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Spin 3/2 Particle in the Coulomb Field: The Non-Relativistic Approximation

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Abstract—In the present paper, we have studied the nonrelativistic problem for a spin 3/2 particle in presence in the external Coulomb field. The known general procedure for performing the nonrelativistic approximation is based on the method of projective operators. This approach is applied directly in the relativistic system of radial equation derived previously for a free spin 3/2 particle for states with the spherical symmetry within the covariant tetrad formalism. The system of two 2-nd order differential equations describing the nonrelativistic particle has been derived. It refers to states with quantum numbers of energy, square and the third projection of the total angular momentum, and spatial parity. Solutions of radial equations have been constructed in terms of confluent hypergeometric functions, the corresponding energy spectra are found.

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1. INTRODUCTION

In the present paper, we will study the non-relativistic approximation for a spin 3/2 particle, within the Pauli-Fierz and Rarita-Schwinger theory [1–3], in presence of the external Coulomb field. In the previous [4], the general structure of the Pauli-like equation for a spin 3/2 particle was derived, with the use of the Cartesian coordinates. This equation has the structure

$$iD_0\Psi = -\frac{1}{2M}\Delta\Psi + \frac{e}{3M}(F_{23}S_1 + F_{31}S_2 + F_{12}S_3)\Psi,$$

where $D_0 = \partial_0 + ieA_0$, $\Delta = (\partial_1 + ieA_1)^2 + (\partial_2 + ieA_2)^2 + (\partial_3 + ieA_3)^2$; the wave function is 4-component, the quantities S_i stand for 4-dimensional matrices of the spin operator for the particle. In the present paper, the system of radial equations describing the non-relativistic spin 3/2 particle in the Coulomb field will be obtained. In this case, we start with the previously obtained system of radial equations for the relativistic spin 3/2 free particle in the case of spherical symmetry [5], when the operators of energy, square and third projection of the total angular momentum, and spatial reflection were diagonalized.

The general procedure for performing the non-relativistic approximation is based on the technique of projective operators; in the case under consideration, this approach will be applied to the system of radial equations. The presence of the Coulomb field is taken into account by the standard change $E \Rightarrow E + \alpha/r$.

After making the needed calculations, we obtain a system of two coupled 2nd order differential equations is derived, this system describes the nonrelativistic particle with spin 3/2 in states with quantum numbers of energy, square and third projection of the total angular momentum, and the spatial parity. This system leads to two separate equations for the new radial functions. Their solutions are found in terms of the confluent hypergeometric functions, and the corresponding energy spectra are derived.

2. SEPARATION OF THE VARIABLES

The basic equation for a spin 3/2 particle used in [5], has the form (it is equivalent to the known Rarita–Schwinger equation [3])

$$\begin{aligned}
&\gamma^5\left(\gamma^1\otimes\mu^{[01]} + \gamma^2\otimes\mu^{[02]} + \gamma^3\otimes\mu^{[03]}\right)iD_0\Psi \\
&+ \gamma^5\left(\gamma^0\otimes\mu^{[01]} + \gamma^2\otimes\mu^{[12]} - \gamma^3\otimes\mu^{[31]}\right)iD_1\Psi \\
&+ \gamma^5\left(\gamma^0\otimes\mu^{[02]} + \gamma^3\otimes\mu^{[23]} - \gamma^1\otimes\mu^{[12]}\right)iD_2\Psi \\
&+ \gamma^5\left(\gamma^0\otimes\mu^{[03]} + \gamma^1\otimes\mu^{[31]} - \gamma^2\otimes\mu^{[23]}\right)iD_3\Psi \\
&- \gamma^5 M\{s_{01}\otimes\mu^{[01]} + s_{02}\otimes\mu^{[02]} + s_{03}\otimes\mu^{[03]} \\
&+ s_{23}\otimes\mu^{[23]} + s_{31}\otimes\mu^{[31]} + s_{12}\otimes\mu^{[12]}\}\Psi = 0,
\end{aligned} \tag{1}$$

where γ^a, γ^5 denote the Dirac matrices, $s_{ab} = \gamma_a\gamma_b - \gamma_b\gamma_a$, square brackets stand for the antisymmetry, symbol $\otimes$ designates the direct product of the matrices; the wave function is presented as a matrix with two indices, the first index is bispinor, and the second is vector one. The system of equations refers to the cyclic basis (in which the 16-dimensional generator J^{12} is diagonal, expressions for operators D_a will be given below. Six μ-matrices are defined by relations

$$\mu^{01} = \frac{1}{\sqrt{2}}\begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & -i \\ 0 & 0 & -i & 0 \end{vmatrix}, \quad \mu^{02} = \frac{1}{\sqrt{2}}\begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{vmatrix},$$

$$\mu^{03} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \end{vmatrix},$$

$$\mu^{23} = \frac{1}{\sqrt{2}}\begin{vmatrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}, \quad \mu^{31} = \frac{1}{\sqrt{2}}\begin{vmatrix} 0 & -i & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{vmatrix},$$

$$\mu^{12} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}.$$

Equation (1) was considered [5] in spherical coordinates $x^\alpha = (t, r, \theta, \phi)$ and corresponding tetrad [6]. For components of the operator $D_c = e^\alpha_{(c)}\partial_\alpha + \frac{1}{2}(\sigma^{nm} \otimes I + I \otimes j^{nm})\gamma_{[nm]c}$ explicitly read

$$\begin{gathered} D_0 = \partial_t, \quad D_3 = \partial_r, \\ D_1 = \frac{1}{r}\partial_\theta + \frac{1}{r}(\sigma^{31} \otimes I + I \otimes j^{31}), \\ D_2 = \frac{1}{r}(\sigma^{32} \otimes I + I \otimes j^{32}) \\ + \frac{1}{r}\frac{\partial_\phi + \cos\theta(\sigma^{12} \otimes I + I \otimes j^{12})}{\sin\theta}, \end{gathered} \tag{2}$$

where σ^{ab}, j^{ab} define generators for the bispinor and vector representations respectively.

The substitution for the wave function has the form [7, 8]

$$\Psi = \Psi_{A(l)} = e^{-i\epsilon t}\begin{vmatrix} f_0 D_{-1/2} & f_1 D_{-3/2} & f_2 D_{-1/2} & f_3 D_{+1/2} \\ g_0 D_{+1/2} & g_1 D_{-1/2} & g_2 D_{+1/2} & g_3 D_{+3/2} \\ h_0 D_{-1/2} & h_1 D_{-3/2} & h_2 D_{-1/2} & h_3 D_{+1/2} \\ d_0 D_{+1/2} & d_1 D_{-1/2} & d_2 D_{+1/2} & d_3 D_{+3/2} \end{vmatrix}, \tag{3}$$

here we use the Wigner functions, $D_\sigma = D^j_{-m,\sigma}(\phi, \theta, 0)$; $j = 1/2, 3/2, 5/2, \ldots$ This substitution contains 16 radial functions f_a, g_a, h_a, d_a. At $j = 1/2$ it simplifies, $f_1 = 0$, $g_3 = 0$, $h_1 = 0$, $d_3 = 0$. From requirement of diagonalization of the space reflection operator [8], only 8 independent functions remain

$$\Psi = \begin{vmatrix} f_0 D_{-1/2} & f_1 D_{-3/2} & f_2 D_{-1/2} & f_3 D_{+1/2} \\ g_0 D_{+1/2} & g_1 D_{-1/2} & g_2 D_{+1/2} & g_3 D_{+3/2} \\ \delta g_0 D_{-1/2} & \delta g_3 D_{-3/2} & \delta g_2 D_{-1/2} & \delta g_1 D_{+1/2} \\ \delta f_0 D_{+1/2} & \delta f_3 D_{-1/2} & \delta f_2 D_{+1/2} & \delta f_1 D_{+3/2} \end{vmatrix}. \tag{4}$$

Using the known relations for the Wigner functions [7],

$$\begin{gathered} \partial_\theta D_{+1/2} = \frac{1}{2}(aD_{-1/2} - bD_{+3/2}), \\ \partial_\theta D_{-1/2} = \frac{1}{2}(bD_{-3/2} - aD_{+1/2}), \\ \partial_\theta D_{+3/2} = \frac{1}{2}(bD_{+1/2} - cD_{+5/2}), \\ \partial_\theta D_{-3/2} = \frac{1}{2}(cD_{-5/2} - bD_{-1/2}), \\ \frac{1}{\sin\theta}\left(-m - \frac{1}{2}\cos\theta\right)D_{+1/2} = \frac{1}{2}(-aD_{-1/2} - bD_{+3/2}), \\ \frac{1}{\sin\theta}\left(-m + \frac{1}{2}\cos\theta\right)D_{-1/2} = \frac{1}{2}(-bD_{-3/2} - aD_{+1/2}), \\ \frac{1}{\sin\theta}\left(-m - \frac{3}{2}\cos\theta\right)D_{+3/2} = \frac{1}{2}(-bD_{+1/2} - cD_{+5/2}), \\ \frac{1}{\sin\theta}\left(-m + \frac{3}{2}\cos\theta\right)D_{-3/2} = \frac{1}{2}(-cD_{-5/2} - bD_{-1/2}), \end{gathered} \tag{5}$$

$$a = j + 1/2, \quad b = \sqrt{(j - 1/2)(j + 3/2)}, \quad c = \sqrt{(j - 3/2)(j + 5/2)},$$

after separating the variables, in [5] where found 8 equations (the equations for states with opposite parities differ only in sign at the mass parameter, so we follow only the case $\delta = +1$; radial equations for states with opposite parities differ only in sign at the mass parameter M)

$$\sqrt{2}\frac{d}{dr}g_1 + \frac{1}{r}\left(f_2 + \frac{3}{\sqrt{2}}g_1\right) + \frac{1}{\sqrt{2}r}(bf_1 - af_3 + a\sqrt{2}g_2) + iM(g_2 - \sqrt{2}f_3) = 0,$$

$$\sqrt{2}\frac{d}{dr}f_3 + \frac{1}{r}\left(g_2 + \frac{3}{\sqrt{2}}f_3\right) + \frac{1}{\sqrt{2}r}(-ag_1 + bg_3 + a\sqrt{2}f_2) + iM(\sqrt{2}g_1 - f_2) = 0,$$

$$-i\epsilon f_1 + \frac{d}{dr}f_1 + \frac{1}{r}f_1 + \frac{1}{\sqrt{2}r}(bf_2 + bf_0) + iMg_3 = 0,$$

$$-i\epsilon(\sqrt{2}f_2 - g_1) + \left(-\sqrt{2}\frac{d}{dr}f_0 + \frac{}{dr}g_1\right)$$
$$-\frac{1}{r}\left(\frac{1}{\sqrt{2}}(f_0 - f_2) - g_1\right) + \frac{1}{\sqrt{2}r}(ag_2 - ag_0)$$
$$- iM(f_3 + \sqrt{2}(g_0 - g_2)) = 0, \tag{6}$$
$$- \ i\epsilon\sqrt{2}g_1 + \frac{1}{r}\left(f_0 - \frac{1}{\sqrt{2}}g_1\right)$$
$$+ \frac{1}{\sqrt{2}r}(-bf_1 + af_3 + a\sqrt{2}g_0) + iM(\sqrt{2}f_3 + g_0) = 0,$$
$$-i\epsilon\sqrt{2}f_3 + \frac{1}{r}\left(g_0 + \frac{1}{\sqrt{2}}f_3\right) + \frac{1}{\sqrt{2}r}$$
$$\times (-ag_1 + bg_3 + a\sqrt{2}f_0) + iM(\sqrt{2}g_1 - f_0) = 0,$$
$$-i\epsilon(\sqrt{2}g_2 - f_3) + \left(-\sqrt{2}\frac{d}{dr}g_0 - \frac{d}{dr}f_3\right)$$
$$-\frac{1}{r}\left(\frac{1}{\sqrt{2}}(g_0 + g_2) + f_3\right) + \frac{1}{\sqrt{2}r}(-af_2 - af_0)$$
$$+ iM(\sqrt{2}f_0 + \sqrt{2}f_2 - g_1) = 0,$$
$$-i\epsilon g_3 - \frac{}{dr}g_3 - \frac{1}{r}g_3 + \frac{1}{\sqrt{2}r}(-bg_2 + bg_0) + iMf_1 = 0.$$

3. LARGE AND SMALL COMPONENTS, PROJECTION OPERATORS

Exists the method for performing the nonrelativistic approximation in equations for spin 3/2 particle in Cartesian coordinates [4]:

$$[\Gamma^0\partial_0 + i\Gamma^1\partial_1 + i\Gamma^2\partial_2 + i\Gamma^3\partial_3 + iM\Gamma]\Psi^{\text{cart}} = 0; \tag{7}$$

After changing notation, this equation is written as

$$[Y^0\partial_0 + iY^1\partial_1 + Y^2\partial_2 + Y^3\partial_3 + iM]\Psi^{\text{cart}} = 0. \tag{8}$$

On the base of 3-order minimal equation for the matrix Y^0, three projective operators are introduced which allow us to decompose the wave function into the sum of three components: one large and two small.

The needed 16-dimensional representations for the wave function (it consists of the four 4-columns) and the matrix $Y^0 = Y_0$:

$$Y^0 = \left|\begin{array}{cccccccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{2\sqrt{2}}{3} & \cdot & \cdot & \frac{2}{3} & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{2}{3} & \cdot & \cdot & \frac{2\sqrt{2}}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{2\sqrt{2}}{3} & \cdot & \cdot & -\frac{2}{3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{2}{3} & \cdot & \cdot & -\frac{2\sqrt{2}}{3} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \frac{\sqrt{2}}{3} & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \frac{\sqrt{2}}{3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{\sqrt{2}}{3} & \cdot & \cdot & \frac{2}{3} & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{2}{3} & \cdot & \cdot & \frac{\sqrt{2}}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{\sqrt{2}}{3} & \cdot & \cdot & \frac{2}{3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{2}{3} & \cdot & \cdot & \frac{\sqrt{2}}{3} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{\sqrt{2}}{3} & \cdot & \cdot & \frac{1}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{\sqrt{2}}{3} & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot
\end{array}\right|.$$

Its minimal equation is $Y_0^2(Y_0^2-1)=0$, we can introduce three projective operators

$$
\begin{gathered}
P_0 = 1 - Y_0^2, \;\; P_+ = +\frac{1}{2}Y_0^2(Y_0+1), \\
P_- = -\frac{1}{2}Y_0^2(Y_0-1);
\end{gathered}
\tag{9}
$$

with the properties $P_0+P_++P_-=1$, $P_0^2=P_0$, $P_+^2=P_+$, $P_-^2=P_-$.

The explicit expressions for these operators are needed, due to its bulkiness we do not present them here. Further we obtain the expressions for the projective constituents of the wave (applying the shortening notations for large and small components)

$$
\Psi_+ = \begin{vmatrix}
0 \\ 0 \\ 0 \\ 0 \\
\frac{1}{2}(f_1+h_1) \\
\frac{1}{6}\left(d_1+\sqrt{2}f_2+g_1+\sqrt{2}h_2\right) \\
\frac{1}{2}(f_1+h_1) \\
\frac{1}{6}\left(d_1+\sqrt{2}f_2+g_1+\sqrt{2}h_2\right) \\
\frac{1}{6}\left(\sqrt{2}d_1+2f_2+\sqrt{2}g_1+2h_2\right) \\
\frac{1}{6}\left(2d_2+\sqrt{2}f_3+2g_2+\sqrt{2}h_3\right) \\
\frac{1}{6}\left(\sqrt{2}d_1+2f_2+\sqrt{2}g_1+2h_2\right) \\
\frac{1}{6}\left(2d_2+\sqrt{2}f_3+2g_2+\sqrt{2}h_3\right) \\
\frac{1}{6}\left(\sqrt{2}d_2+f_3+\sqrt{2}g_2+h_3\right) \\
\frac{1}{2}(d_3+g_3) \\
\frac{1}{6}\left(\sqrt{2}d_2+f_3+\sqrt{2}g_2+h_3\right) \\
\frac{1}{2}(d_3+g_3)
\end{vmatrix}
= \begin{vmatrix}
0 \\ 0 \\ 0 \\ 0 \\ L_1 \\ L_2 \\ L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_5 \\ L_6
\end{vmatrix},
$$

$$\Psi_0 = \begin{vmatrix} f_0 \\ g_0 \\ h_0 \\ d_0 \\ 0 \\ \frac{1}{3}\left(2g_1 - \sqrt{2}f_2\right) \\ 0 \\ \frac{1}{3}\left(2d_1 - \sqrt{2}h_2\right) \\ -\frac{1}{3\sqrt{2}}\left(2g_1 - \sqrt{2}f_2\right) \\ \frac{1}{3}\left(g_2 - \sqrt{2}f_3\right) \\ -\frac{1}{3\sqrt{2}}\left(2d_1 - \sqrt{2}h_2\right) \\ \frac{1}{3}\left(d_2 - \sqrt{2}h_3\right) \\ -\frac{\sqrt{2}}{3}\left(g_2 - \sqrt{2}f_3\right) \\ 0 \\ -\frac{\sqrt{2}}{3}\left(d_2 - \sqrt{2}h_3\right) \\ 0 \end{vmatrix} = \begin{vmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ 0 \\ S_5 \\ 0 \\ S_6 \\ -\frac{1}{\sqrt{2}}S_5 \\ S_7 \\ -\frac{1}{\sqrt{2}}S_6 \\ S_8 \\ -\sqrt{2}S_7 \\ 0 \\ -\sqrt{2}S_8 \\ 0 \end{vmatrix}, \quad \Psi_- = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2}(f_1 - h_1) \\ \frac{1}{6}\left(-d_1 + \sqrt{2}f_2 + g_1 - \sqrt{2}h_2\right) \\ \frac{1}{2}(h_1 - f_1) \\ \frac{1}{6}\left(d_1 - \sqrt{2}f_2 - g_1 + \sqrt{2}h_2\right) \\ \frac{1}{6}\left(-\sqrt{2}d_1 + 2f_2 + \sqrt{2}g_1 - 2h_2\right) \\ \frac{1}{6}\left(-2d_2 + \sqrt{2}f_3 + 2g_2 - \sqrt{2}h_3\right) \\ \frac{1}{6}\left(\sqrt{2}d_1 - 2f_2 - \sqrt{2}g_1 + 2h_2\right) \\ \frac{1}{6}\left(2d_2 - \sqrt{2}f_3 - 2g_2 + \sqrt{2}h_3\right) \\ \frac{1}{6}\left(-\sqrt{2}d_2 + f_3 + \sqrt{2}g_2 - h_3\right) \\ \frac{1}{2}(g_3 - d_3) \\ \frac{1}{6}\left(\sqrt{2}d_2 - f_3 - \sqrt{2}g_2 + h_3\right) \\ \frac{1}{2}(d_3 - g_3) \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P_1 \\ P_2 \\ -P_1 \\ -P_2 \\ P_3 \\ P_4 \\ -P_3 \\ -P_4 \\ P_5 \\ P_6 \\ -P_5 \\ -P_6 \end{vmatrix}.$$

As known from the general theory, the component Ψ_+ should be considered as the large one, and two other as small, $P_i \ll L_i$, $S_i \ll L_i$. Three projective constituents consist of following variables

$$\Psi_+, \ \{L_1,\ldots,L_6\}, \ \Psi_0, \ \{S_1,\ldots,S_8\}, \ \Psi_-, \ \{P_1,\ldots,P_6\}.$$

Since the initial wave function contains 16 components, we should expect existence of 4 restrictions. After preserving only independent variables among $L_{..}$, $P_{..}$, $S_{..}$, and taking into account parity restrictions, we obtain

$$\Psi_{\delta=+1} = \begin{vmatrix} f_0 \\ g_0 \\ g_0 \\ f_0 \\ f_1 \\ g_1 \\ g_3 \\ f_3 \\ f_2 \\ g_2 \\ g_2 \\ f_2 \\ f_3 \\ g_3 \\ g_1 \\ f_1 \end{vmatrix} = \begin{vmatrix} S_1 \\ S_2 \\ +S_2 \\ +S_1 \\ L_1 + y_1 \\ L_2 + y_2 \\ L_1 - y_1 \\ L_2 - y_2 - \sqrt{2}y_4 - \sqrt{2}y_5 \\ \sqrt{2}L_2 + y_4 \\ \sqrt{2}L_2 + y_5 \\ \sqrt{2}L_2 + y_5 \\ \sqrt{2}L_2 + y_4 \\ L_2 - y_2 - \sqrt{2}y_4 - \sqrt{2}y_5 \\ +L_1 - y_1 \\ L_2 + y_2, \\ +L_1 + y_1 \end{vmatrix},$$

Further, we can pass to truncated 8-component columns (recall that when separating the variables for states with fixed parities, two systems of 8 equations were derived; and they differ only in sign at the mass parameter)

$$\Psi_{\delta=+1} = \begin{vmatrix} f_0 \\ g_0 \\ f_1 \\ g_1 \\ g_3 \\ f_3 \\ f_2 \\ g_2 \\ g_2 \\ f_2 \\ f_3 \\ g_3 \end{vmatrix} = \begin{vmatrix} S_1 \\ S_2 \\ L_1 + y_1 \\ L_2 + y_2 \\ L_1 - y_1 \\ L_2 - y_2 - \sqrt{2}y_4 - \sqrt{2}y_5 \\ \sqrt{2}L_2 + y_4 \\ \sqrt{2}L_2 + y_5 \\ \sqrt{2}L_2 + y_5 \\ \sqrt{2}L_2 + y_4 \\ L_2 - y_2 - \sqrt{2}y_4 - \sqrt{2}y_5 \\ +L_1 - y_1 \end{vmatrix},$$

The truncated 8-dimensional column (it is composed of functions included in the 8-dimensional radial system) turns out to be the same (the difference for the parity will be only in sign at the mass parameter). The inverse transformation has the form

$$\begin{gathered} L_1 = \frac{1}{2}(f_1 + g_3), \quad L_2 = \frac{1}{6}(\sqrt{2}f_2 + f_3 + g_1 + \sqrt{2}g_2), \\ S_1 = f_0, \quad S_2 = g_0, y_1 = \frac{1}{2}(f_1 - g_3), \\ y_2 = \frac{1}{6}(-\sqrt{2}f_2 - f_3 + 5g_1 - \sqrt{2}g_2), \\ y_4 = \frac{1}{6}(4f_2 - \sqrt{2}f_3 - \sqrt{2}g_1 - 2g_2), \\ y_5 = \frac{1}{6}(-2f_2 - \sqrt{2}f_3 - \sqrt{2}g_1 + 4g_2). \end{gathered} \tag{10}$$

We turn to the system of 8 radial equations (the presence of external Coulomb field is taken into account by formal change $E \Rightarrow E + \alpha/r$), and substitute expressions for functions through independent large and small components.

Then, we should separate the rest energy by the formal change $\epsilon = (M + E)$, $E \ll M$. Besides, when performing the no-relativistic approximation in the resulting radial system, we should follow the known prescriptions [4] for smallness orders of various quantities

$$\begin{gathered} L \sim 1, \quad y \sim x, \quad S \sim x, \quad \frac{1}{M}\frac{d}{dr} \sim x, \quad \frac{1}{rM} \sim x, \\ rM \sim \frac{1}{x}, \quad r\frac{1}{M}\frac{d^2}{dr^2} \sim x, \quad \frac{E}{M} \sim x^2, \\ rE \sim x, \quad r\frac{d}{dr} \sim 1. \end{gathered} \tag{11}$$

Then, after manipulation with the equations, in order to correctly take into account existence of large and small variables, and eliminating the all small variables, we can derive equations for two large variables L_1, L_2:

$$\begin{gathered} \left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + 2M\left(E + \frac{\alpha}{r}\right)\right]L_1 \\ = \frac{1}{r^2}[b^2 L_1 + 3bL_2], \\ \left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + 2M\left(E + \frac{\alpha}{r}\right)\right]L_2 \\ = \frac{1}{r^2}\left[\frac{2a(a+3) + b^2 + 10}{3}L_2 + bL_1\right]. \end{gathered} \tag{12}$$

The last system, after performing the relevant linear transformation $\bar{L} = SL$, can be reduced to separate equations for new variables $\bar{L}_1, \bar{L}_2$:

$$\begin{gathered} \left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + 2M\left(E + \frac{\alpha}{r}\right) - \frac{(j+2)^2 - 1/4}{r^2}\right]\bar{L}_1 = 0, \\ \left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + 2M\left(E + \frac{\alpha}{r}\right) - \frac{j^2 - 1/4}{r^2}\right]\bar{L}_2 = 0. \end{gathered} \tag{13}$$

These equation are solved in hypergeometric functions, they provide us with two energy spectra (let $2Br = -x$)

$$E_2 = -\frac{\alpha^2 M}{2N^2} = -\frac{\alpha^2 M}{2\left(j + \frac{1}{2} + n\right)^2},$$
$$\bar{L}_2 = r^{j-1/2} e^{-\sqrt{-2MEr}} F(-n, 2j+1, x). \tag{14}$$
$$E_1 = -\frac{\alpha^2 M}{2N^2} = -\frac{\alpha^2 M}{2\left(j + \frac{5}{2} + n\right)^2},$$
$$\bar{L}_1 = r^{j+3/2} e^{-\sqrt{-2MEr}} F(-n, 2j+5, x).$$

The initial variables L_1, L_2 are determined by the rule $L = S^{-1}\bar{L}$. The non-relativistic wave function with quantum numbers of the energy, square and the third projection of the total momentum, and parity is given as

$$\Psi_{E,j,m,\delta}(t,r,\theta,\phi) = e^{-iEt} \begin{vmatrix} L_1(r) D^j_{-m,-3/2}(\phi,\theta,0) \\ L_2(r) D^j_{-m,-1/2}(\phi,\theta,0) \\ \delta L_2(r) D^j_{-m,+1/2}(\phi,\theta,0) \\ \delta L_1(r) D^j_{-m,+3/2}(\phi,\theta,0) \end{vmatrix}, \quad \delta = \pm 1. \tag{15}$$

4. CONCLUSIONS

In the present paper, the system of radial Eqs. (6) describing the nonrelativistic spin 3/2 particle in presence if the external Coulomb field has been derived from the relativistic equations. As a result, the system of two 2nd order differential Eqs. (12), (13) describing the nonrelativistic particle has been derived. It refers to states with quantum number of energy (14), square and the third projection of the total angular momentum (3), and parity (4). Solutions of these equations have been constructed in terms of the confluent hypergeometric functions, the corresponding energy spectra are found.

SPELL: 1. ok

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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