Principles and Solutions for Integrating Computer Algebra Tools and Applications Based on Open Semantic Technologies

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***Abstract*—The principles, methodical and technical solutions for the integration of the Ecosystem of intelligent computer systems of the new generation (OSTIS ecosystem) and computer algebra systems are considered. The expediency of such integration is shown; various integration options, advantages and disadvantages of these options are noted. The implementation of the proposed approach is considered using the example of an intelligent learning system in discrete mathematics with illustrations of studying the topic, performing visualization tasks and solving graph theory problems.**

***Keywords*—OSTIS Ecosystem, Wolfram Mathematica computer algebra system, intelligent learning system for discrete mathematics.**

I. Introduction

At the current stage of information technology development, the transition from modern computer systems to new-generation computer systems, which should have a sufficiently high level of intelligence, is relevant. This means a transition to a fundamentally new technological order in automation, and also makes it necessary to develop a comprehensive Technology to support the life cycle of intelligent computer systems (ICS) of a new generation. The systems developed on the basis of such technology are called ostis-systems [1].

A special place in the technology of ICS development and modernization is occupied by approaches that ensure the connection and use of the most successful modern solutions already used in various fields. Such solutions currently include computer algebra (CAS) systems. It is important to transform the modern variety of tools (frameworks) for the development of various ICS components into a single technology for integrated design and support of the full lifecycle of these systems, ensuring the compatibility of all components being developed, as well as the compatibility of the ICS themselves. Convergence and unification of a new generation of ICS and their components is necessary, and it must be possible to solve certain tasks with optimization and maximum performance requirements. Similar problems are being solved during the development, improvement, updating of the content and expansion of the capabilities of computer algebra systems. The integration of CAS and the OSTIS Ecosystem is an important and urgent task.

II. About the integration of computer algebra systems with applications within the OSTIS Ecosystem

One of the options for the interaction of the OSTIS Ecosystem and CAS may be approaches similar to those implemented within the framework of integration into ostis systems of artificial neural networks ([1] Chapter 3.6). It is advisable to consider the following methodological and technical solutions (in detail – [1] § 7.4.2):

- “Black box" integration.
- Closer integration, in which a specific function remains part of a third-party CAS, when not only the result of its implementation, but also all possible specifications are immersed in the ostis system’s knowledge base.
- Full integration, which translates the used functions of the computer algebra system from the internal language of this system into the ostis system.

The expediency of using one or another integration option is primarily due to the difference in their complexity, which, in turn, makes step-by-step integration preferable as the project develops. So, at the stage of testing the idea of integration, it is advisable to use the "black box" option as the least labor-intensive, but at the same time making it possible to determine the advantages of such integration.

From the point of view of practice, at this stage of development and application of the OSTIS Ecosystem, *integration into a single set of CAS capabilities and intelligent learning systems* built within the OSTIS Ecosystem seems to be the most promising. The expediency of such integration is due to the fact that CAS has an undoubted advantage and wide possibilities in solving problems relevant to teaching systems in almost all natural science and technical disciplines involving the use of complex mathematical apparatus.

Despite the popularity of topics related to the automation and intellectualization of educational activities in natural sciences and the development of appropriate computer systems, at the moment there are practically no proven intelligent learning systems in the public domain that have the ability to independently generate and solve various tasks, as well as verify the correctness of the resulting solution. The approach to solving the problems of intellectualization of educational activities, based on the integration of ostis systems and CAS, has a number of advantages:

- When developing ostis systems, the need to program many functions that are implemented, tested and tested in CAS is eliminated. This is essential, as CASs are developed by highly qualified specialists in their respective fields. The implementation of similar functions in ostis systems may require significant financial and time costs.
- A specific ostis system using individual CAS functions, thanks to the approach to developing hybrid problem solvers in OSTIS Technology ([1] § 1.1.1), gets the opportunity to plan the course of solving problems when individual stages are implemented using attached functions.

It should be emphasized that the integration options do not exclude each other and can be combined. The deepening of integration can be carried out in stages, taking into account the advantages and disadvantages, the relevance of using certain CAS functions in solving specific tasks within the framework of the relevant ostis systems [2].

The step–by-step integration of CAS with the OSTIS Ecosystem involves, at a minimum, a description of the specification of the main functions of the selected computer algebra system using OSTIS Technology, in other words, the development of an ontology of external functions. In the case of systems of the Wolfram Mathematica family, the process of developing such an ontology can be automated due to the presence of the formal Wolfram Language and the well-documented functions of the system.

III. An example of integration of a prototype of the ostis learning system for discrete mathematics and Wolfram Mathematica

We emphasize the fundamental positions and basic capabilities that dictate the expediency of integrating CAS with the OSTIS Ecosystem:

- work with mathematical expressions in symbolic form, performing analytical transformations, presenting results in mathematical notation;
- numerical operations of any specified precision;
- availability of various types of data processing;
- an interactive graphical visualization;
- registration of results and preparation for publication;
- the use of special-purpose expansion packs;
- programming in an embedded language, program synthesis.

Here are illustrations of the joint use of the Wolfram Mathematica CAS [3] and a prototype of a semantic electronic textbook on discrete mathematics (SET DM) developed on the basis of OSTIS Technology.

In the examples below, the source data (graph) is received (imported) from the SET DM. The parameters and properties of the graph can be derived, and the graph can be modified [4], [5]. Users have the opportunity to obtain solutions to typical graph theory problems; Mathematica's tools allow them to export preferred final results back to the SET DM with different graphical visualization options.

The following illustrations were obtained in Mathematica, while the original sc.g-text from the SET DM was used to define the graph. For an imported graph in CAS, you can get and output general information, for example: the number of vertices, arcs, and a list of edges; using Mathematica graphics, you can visualize vertices and edges in different stacking options, vertex and edge signatures [6].

It should be noted that for understanding the content and analyzing the parameters of a graph, its graphical visualization is important. According to this position, Wolfram Language provides users of the Mathematica system with a wide range of possibilities – more than 30 variants of typical graph views in different layouts, various user settings tools for creating a preferred layout [7], [8].

Graphs provide excellent visualization of information. Highlighting the graph elements will allow you to emphasize the features of the information and its interpretation.

Using the options, you can design, change, and connect: Highlight Graphic Elements, Overall Look and Feel, Graph Layout, Vertex and Edge Styles, Vertex and Edge Labels, Vertex and Edge Shapes, Interactive Effects.

Using algorithmic layouts (stacking) of the graph, most of the structure in the graph will be understandable, for example, the connected components.

By attaching additional interactive effects to graph elements, you can provide detailed information.

The figures below show the most spectacular views for the graph under study.

The Wolfram Language provides extensive collections of carefully designed graph styles, highlight styles, and layout algorithms. The Wolfram Language provides also in-depth support for every aspect of styling, labeling, and shape generation for graphs, as well as carefully designed libraries of edge and vertex shapes.

Figures 1 – 6 below show the most spectacular views for the graph under study.

The Figure 1 illustrates the stacking options CircularEmbedding (vertices on a circle) and StarEmbedding (vertices on a circle with a center).

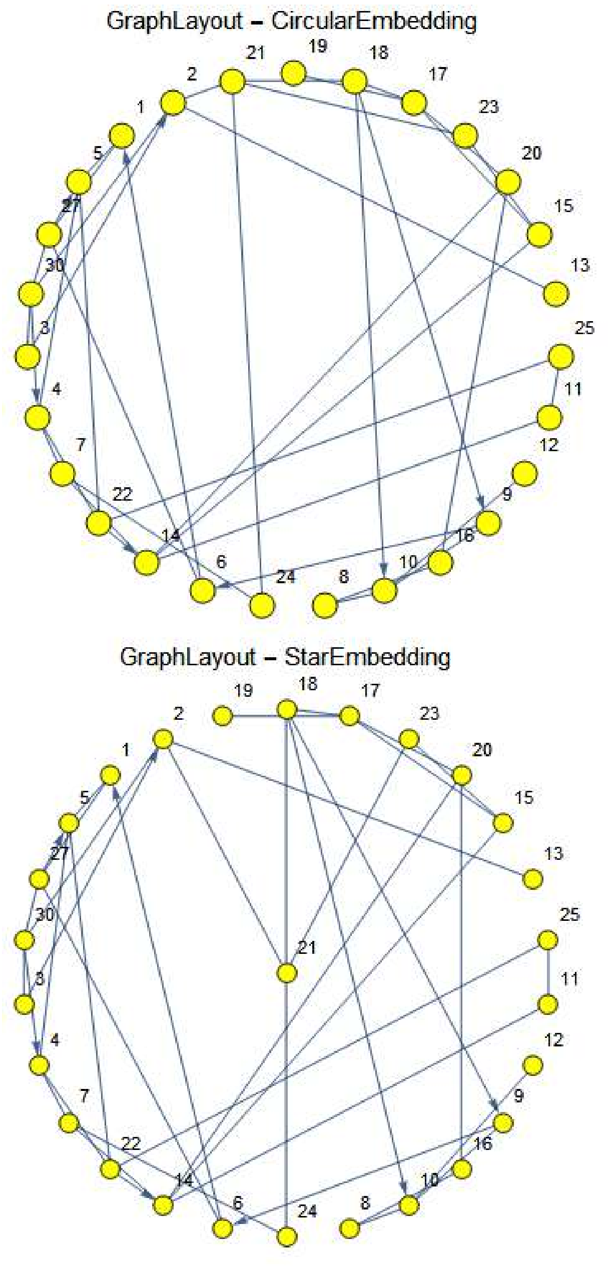

Figure 1. Views (layouts) of the studied graph of the category circle embeddings.

The Figure 2 shows the stacking options for DiscreteSpiralEmbedding (vertices on a discrete spiral) and SpiralEmbedding (vertices on a 3D spiral projected to 2D).

The Figure 3 shows options for layouts in the category structured embeddings for layered graphs such as

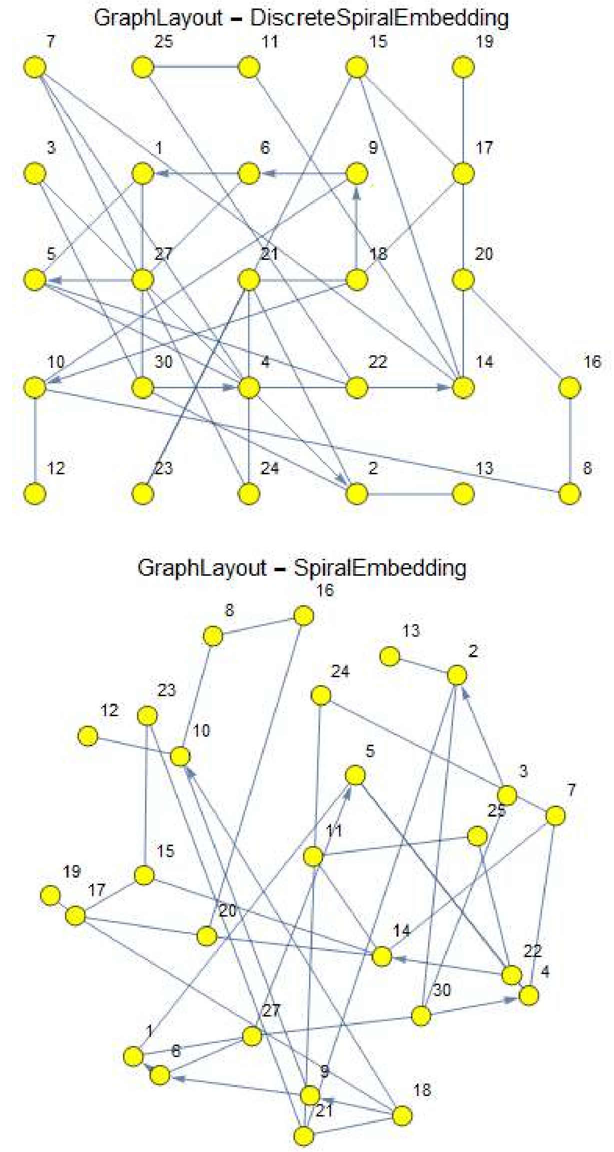

Figure 2. Views (layouts) of the studied graph of the category spiral embeddings.

trees and directed acyclic graphs: RadialEmbedding (vertices on a circular segment), LayeredDigraphEmbedding (vertices on parallel lines for directed acyclic graphs), LayeredEmbedding (vertices on parallel lines).

The Figure 4 shows the installation options of the categories optimizing embeddings all minimize a quantity PlanarEmbedding (number of edge crossings) and HighDimensionalEmbedding (energy for spring-electrical in high dimension).

The Figure 5 shows the layout options for the categories SpringEmbedding (energy with edges as springs) and SpringElectricalEmbedding (energy with edges as springs and vertices as charges).

Variants of vertex and edges signatures are illustrated

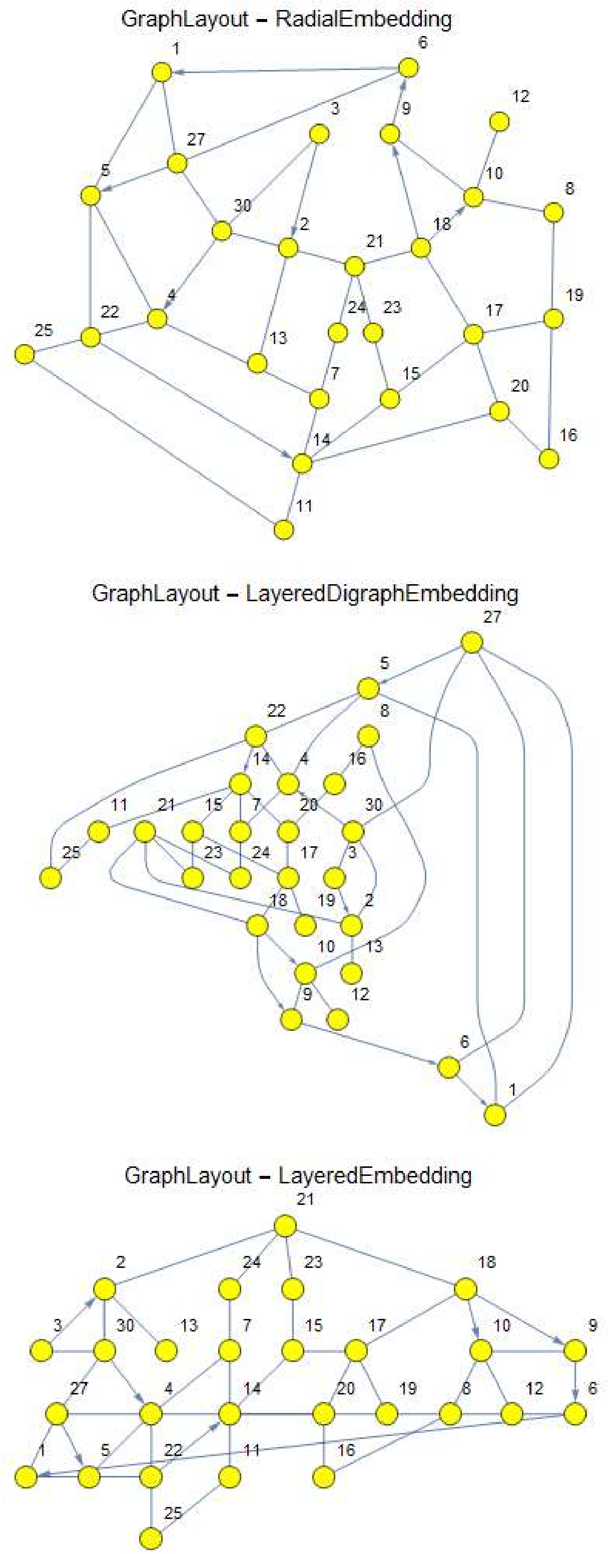

Figure 3. Views (layouts) of the studied graph of the category layered embeddings.

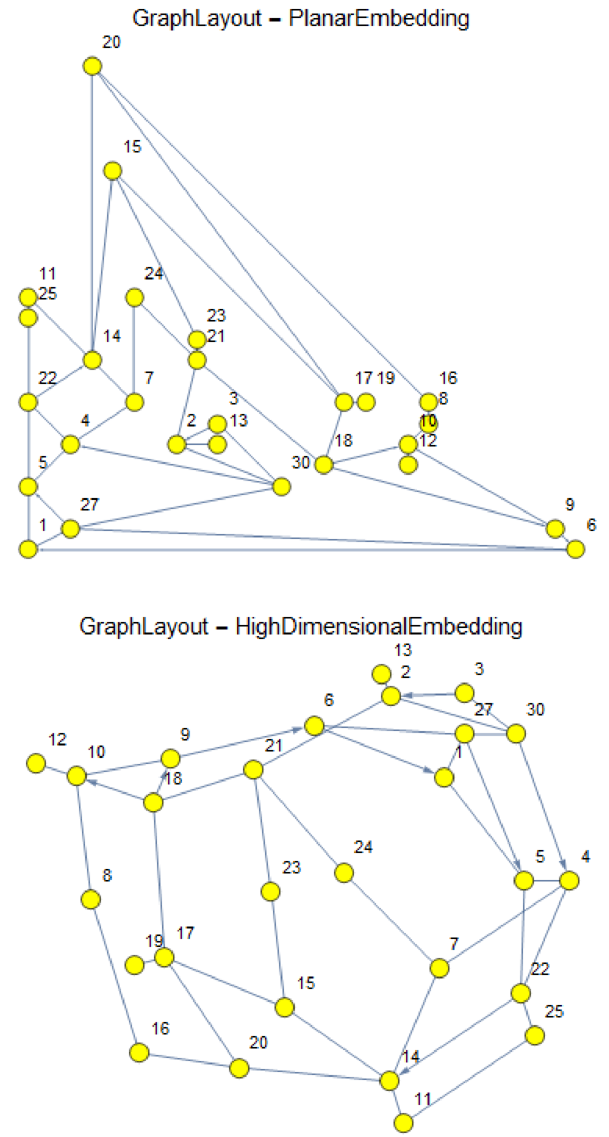

Figure 4. Views (layouts) of the studied graph of the category optimizing embeddings.

in Figure 6 (labels and label positions for edges).

The FIgure 6 shows views using RadialEmbedding layout connections with directions and vertex numbers.

In the top part of the illustration, the output is made in the form of signatures of numbers in the centers of vertices and signatures of edges, indicating their types. In the lower part of the illustration, edges are displayed with a design according to the rules – all edges from a node with a large number to a node with a smaller number are displayed as dotted red lines, and the rest are solid green.

An **example of solving the problem of finding the shortest path between two vertices** is illustrated in Figure 7. Mathematica functions are used in the solution: FindShortestPath [9], HighlightGraph [10]. The output is

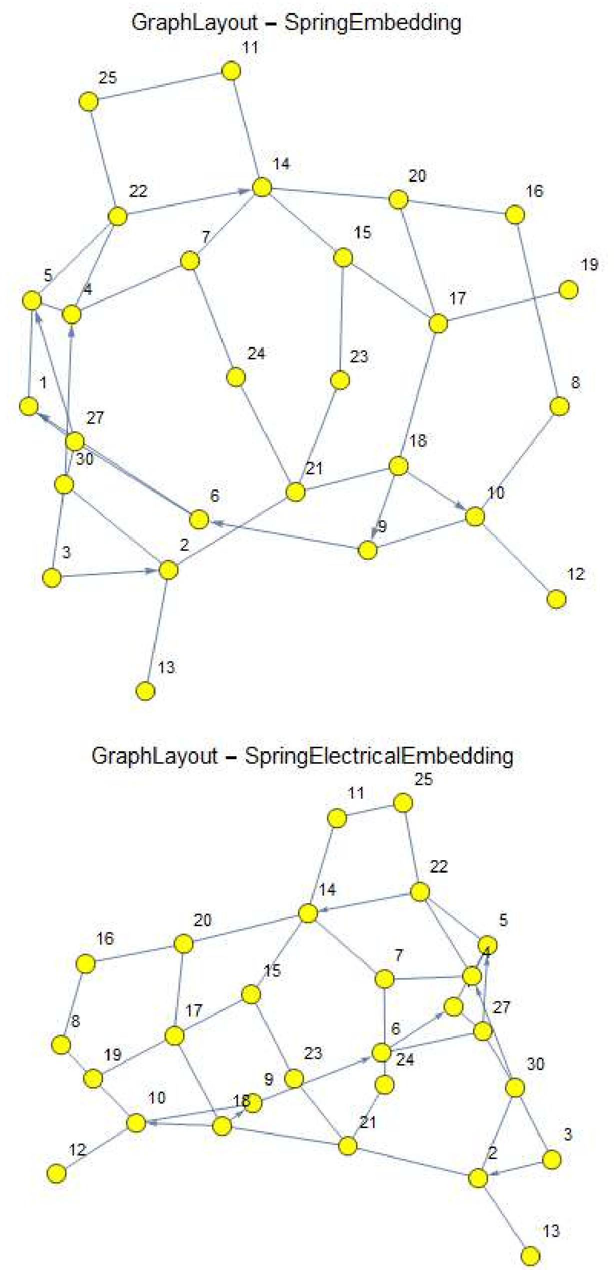

Figure 5. Views (layouts) of the studied graph of the category springs embeddings.

done using the Radial Embedding layout.

The solution for the graph considered and illustrated in Figures 1 – 6 is shown on the top, it is important that movement in any direction is possible along the edge 15-17. The solution for the modified graph is shown on the lower part, in which the path from 15 to 17 vertices is "blocked", it is possible only in the direction from 17 to 15.

Note that the results obtained and considered include graphics tasks that are laborious to implement in programming languages, as well as mathematically and

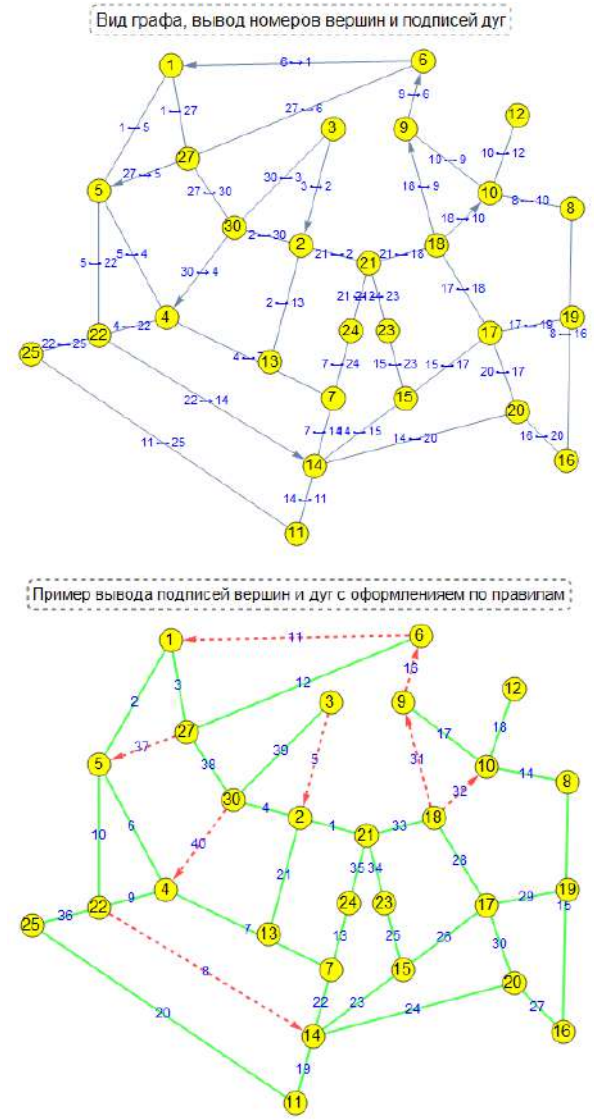

Figure 6. Views of graph under study, design variants.

algorithmically complex tasks in the subject area. The presented visualization options and finding a solution require only careful study of the examples of the Wolfram Mathematica help system, initial programming skills, that is, they are available to most software engineers.

IV. Conclusion

Computer algebra systems currently represent powerful tool complexes, the capabilities of which have long gone beyond algebraic calculations and even classical mathematics in general. CAS provides many computing capabilities, processing algorithms, analysis, visualization. One of the leaders is the Wolfram Mathematica system, the core of which contains more than 6,000 functions [3]. Wolfram has also developed many unique projects, which, in addition to the Mathematica system,

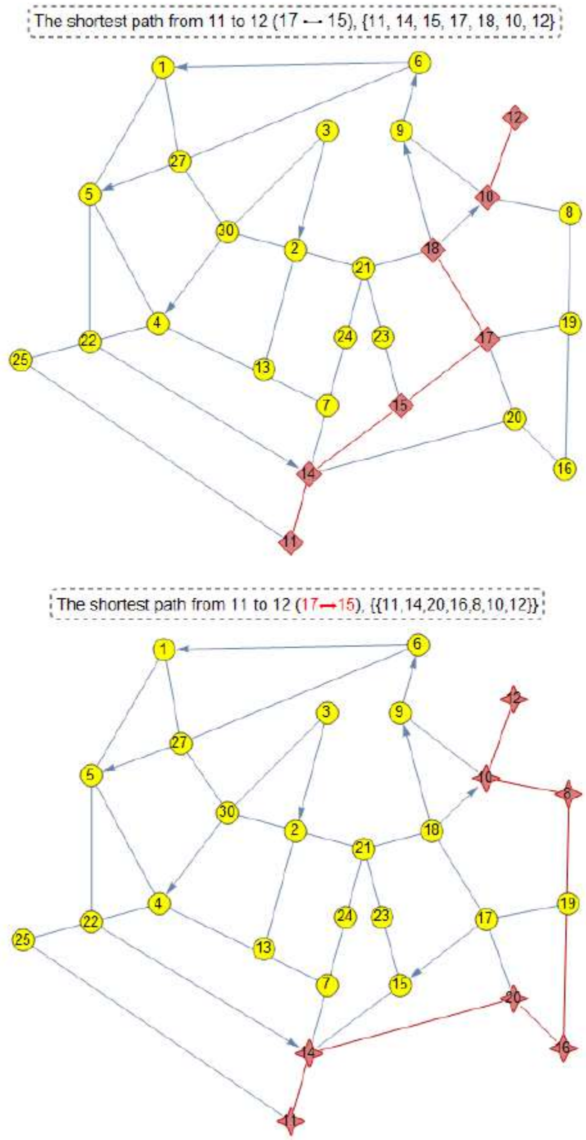

Figure 7. Solutions to the problem of finding the shortest path.

include the Wolfram|Alpha computational knowledge engine, which contains an extensive knowledge base and a set of computational algorithms.

The representation of factual, logical, and procedural knowledge for the Wolfram family's systems is based on the multi-paradigm programming language Wolfram Language. The presence of such an internal language, functions of Wolfram systems and, in general, a high level of documentation of these functions distinguishes Wolfram systems from other services that allow solving general and specific tasks. In many cases, Wolfram allows you not only to solve a problem, but also to explain the course of the solution, as well as to help the user choose a function suitable for solving his specific task, or to suggest a set of functions that can be applied to the input and received data used. Another advantage of the Wolfram family's systems is their complexity, which allows them to solve fairly complex tasks within a single application and without the need to integrate heterogeneous services.

Considering the above, we can definitely conclude that it is advisable to integrate the Wolfram Mathematica computer algebra system with ostis systems that are part of the OSTIS Ecosystem.

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ПРИНЦИПЫ И РЕШЕНИЯ ИНТЕГРАЦИИ ИНСТРУМЕНТОВ КОМПЬЮТЕРНОЙ АЛГЕБРЫ И ПРИЛОЖЕНИЙ НА БАЗЕ ОТКРЫТЫХ СЕМАНТИЧЕСКИХ ТЕХНОЛОГИЙ

Таранчук В. Б.

Рассмотрены принципы, методические и технические решения интеграции Экосистемы интеллектуальных компьютерных систем нового поколения (Экосистемы OSTIS) и систем компьютерной алгебры. Показана целесообразность такой интеграции, отмечаются различные варианты интеграции, преимущества и недостатки указанных вариантов. Реализация предлагаемого подхода рассматривается на примере интеллектуальной обучающей системы по дискретной математике с иллюстрациями изучения темы, выполнения заданий по визуализации и решениям задач теории графов.

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