle b,c\rangle)=1+\mu(\langle a,c\rangle))))\\(\mu(\langle a,b\rangle)=1)\rightarrow\\(\exists c((\mu(\langle c,a\rangle)=1+\mu(\langle c,b\rangle))\vee\\(\mu(\langle c,b\rangle)=1+\mu(\langle c,a\rangle)))).\end{gathered}\tag{81}$$

Definition 78. Total left and right quasi-difference:

$$\begin{gathered}(\langle a,b\rangle\in X\times X)\rightarrow\\(\exists c((\mu(\langle a,c\rangle)=1+\mu(\langle b,c\rangle))\vee\\(\mu(\langle b,c\rangle)=1+\mu(\langle a,c\rangle))))\\(\langle a,b\rangle\in X\times X)\rightarrow\\(\exists c((\mu(\langle c,a\rangle)=1+\mu(\langle c,b\rangle))\vee\\(\mu(\langle c,b\rangle)=1+\mu(\langle c,a\rangle)))).\end{gathered}\tag{82}$$

One of the basic properties of difference relations is irreflexivity.

Other useful properties of binary and difference relations are defined below.

Definition 79. Right quasi-reflexivity:

$$(\mu(\langle a,b\rangle)=0)\to(\mu(\langle a,a\rangle)=0)\,. \tag{83}$$

Definition 80. Right antiquasi-reflexivity:

$$((\mu(\langle a,b\rangle)=1)\wedge(\neg(a=b)))\to(\mu(\langle a,a\rangle)=0)\,. \tag{84}$$

Definition 81. Left quasi-reflexivity:

$$(\mu(\langle a,b\rangle)=0)\to(\mu(\langle b,b\rangle)=0)\,. \tag{85}$$

Definition 82. Left antiquasi-reflexivity:

$$((\mu(\langle a,b\rangle)=1)\wedge(\neg(a=b)))\to(\mu(\langle b,b\rangle)=0)\,. \tag{86}$$

Definition 83. Quasi-reflexivity:

$$(\mu(\langle a,b\rangle)=0)\to((\mu(\langle a,a\rangle)=0)\wedge(\mu(\langle b,b\rangle)=0))\,. \tag{87}$$

Definition 84. Antiquasi-reflexivity:

$$\begin{gathered}((\mu(\langle a,b\rangle)=1)\wedge(\neg(a=b)))\to\\((\mu(\langle a,a\rangle)=0)\wedge(\mu(\langle b,b\rangle)=0))\,.\end{gathered} \tag{88}$$

Definition 85. Antitransitivity:

$$((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle b,c\rangle)=1))\to(\mu(\langle a,c\rangle)=0). \tag{89}$$

Definition 86. Antiantiantitransitivity:

$$((\mu(\langle a,b\rangle)=0)\wedge(\mu(\langle b,c\rangle)=0))\to(\mu(\langle a,c\rangle)=0). \tag{90}$$

Definition 87. Weak antitransitivity:

$$\begin{gathered}(((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle b,c\rangle)=1))\wedge\\((\mu(\langle c,b\rangle)=0)\wedge(\mu(\langle b,a\rangle)=0)))\to\\(\mu(\langle c,a\rangle)=0).\end{gathered} \tag{91}$$

Definition 88. Antiquasi-Euclidean relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=1)\wedge(\mu(\langle c,a\rangle)=1))\wedge\\((\mu(\langle a,b\rangle)=0)\wedge(\mu(\langle a,c\rangle)=0)))\to\\(\mu(\langle b,c\rangle)=0)\\(((\mu(\langle b,a\rangle)=0)\wedge(\mu(\langle c,a\rangle)=0))\wedge\\((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle a,c\rangle)=1)))\to\\(\mu(\langle b,c\rangle)=0).\end{gathered} \tag{92}$$

Definition 89. Left antiquasi-Euclidean relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=1)\wedge(\mu(\langle c,a\rangle)=1))\wedge\\((\mu(\langle a,b\rangle)=0)\wedge(\mu(\langle a,c\rangle)=0)))\to\\(\mu(\langle b,c\rangle)=0).\end{gathered} \tag{93}$$

Definition 90. Right antiquasi-Euclidean relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=0)\wedge(\mu(\langle c,a\rangle)=0))\wedge\\((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle a,c\rangle)=1)))\to\\(\mu(\langle b,c\rangle)=0).\end{gathered} \tag{94}$$

Definition 91. Anti-Euclidean relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=0)\wedge(\mu(\langle c,a\rangle)=0))\vee\\((\mu(\langle a,b\rangle)=0)\wedge(\mu(\langle a,c\rangle)=0)))\to\\(\mu(\langle b,c\rangle)=0).\end{gathered} \tag{95}$$

Definition 92. Left anti-Euclidean relation:

$$((\mu(\langle a,b\rangle)=0)\wedge(\mu(\langle a,c\rangle)=0))\to(\mu(\langle b,c\rangle)=0)\,. \tag{96}$$

Definition 93. Right anti-Euclidean relation:

$$((\mu(\langle b,a\rangle)=0)\wedge(\mu(\langle c,a\rangle)=0))\to(\mu(\langle b,c\rangle)=0)\,. \tag{97}$$

Definition 94. Antisub-Euclidean relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=0)\wedge(\mu(\langle c,a\rangle)=0))\wedge\\((\mu(\langle a,b\rangle)=0)\wedge(\mu(\langle a,c\rangle)=0)))\to\\(\mu(\langle b,c\rangle)=0).\end{gathered} \tag{98}$$

Definition 95. Left divergent relation:

$$((\mu(\langle a,c\rangle)=1)\wedge(\mu(\langle b,c\rangle)=1))\to(\mu(\langle a,b\rangle)=0)\,. \tag{99}$$

Definition 96. Right divergent relation:

$$((\mu(\langle c,a\rangle)=1)\wedge(\mu(\langle c,b\rangle)=1))\to(\mu(\langle a,b\rangle)=0)\,. \tag{100}$$

Definition 97. Left and right divergent relation:

$$\begin{gathered}(((\mu(\langle b,a\rangle)=1)\wedge(\mu(\langle c,a\rangle)=1))\vee\\((\mu(\langle a,b\rangle)=1)\wedge(\mu(\langle a,c\rangle)=1)))\to\\(\mu(\langle b,c\rangle)=0)\,.\end{gathered} \tag{101}$$

Similarity and difference relations can be classified according to the field of definition and its relation to the type of the subject area: similarity and difference relations of biological species, chemical similarity and difference relations, similarity and difference relations of physical processes [32], [34], [35]. From the point of view of knowledge representation, including within the semantic space, the following types of binary relations can be distinguished among similarity and difference relations.

Relations of similarity and difference of ontological structures on the basis of syntactic features: qualitative (structural) and quantitative (including – measures) [55]. For example, similarity and difference by absence of cycles on the incident links of signs, by the number of types of alphabet elements: nodes and edges. A separate example of similarity relation is the relation of coincidence of elements of ontological structures [21].

Relations of similarity and difference of static ontological structures on the basis of semantic attributes (attributes of denotational semantics, primarily related to essentially-systematic properties): qualitative (structural), quantitative (including measures). For example, similarity and difference by the presence of substructures of a certain kind (subgraphs, motifs), morphisms between structures (semantically stable), similarity and difference of signs of sets by their power, similarity and difference of signs by the power of extensional closures (essential, fully-connected, etc.). Similarity and difference of (semantically stable) structures (extensional closures) by the number of elements of a given kind, by the number of morphisms, by the number of substructures, motifs, by spectral characteristics of structures and other invariants.

The ontology of morphism relations, which are similarity relations, was developed earlier and is part of the ontology of measures and similarity and difference relations [25].

Relations of similarity and difference of dynamic ontological structures on the basis of semantic attributes (attributes of operational semantics, primarily related to relationally-phenomenological properties): qualitative (structural), quantitative (including measures). For example, similarity and difference in properties of operations [7] (determinacy, reversibility, returnability, etc.), spatio-temporal properties [1], [42], possibility of simulation [5], [7], [43]. Similarity and dissimilarity in measures of information quantity, computational complexity (spatio-temporal), period [25], degree of parallelism, etc.

Relations of similarity and difference of ontological structures on the basis of semantic-logical attributes (attributes of semantics related primarily to intensionally-logical properties): qualitative (structural), quantitative (including – measures). For example, similarity relations by the kind and structure of utterance forms, statements and axiomatizations, by logical consequence, by the measure of truth and other quantitative features.

Separate types of similarity and difference relations are meta-relations of similarity and difference.

A. *Meta-relations of similarity and difference*

The meta-relations of similarity include binary meta-relations of equality, equivalence, morphisms of binary relations, binary meta-relations of closures of binary relations: reflexive, symmetric and transitive closures, etc.

The meta-relations of difference include binary meta-relations of non-trivial permutations. Binary meta-relations of nontrivial permutations, in particular, the binary meta-relations of the complement of a binary relation (up to the Cartesian square of the domain of its definition) can be referred to meta-relations of difference.

In the study of semantic space, static ontological structures at the entity-system level (semantic), which are stable to the addition of new fragments of ontologies, can be investigated first of all. Previously, the hereditary finite ontological structures of classical and non-classical [61] kinds were investigated (see fig.1), and their ontology was constructed on the basis of morphisms between them, which are special cases of similarity relations (see fig.2). Topological characterizations of ontological structures exploit the transitive closure (similarity meta-relations) of the membership relations associated with the extensional concepts, elements of ontological structures.

The introduction of the metric on these structures encounters the fact that the topology of semantically stable structures is directional (oriented), so the transition to the metric requires a symmetric closure of the corresponding topological relations, i.e., the application of a similarity meta-relations. In addition, due to the flexibility of the knowledge representation model used, this metric is obtained bounded (maximal value). The only way to ensure metric distinction of elements of ontological structures, while preserving integer values of the metric, is to increase the dimensionality of the space. As it has already been established, under some natural requirements for the distinction of elements of ontological structures, the dimensionality of space in this case will grow linearly with the number of elements in the [56] structure. A metric in a multidimensional space can be obtained by introducing one-dimensional metrics on each dimension and integrating them through the application of Theorem 1. In this case, metrics are introduced for canonical forms of ontological structures in order to ensure unambiguity of semantically stable metrics, allowing to pass to a canonical metric space integrated into a semantic space in accordance with the metamodel of semantic space [25]. Examples of metrics besides the distance between elements in the ontological structure can also be the powers of symmetric differences (sets of elements) of element's extensions, sets of elements of extensional (essential, fully-connected) closures, their sections, etc. The metrics can also be the power of symmetric differences (sets of elements) of extensional elements. However, to be able to fully work with such a metric space, ontological structures must be fully represented (defined).

VI. Measures and similarity in models with operational semantics

A. *Operational information space*

Definition 98. An operation-information space [25] is given by a transition relation R between configurations from set C, set of operations O, set of operators (command or instructions) K, set of parameters P, and set of values V.

$$\begin{array}{l} R \subseteq C \times C \\ O \subseteq 2^{V^2 \times V} \cup 2^{V \times V^2} \\ C \subseteq 2^{K \cup P \cup (P \times (V \cup K)) \cup (K \times (O \cup P))} \end{array} . \tag{102}$$

The introduced space can be considered as texts of a generalized formal language [25] in some alphabet A.

$$K \cup O \cup P \cup V \subseteq A^{(*^*)}. \tag{103}$$

Let us enumerate the structural (syntactic) properties of configurations.

Statement 1. Operators and parameter values are distinguishable in each configuration.

$$(c \in C) \to (c \cap (P \times (K \cap V)) = \emptyset) . \tag{104}$$

Statement 2. In each configuration, operations and parameters are distinguishable.

$$(c \in C) \to (c \cap (K \times (O \cap P)) = \emptyset) . \tag{105}$$

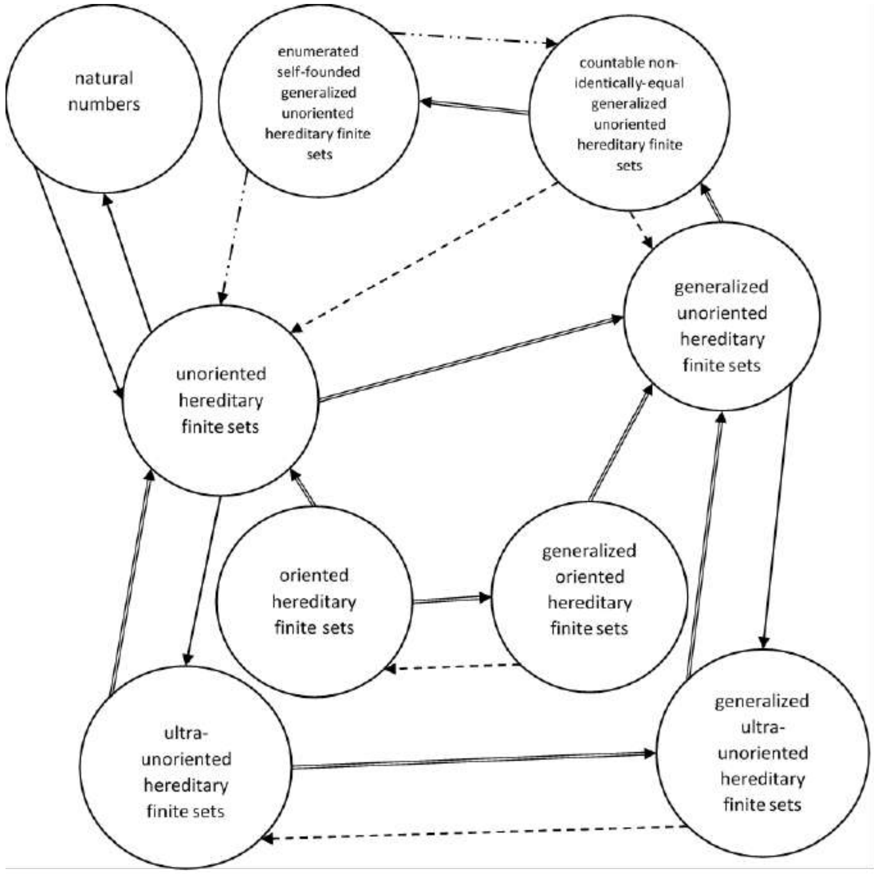

Figure 1. General classes of finite structures

Statement 3. If an operator has two input parameters, it has one output parameter.

$$(c \in C) \to ((k \in c \cap K) \to \\ ((|c \cap (\{k\} \times P)| = 2) \to (|c \cap (P \times \{k\})| = 1))). \tag{106}$$

Statement 4. If an operator has one input parameter, it has at most one output parameter.

$$(c \in C) \to ((k \in c \cap K) \to \\ ((|c \cap (\{k\} \times P)| = 1) \to (1 \le |c \cap (P \times \{k\})| \le 2))). \tag{107}$$

Statement 5. Each operator has at least one and no more than two input and output parameters.

$$(c \in C) \to ((k \in c \cap K) \to \\ ((1 \le |c \cap (\{k\} \times P)| \le 2) \wedge (1 \le |c \cap (P \times \{k\})| \le 2))). \tag{108}$$

Relation of the operational semantics of "big" and "small step". Operational (semantic) properties of configurations:

$$(\langle c, s \rangle \in R) \sim \exists q(\left(q \in 2^{(c \cap K)} / \{\emptyset\}\right) \wedge (s \cap (P \times V) = \\ \left(c \cap (P \times V) / \bigcup_{k \in q} (\delta(\langle c, k \rangle))_1\right) \cup \bigcup_{k \in q} (\delta(\langle c, k \rangle))_2 t)). \tag{109}$$

i.e., the transition can be made only to the next configuration that contains results of all operations of some non-empty set of commands and does not contain input data processed by operations of commands from this set.

Definition 99. Changes (possible) resulting from an operation on command k in configuration c:

$$\delta(\langle c, k \rangle) \in \Delta(\langle c, k \rangle). \tag{110}$$

Definition 100. The set of (possible) changes resulting from an operation on command k in configuration c:

$$\Delta(\langle c, k \rangle) \overset{def}{=} \\ \{\langle r, \{(c \cap (\{k\} \times P))(k)\} \times \{((c \cap (\{k\} \times O))(k))(v)\}\rangle \\ \left|\left(v \in \times_{p \in r} \{p_2\}\right) \wedge (r \in \Lambda(\langle c, k \rangle))\right.\} \tag{111}$$

Definition 101. The set of values of k command inputs in configuration c:

$$\Lambda(\langle c, k \rangle) \overset{def}{=} \\ \left\{x \left| x = \bigcup\nolimits_{p \in \Gamma(\langle c, k \rangle)} \{\langle p, (c \cap (\Gamma(\langle c, k \rangle) \times V))(p) \rangle\}\right.\right\}. \tag{112}$$

Definition 102. The set of command inputs k in configuration c:

$$\Gamma(\langle c, k \rangle) \overset{def}{=} \left\{x \left| x = (c \cap (P \times \{k\}))^{-1}(k)\right.\right\}. \tag{113}$$

Statement 6. The number of configurations to which a transition is possible is equal to the number of all possible

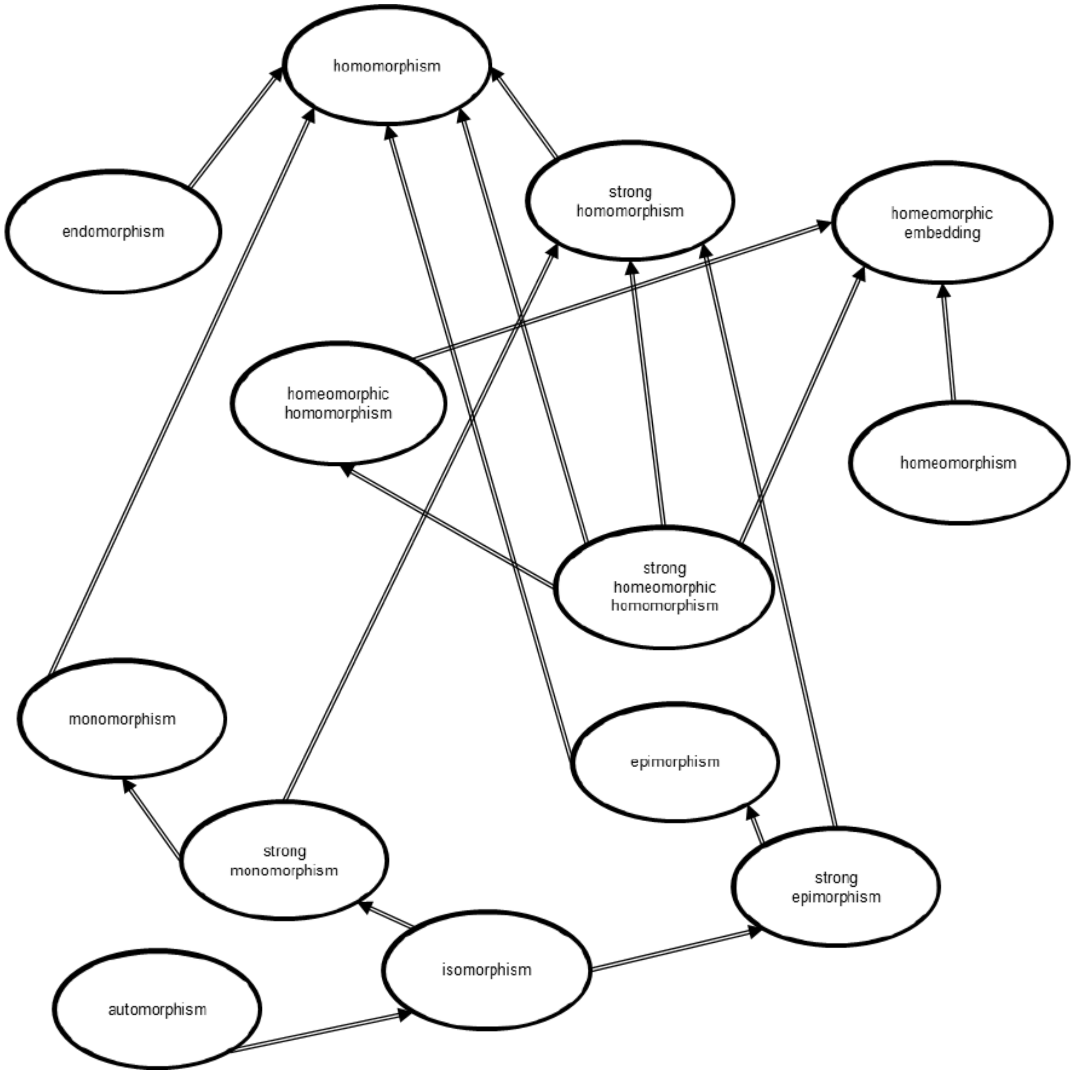

Figure 2. Ontology of morphisms

different changes caused by the operations of all possible non-empty instruction sets:

$$(c \in C) \rightarrow \left(\left| \{ R(c) \} \right| = \left| \left\{ \bigcup_{k \in q} \Delta(\langle c, k \rangle) \,\middle|\, q \in \left(2^{c \cap K} / \{\emptyset\} \right) \right\} \right| \right). \quad (114)$$

Operational-information space allows to connect the operational semantics of “small step” with the operational semantics of “big step”. The operational-information space can be viewed as a topological space on a graph with a transitive closure R or a metric space with a symmetric closure R. Similarly as a topological or metric space each of the configurations can be considered.

The operational-information space, having similarities with the model-parametric spac [60], is oriented to support the implementation of knowledge base components with complex operational semantics (see Reo Coordination Language [26]) and the design of problem-solving models for intelligent systems based on reusable and reusable components as a unified semantic model [29]. On the other hand, the high degree of unification of ontological structures of the operational-information space allows us to consider it as a subspace of the semantic space [25], [44].

The design of problem-solving models for intelligent systems relies on the previously discussed technological principles, which include the typology of reusable components, consistent with the general classification of tasks [7]. It is also important to observe the properties of coreturnability and parareversibility in the development and adaptation of the intellectual system (and its components) [54], which, if necessary, allows to pass to the returnability of operations realized by the system (component), along with the coreturnability being the basis of its repeated and multiple application for solving tasks of the same class.

Since the operational semantics within the framework of the operational-information space is reduced to the relation of becoming [25] coordinated with the denotational semantics, the operational-information space is a model not only for studying the properties of internal operations and corresponding processes, but also the phenomena and processes conditioned by the interaction of the intellectual system with the external environment, i.e. related to cognition. Thus, on this basis by means of mechanisms of semantic logging of events (repeated or not), methods of analyzing protocols of processes and phenomena, by revealing relations of similarity and difference, the study of spatial-temporal, causal relations and attributes between phenomena, subjects and objects [43] is carried out. The introduction of corresponding order relations, which allow to specify and investigate topological properties, as well as the introduction of measures, which allow to specify and investigate metrical, including spatio-temporal, properties, complements the topology and metrics of the semantic space [25].

B. *Measures of dynamic structures*

Let us consider pseudo-orgraphs corresponding to some orgraphs of configurations of information-operational space. Let us introduce a binary (meta)operation of the tensor product of pseudo-orgraphs and the relation to be a divisor of the tensor product of pseudo-orgraphs.

Definition 103. Tensor product of pseudo-orgraphs [7]:

$$\begin{gathered} G \times_G H = \langle V_G, E_G \rangle \times_G \langle V_H, E_H \rangle \overset{def}{=} \\ \langle V_G \times V_H, \{ \langle \langle y, v \rangle, \langle u, w \rangle \rangle | \langle \langle y, u \rangle, \langle v, w \rangle \rangle \in E_G \times E_H \} \rangle . \end{gathered} \tag{115}$$

Definition 104. The divisor of the tensor product of pseudo-orgraphs.

$$\begin{gathered} (G \upharpoonright_G H) \sim \\ (\exists X ((H = (G \times_G X)) \vee (H = (X \times_G G)))) . \end{gathered} \tag{116}$$

Let us consider the time-averaged entropy E of [25] wave fronts of the equilibrium state of a knowledge processing model (system) with a pseudo-orgraphic structure of returnable operations. Returnable operations are one of the three key features for constructing a general classification of operations and the tasks (problems) solved with their help [7].

Theorem 2: Entropy E is a $\upharpoonright_G$-monotone measure:

$$(G \upharpoonright_G H) \to (E(G) \leq E(H)) . \tag{117}$$

The entropy E satisfies four properties of the Hinchin entropy.

Property 1. The entropy of a system s is a function only of the probabilities of its n states $P(s) = \{p_1(s)\} \times \{p_2(s)\} \times ... \times \{p_n(s)\}$:

$$E \in (Range(E))^{\bigcup_s P(s)}. \tag{118}$$

Property 2. For any system s with n states it is true that:

$$E(P(s)) \leq E\left(\left\{n^{-1}\right\}^n\right), \tag{119}$$

where $E\left(\left\{n^{-1}\right\}^n\right)$ is the entropy of a system with uniform probability distribution of n states.

Property 3. If $n+1$th state $p_{n+1}(s) = 0$ is added to the system s, the entropy will not change.

Property 4. The entropy of the set of two independent systems s and m is of the form:

$$E(P(s \otimes m)) = E(P(s)) + E(P(m)) . \tag{120}$$

Statement 7. The entropy for classes of isomorphic (multi)pseudo-orgraphs is $[\times_G]_{\cong}$-additive measure in the algebra of the complement $\overline{[\,]}_{\cong}$, the GCD $[\curlywedge_G]_{\cong}$ and the LCM $[\curlyvee_G]_{\cong}$ of some class of isomorphic (multi)pseudo-orgraphs $\overline{[\bigcirc]}_{\cong}$ and the classes of their tensor product divisors:

$$E\left([G]_{\cong}[\times_G]_{\cong}[H]_{\cong}\right) = E\left([G]_{\cong}\right) + E\left([H]_{\cong}\right) . \tag{121}$$

Some properties of the operations of this algebra:

$$\begin{gathered} \left([G]_{\cong}[\upharpoonright_G]_{\cong}\left([G]_{\cong}[\curlyvee_G]_{\cong}[H]_{\cong}\right)\right), \\ \left(\left([G]_{\cong}[\curlywedge_G]_{\cong}[H]_{\cong}\right)[\upharpoonright_G]_{\cong}[G]_{\cong}\right), \\ \left(\left([G]_{\cong}[\curlyvee_G]_{\cong}[H]_{\cong}\right)[\times_G]_{\cong}\left([G]_{\cong}[\curlywedge_G]_{\cong}[H]_{\cong}\right)\right) = \\ \left([G]_{\cong}[\times_G]_{\cong}[H]_{\cong}\right), \\ \left(\overline{[G]}_{\cong}[\curlyvee_G]_{\cong}\overline{[H]}_{\cong}\right) = \overline{\left[\left([G]_{\cong}[\curlywedge_G]_{\cong}[H]_{\cong}\right)\right]}_{\cong}, \\ \overline{\overline{[G]}}_{\cong} = [G]_{\cong}. \end{gathered} \tag{122}$$

The proposed measure differs from the von Neumann entropy [30], the Laplacian entropy (Kirchhoff matrix [32]) and the normalized Laplacian [32] of an oriented graph (multipseudo-orgraph).

$$E\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & 0 \end{pmatrix} = (0.8 * \ln(2) - \ln(5)) / \ln(2) \tag{123}$$

There are similarities and differences between models with operational semantics based on the introduced measure, also based on it we can consider convergence and divergence measures [35].

The following transformations allow us to introduce meta-operations and specify the change of a dynamic system, analogous to the way a semiotic system specifies the change of a formal system [13]. Thus, the dynamics of the system can be described along more than one temporal axis.

Addition of multipseudo-orgraphs. Examples:

$$\begin{gathered} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 \end{pmatrix} \end{gathered} \tag{124}$$

Vertex permutation (permutation matrix is used). Example:

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} * \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} * \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{125}$$

Splitting vertices of a multipseudo-orgraph using diagonal rectangular matrices. Examples:

$$\begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} * \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & 0 \end{pmatrix} * \begin{pmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{3}/\sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & 0 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{3}/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 \end{pmatrix} \tag{126}$$

Inverse transformation (gluing) of vertices using diagonal rectangular matrices. Example:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & \sqrt{2}/\sqrt{3} \end{pmatrix} * \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 \end{pmatrix} * \begin{pmatrix} \sqrt{2}/\sqrt{3} & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & 0 \end{pmatrix} \tag{127}$$

VII. Logic models. Integration, parametrization, generalization

A. Similarity, inductive logical inference and multi-valued logics

Logic models aimed at supporting and integrating deductive, abductive, and inductive logical inference mechanisms citebTaxonomy, in particular those based on the application of the DSM-method and the JSM-method, use multi-valued [4] logics. DSM-method and JSM-method, multi-valued [4] logics are used. These logics are based on four-valued logic using non-associative binary logic operations of conjunction and disjunction, which requires consideration of non-binary logic operations. Increasing the number of values allows us to rebalance the complexity between the operational and denotational semantics of logics of the corresponding [40] class. For example, we can eliminate the use of non-binary logical operations in such logics, by switching from four-valued logics to six-valued logics. The DSM method and the JSM method are based on mechanisms that utilize similarity and dissimilarity features and relations.

B. Integration of logical models of knowledge processing

Previously, a model and principles for integrating logical models of knowledge processing have been proposed [5].

An important property during integration is the possibility of bisimulation [58].

It should be noted that when integrating logical models of knowledge processing, it is important to consider the support of different types of semantics [4], [37], [38], [38]–[41]: axiomatic, algebraic, relational and others. In this regard, let us consider the integration of temporal logics and fuzzy logics on the basis of parameterized fuzzy logics.

Table I
Integrated logic models

Semantics	Double-valued logic	Multi-valued logic	
			Fuzzy logic
Predicate Logic	+	+	+
Modal Logic	+	+	+
Temporal logic	+	+	

C. Parameterized temporal fuzzy logic

Let us consider the generalization of parameterized fuzzy logics [53] to temporal logics.

Definition 105. Fuzzy negation:

$$\mu_{\sim\alpha}\left(\langle\chi,\pi\rangle\right) \stackrel{def}{=} \sim^P_\lambda \mu_\alpha\left(\langle\chi,\pi\rangle\right). \tag{128}$$

It is usually true for fuzzy negation:

$$\mu_{\sim\alpha}\left(\langle\chi,\pi\rangle\right) \stackrel{def}{=} 1-\mu_\alpha\left(\langle\chi,\pi\rangle\right). \tag{129}$$

Definition 106. Fuzzy conjunction:

$$\mu_{\alpha\tilde{\wedge}\beta}\left(\langle\chi,\pi\rangle\right) \stackrel{def}{=} \Big(\mu_\alpha\left(\langle\chi,\pi\rangle\right)\tilde{\wedge}^P_\lambda \mu_\beta\left(\langle\chi,\pi\rangle\right)\Big). \tag{130}$$

In particular, for a fuzzy disjunction of causal properties (case when triangular norm [8] is minimum) it is true:

$$\mu_{\alpha\tilde{\wedge}\beta}\left(\langle\chi,\pi\rangle\right) = \inf\left(\{\mu_\alpha\left(\langle\chi,\pi\rangle\right)\}\cup\{\mu_\beta\left(\langle\chi,\pi\rangle\right)\}\right). \tag{131}$$

Definition 107. Fuzzy disjunction:

$$\mu_{\alpha\tilde{\vee}\beta}\left(\langle\chi,\pi\rangle\right) \stackrel{def}{=} \Big(\mu_\alpha\left(\langle\chi,\pi\rangle\right)\tilde{\vee}^P_\lambda \mu_\beta\left(\langle\chi,\pi\rangle\right)\Big) = \mu_{\sim((\sim\alpha)\tilde{\wedge}(\sim\beta))}\left(\langle\chi,\pi\rangle\right). \tag{132}$$

Definition 108. Fuzzy strict disjunction:

$$\mu_{\alpha \veebar \beta}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \left(\mu_\alpha\left(\langle \chi, \pi \rangle\right) \veebar_\lambda^P \mu_\beta\left(\langle \chi, \pi \rangle\right)\right) = \mu_{(\alpha \tilde{\vee} \beta) \tilde{\wedge} ((\sim \alpha) \tilde{\vee} (\sim \beta))}\left(\langle \chi, \pi \rangle\right). \tag{133}$$

Definition 109. Fuzzy implication:

$$\mu_{\alpha \rightsquigarrow \beta}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \left(\mu_\alpha\left(\langle \chi, \pi \rangle\right) \rightsquigarrow_\lambda^P \mu_\beta\left(\langle \chi, \pi \rangle\right)\right). \tag{134}$$

Definition 110. Fuzzy equivalence:

$$\mu_{\alpha \simeq \beta}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \left(\mu_\alpha\left(\langle \chi, \pi \rangle\right) \simeq_\lambda^P \mu_\beta\left(\langle \chi, \pi \rangle\right)\right) = \mu_{(\alpha \rightsquigarrow \beta) \tilde{\wedge} (\beta \rightsquigarrow \alpha)}\left(\langle \chi, \pi \rangle\right). \tag{135}$$

Definition 111. Fuzzy existence:

$$\mu_{\tilde{\exists} \gamma \alpha(\gamma)}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \tilde{\exists}_\lambda^P\left(\left\{\mu_{\alpha(\tau)}\left(\langle \chi, \pi \rangle\right) | \tau\right\}\right) = \tilde{\bigvee}_\lambda^P\left(\left\{\mu_{\alpha(\tau)}\left(\langle \chi, \pi \rangle\right) | \tau\right\}\right). \tag{136}$$

In particular, it is true for the fuzzy disjunction of causal properties (the case of minimum as triangular norm):

$$\mu_{\tilde{\exists} \gamma \alpha(\gamma)}\left(\langle \chi, \pi \rangle\right) = \sup\left(\left\{\mu_{\alpha(\tau)}\left(\langle \chi, \pi \rangle\right) | \tau\right\}\right). \tag{137}$$

Definition 112. Fuzzy univeral quantor:

$$\mu_{\tilde{\forall} \gamma \alpha(\gamma)}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \tilde{\forall}_\lambda^P\left(\left\{\mu_{\alpha(\tau)}\left(\langle \chi, \pi \rangle\right) | \tau\right\}\right) = \tilde{\bigwedge}_\lambda^P\left(\left\{\mu_{\alpha(\tau)}\left(\langle \chi, \pi \rangle\right) | \tau\right\}\right). \tag{138}$$

In particular, this is true for the fuzzy conjunction of causal properties (case the triangular norm is the minimum):

$$\mu_{\tilde{\forall} \gamma \alpha(\gamma)}\left(\langle \chi, \pi \rangle\right) = \inf\left(\left\{\mu_{\alpha(\tau)}\left(\langle \chi, \pi \rangle\right) | \tau\right\}\right). \tag{139}$$

Definition 113. Maximal routes through a vertex in a model:

$$\Phi\left(\langle \chi, \iota \oplus \tau, \tau, E, V \rangle\right) \stackrel{def}{=} ((\Psi\left(\left\langle \chi, \pi, E^{-1}, V \right\rangle\right) \wedge \Psi\left(\langle \chi, \tau, E, V \rangle\right)) \wedge \left(\iota \in \times_{k<|\pi|}^{k=0}\left(\left\{\pi_{k+1}\right\}^{-1}\right)\right)). \tag{140}$$

Definition 114. Maximal half-routes from a vertex in a finite returnable model:

$$\hat{\Psi}\left(\langle \chi, \pi, E, V \rangle\right) \stackrel{def}{=} ((\Theta\left(\langle \chi, \pi, E, V \rangle\right) \wedge \left(\exists q\left(\left(\pi_{|\pi|}\right)_2 = \left(\pi_q\right)_1\right)\right)) \wedge \left(\bigcup_{k=0}^{k<|\pi|}\left\{\pi_{k+1}\right\} \in \left(V_+{}^V\right)\right)). \tag{141}$$

Definition 115. Maximal half-route from a vertex in the model:

$$\Psi\left(\langle \chi, \pi, E, V \rangle\right) \stackrel{def}{=} \left(\Theta\left(\langle \chi, \pi, E, V \rangle\right) \wedge \Xi\left(\langle \chi, \pi, E, V \rangle\right)\right). \tag{142}$$

Definition 116. Half-route from a vertex in the model:

$$\Theta\left(\langle \chi, \pi, E, V \rangle\right) \stackrel{def}{=} \left(\left(\chi = \left(\pi_1\right)_1\right) \wedge \left(\pi \in E^{|\pi|}\right)\right) \wedge \left(\left((j \in \mathbb{N} / \{0\}) \wedge (j < |\pi|)\right) \rightarrow \left(\left(\pi_j\right)_2 = \left(\pi_{j+1}\right)_1\right)\right). \tag{143}$$

Definition 117. Maximality of half-routes from a vertex in the model:

$$\Xi\left(\langle \chi, \pi, E, V \rangle\right) \stackrel{def}{=} ((|\pi| \notin \mathbb{N}) \vee (\neg \tau\left(\Theta\left(\langle \chi, \tau, E, V \rangle\right) \wedge \left((\tau = \pi \oplus \langle \varepsilon \rangle) \wedge (\varepsilon \in E)\right)\right))). \tag{144}$$

Definition 118. Fuzzy operator "sometimes in the future":

$$\mu_{\tilde{\mathrm{F}} \alpha}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \tilde{\mathrm{F}}_\lambda^P\left(\left\{\mu_\alpha\left(\langle \gamma, \pi \rangle\right) | \Phi\left(\langle \chi, \pi, \tau, E, V \rangle\right) \wedge \Phi\left(\langle \gamma, \pi, \tau, E, V \rangle\right)\right\}\right). \tag{145}$$

The following expression is valid for models of returnable operations:

$$\mu_{\tilde{\mathrm{F}} \alpha}\left(\langle \chi, \pi \rangle\right) = \sup\left(\left\{\mu_\alpha\left(\langle \gamma, \pi \rangle\right) | \Phi\left(\langle \chi, \pi, \tau, E, V \rangle\right) \wedge \Phi\left(\langle \gamma, \pi, \tau, E, V \rangle\right)\right\}\right), \tag{146}$$

and instead of Ψ, just check $\hat{\Psi}$.

CTL* [59] operator version.

$$\mu_{\tilde{\mathbf{F}} \alpha}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \mu_{\alpha \tilde{\vee} \tilde{\mathrm{F}} \alpha}\left(\langle \chi, \pi \rangle\right). \tag{147}$$

Definition 119. Fuzzy operator "always in the future":

$$\mu_{\tilde{\mathrm{G}} \alpha}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \tilde{\mathrm{G}}_\lambda^P\left(\left\{\mu_\alpha\left(\langle \gamma, \pi \rangle\right) | \Phi\left(\langle \chi, \pi, \tau, E, V \rangle\right) \wedge \Phi\left(\langle \gamma, \pi, \tau, E, V \rangle\right)\right\}\right). \tag{148}$$

The following expression is valid for models of returnable operations:

$$\mu_{\tilde{\mathrm{G}} \alpha}\left(\langle \chi, \pi \rangle\right) = \inf\left(\left\{\mu_\alpha\left(\langle \gamma, \pi \rangle\right) | \Phi\left(\langle \chi, \pi, \tau, E, V \rangle\right) \wedge \Phi\left(\langle \gamma, \pi, \tau, E, V \rangle\right)\right\}\right), \tag{149}$$

and instead of Ψ, just check $\hat{\Psi}$.

CTL* operator version.

$$\mu_{\tilde{\mathbf{G}} \alpha}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \mu_{\alpha \tilde{\wedge} \tilde{\mathrm{G}} \alpha}\left(\langle \chi, \pi \rangle\right). \tag{150}$$

Definition 120. Fuzzy operator "sometimes in the past":

$$\mu_{\tilde{\mathrm{P}} \alpha}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \tilde{\mathrm{P}}_\lambda^P(\{\mu_\alpha\left(\langle \gamma, \pi \rangle\right) | \Phi\left(\left\langle \chi, \pi, \tau, E^{-1}, V \right\rangle\right) \wedge \Phi\left(\left\langle \gamma, \pi, \tau, E^{-1}, V \right\rangle\right)\}). \tag{151}$$

The following expression is valid for models of returnable operations:

$$\mu_{\tilde{\mathrm{P}} \alpha}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \sup(\{\mu_\alpha\left(\langle \gamma, \pi \rangle\right) | \Phi\left(\left\langle \chi, \pi, \tau, E^{-1}, V \right\rangle\right) \wedge \Phi\left(\left\langle \gamma, \pi, \tau, E^{-1}, V \right\rangle\right)\}), \tag{152}$$

and instead of Ψ, just check $\hat{\Psi}$.

Definition 121. Fuzzy operator "always in the past":

$$\mu_{\tilde{\mathrm{H}} \alpha}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \tilde{\mathrm{H}}_\lambda^P(\{\mu_\alpha\left(\langle \gamma, \pi \rangle\right) | \Phi\left(\left\langle \chi, \pi, \tau, E^{-1}, V \right\rangle\right) \wedge \Phi\left(\left\langle \gamma, \pi, \tau, E^{-1}, V \right\rangle\right)\}). \tag{153}$$

The following expression is valid for models of returnable operations:

$$\mu_{\tilde{\mathrm{H}} \alpha}\left(\langle \chi, \pi \rangle\right) \stackrel{def}{=} \inf(\{\mu_\alpha\left(\langle \gamma, \pi \rangle\right) | \Phi\left(\left\langle \chi, \pi, \tau, E^{-1}, V \right\rangle\right) \wedge \Phi\left(\left\langle \gamma, \pi, \tau, E^{-1}, V \right\rangle\right)\}), \tag{154}$$

and instead of Ψ, just check $\hat{\Psi}$.

Definition 122. Fuzzy possibility operator:

$$\mu_{\tilde{\Diamond}\alpha}\left(\langle\chi,\pi\rangle\right) \overset{def}{=} \tilde{\Diamond}_{\lambda}^{P}\left(\left\{\mu_{\alpha}\left(\langle\gamma,\iota\rangle\right)|\Phi\left(\langle\chi,\iota,\tau,E,V\rangle\right)\right\}\right). \tag{155}$$

The following expression is valid for models of returnable operations:

$$\mu_{\tilde{\Diamond}\alpha}\left(\langle\chi,\pi\rangle\right) = \sup\left(\left\{\mu_{\alpha}\left(\langle\chi,\iota\rangle\right)|\Phi\left(\langle\chi,\iota,\tau,E,V\rangle\right)\right\}\right), \tag{156}$$

and instead of Ψ, just check $\hat{\Psi}$.

Definition 123. Fuzzy necessity operator:

$$\mu_{\tilde{\Box}\alpha}\left(\langle\chi,\pi\rangle\right) \overset{def}{=} \tilde{\Box}_{\lambda}^{P}\left(\left\{\mu_{\alpha}\left(\langle\chi,\iota\rangle\right)|\Phi\left(\langle\chi,\iota,\tau,E,V\rangle\right)\right\}\right). \tag{157}$$

The following expression is valid for models of returnable operations:

$$\mu_{\tilde{\Box}\alpha}\left(\langle\chi,\pi\rangle\right) = \inf\left(\left\{\mu_{\alpha}\left(\langle\chi,\iota\rangle\right)|\Phi\left(\langle\chi,\iota,\tau,E,V\rangle\right)\right\}\right). \tag{158}$$

Definition 124. Fuzzy operator "next":

$$\begin{gathered}\mu_{\tilde{\bigcirc}\alpha}\left(\langle\chi,\pi\rangle\right) \overset{def}{=} \tilde{\bigcirc}_{\lambda}^{P} \\ \left(\left\{\mu_{\alpha}\left(\langle(\tau_1)_2,\pi\rangle\right)|\Phi\left(\langle\chi,\pi,\tau,E,V\rangle\right)\right\}\right),\end{gathered} \tag{159}$$

i.e.

$$\mu_{\tilde{\bigcirc}\alpha}\left(\langle\chi,\pi\rangle\right) \in \left(\left\{\mu_{\alpha}\left(\langle(\tau_1)_2,\pi\rangle\right)|\Phi\left(\langle\chi,\pi,\tau,E,V\rangle\right)\right\}\right). \tag{160}$$

Let it:

$$\mu_{\tilde{0}\alpha}\left(\langle\chi,\pi\rangle\right) \overset{def}{=} 1, \tag{161}$$

and

$$\begin{gathered}\mu_{\widetilde{(k+1)}\alpha}\left(\langle\chi,\pi\rangle\right) \overset{def}{=} \widetilde{(k+1)}_{\lambda}^{P} \\ \left(\left\{\left(\mu_{\alpha}\left(\langle\chi,\pi\rangle\right)\tilde{\wedge}_{\lambda}^{P}\mu_{\tilde{k}\alpha}\left(\langle(\tau_1)_2,\pi\rangle\right)\right)|\Phi\left(\langle\chi,\pi,\tau,E,V\rangle\right)\right\}\right),\end{gathered} \tag{162}$$

i.e.

$$\begin{gathered}\mu_{\widetilde{(k+1)}\alpha}\left(\langle\chi,\pi\rangle\right) \in \\ \left(\left\{\left(\mu_{\alpha}\left(\langle\chi,\pi\rangle\right)\tilde{\wedge}_{\lambda}^{P}\mu_{\tilde{k}\alpha}\left(\langle(\tau_1)_2,\pi\rangle\right)\right)|\Phi\left(\langle\chi,\pi,\tau,E,V\rangle\right)\right\}\right).\end{gathered} \tag{163}$$

Definition 125. Fuzzy operator "until":

$$\begin{gathered}\mu_{\alpha\tilde{\multimap}\beta}\left(\langle\chi,\pi\rangle\right) \overset{def}{=} \\ \tilde{\multimap}_{\lambda}^{P}(\{\left(\mu_{\beta}\left(\langle(\tau_k)_1,\pi\rangle\right)\tilde{\wedge}_{\lambda}^{P}\mu_{\widetilde{(k-1)}\alpha}\left(\langle\chi,\pi\rangle\right)\right) \\ |\Phi\left(\langle\chi,\pi,\tau,E,V\rangle\right)\wedge\left((k\in\mathbb{N})\wedge(k\leq|\tau|)\right)\}),\end{gathered} \tag{164}$$

i.e.

$$\begin{gathered}\mu_{\alpha\tilde{\multimap}\beta}\left(\langle\chi,\pi\rangle\right) \in \\ \tilde{\exists}_{\lambda}^{P}(\{\left(\mu_{\beta}\left(\langle(\tau_k)_1,\pi\rangle\right)\tilde{\wedge}_{\lambda}^{P}\mu_{\widetilde{(k-1)}\alpha}\left(\langle\chi,\pi\rangle\right)\right) \\ |\Phi\left(\langle\chi,\pi,\tau,E,V\rangle\right)\wedge\left((k\in\mathbb{N})\wedge(k\leq|\tau|)\right)\}),\end{gathered} \tag{165}$$

Examples (see figure.3 and tables II, III):

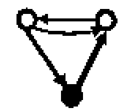

Figure 3. Structure example

Table II
Fuzzy temporal logic formula "sometimes" operators examples

χ	π	α	$\tilde{F}\alpha$	$\tilde{G}\alpha$	$\tilde{P}\alpha$	$\tilde{H}\alpha$	$\tilde{\bigcirc}\alpha$
1	< <1,2>,<2,1> >	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
1	<<1,2>,<2,3>,<3,1>>	$\perp$	$\top$	$\perp$	$\top$	$\perp$	$\perp$
2	< <2,1>,<1,2> >	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
2	<<2,3>,<3,1>,<1,2>>	$\perp$	$\top$	$\perp$	$\top$	$\perp$	$\top$
3	< <3,1>,<1,2>, <2,1>, <1,2>,<2,3> >	$\top$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
3	<<3,1>,<1,2>,<2,3>>	$\top$	$\top$	$\perp$	$\top$	$\perp$	$\perp$

D. Metrics in logical models

For logical models, one can identify: their order relations and topology associated with a logical consequence relation, corresponding measures associated with classes of logical formulas and propositions. Thus, a (pseudo)metric space formed by classes of logically equivalent formulas and a metric expressing the difference of values of propositions (can be computed as a fuzzy strict disjunction) can be considered. This metric induces pseudometrics on instances of these classes.

Integration of pseudometric spaces of the intensionally-logical level can be realized similarly to the integration of metric and pseudometric spaces of the essentially-systematic and relationally-phenomenological levels with the help of Theorem 1.

For the technical realization of metric subspaces of the semantic space, we propose to consider structures based on k-d trees, in which dimensions are selected according to the highest value of the informativeness feature. The number of selected dimensions for indexing elements of ontology structures using k-d trees should be sublinear. In the presence of non-factors that do not allow the formation of a replenishment-resistant metric space for ontological structures, it is proposed to use conditionally stable attributes of metacognition structures of ignorance with respect to these ontological structures. Thus, these attributes will not be absolutely stable. However, in the process of accumulation of additional knowledge about ontological structures, elimination of non-factors (ignorance) and integration of replenishment-resistant metrics within the framework of additional dimensions, it is proposed to ensure

Table III
Fuzzy temporal logic formula "until" and necessity examples

χ	π	$(\sim\alpha)\tilde{\multimap}\alpha$	$\hat{\Diamond}\alpha$	$\tilde{\Box}\alpha$	$\hat{\Diamond}\tilde{F}\alpha$	$\tilde{\Box}\tilde{F}\alpha$
1	< <1,2>,<2,1> >	$\perp$	$\perp$	$\perp$	$\top$	$\perp$
1	<<1,2>,<2,3>,<3,1>>	$\perp$	$\perp$	$\perp$	$\top$	$\perp$
2	< <2,1>,<1,2> >	$\perp$	$\perp$	$\perp$	$\top$	$\perp$
2	<<2,3>,<3,1>,<1,2>>	$\perp$	$\perp$	$\perp$	$\top$	$\perp$
3	< <3,1>,<1,2>, <2,1>, <1,2>,<2,3> >	$\top$	$\top$	$\top$	$\top$	$\perp$
3	<<3,1>,<1,2>,<2,3>>	$\top$	$\top$	$\top$	$\top$	$\perp$

convergence of the corresponding formed metric space to the metric space based on stable attributes by means of parameterization.

VIII. Conclusion

The review of existing approaches to analyzing semantic relations, identifying meaning in data and knowledge, similarity and proximity of knowledge structures has been carried out. The ontology of measures of similarity and similarity relations of elements and knowledge structures in ontologies was developed. The model of operational information space oriented to the creation of intelligent systems based on reusable and reusable components was proposed, which provides semantic unification and integration of models with operational semantics of "big step" and "small step" within the semantic space. Also, the monotone measure expressing the entropy of the equilibrium state of information processing models using the model of returnable computation was proposed. The algebra of transformations of the models on which this measure is defined was considered. The model of semantics of the parametrized fuzzy temporal logic language was developed and proposed which provides the computation of fuzzy truth values for fuzzy CTL* language operators and other temporal logic operators on the models coordinated with denotational and operational semantics of the unified semantic knowledge representation model. The principles of construction on the basis of generalized measures and relations and similarity of semantic space models that take into account topological and metric properties of ontological structures at the essentially-systematic, relationally-phenomenological and intensionally-logical levels are formulated.

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ПОДХОДЫ К ИССЛЕДОВАНИЮ СЕМАНТИЧЕСКОГО ПРОСТРАНСТВА И МОДЕЛЕЙ ИНТЕГРИРОВАННОГО ЛОГИЧЕСКОГО ВЫВОДА С ИСПОЛЬЗОВАНИЕМ МЕР СХОДСТВА, РАЗЛИЧИЯ И ДРУГИХ МЕР

Ивашенко В.П.

Рассмотрены подходы к анализу близости, сходств и различий стуктур знаний для выявления смысла в данных и знаниях, подходы и модели представления знаний с операционной семантикой для компонентного проектирования интеллектуальных систем. Разработана онтология обобщённых мер и отношений различия и сходства, включая отношения подобия. Предложен класс предпорядковых шкал, меры на структурах с операционной семантикой, проведено исследование их алгебраических свойств. Уточнено понятие операционно-информационного пространства. Предложена модель семантики нечёткой временной логики для спецификации структур знаний с операционной семантикой на основе ранее предложенной параметризованной нечёткой логики. Для онтологических структур, соответствующих полученным результатам и трём уровням семантического анализа, сформулированы принципы их интеграции как метрических подпространств смыслового пространства.

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