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Nonrelativistic Approximation for a Spin-3/2 Particle in Combined Electromagnetic and Gravitational Fields

A. V. Ivashkevich[a], V. M. Red'kov[a], M. K. Margaryan[b], and A. M. Ishkhanyan[b, *]

[a] *Stepanov Institute of Physics of NAS of Belarus, Minsk, Belarus*
[b] *Institute for Physical Research, NAS of Armenia, Ashtarak, Armenia*
**e-mail: aishkhanyan@gmail.com*

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Abstract—This paper investigates the nonrelativistic approximation in the first-order 39-component theory for a spin-3/2 particle in curved spacetime under external electromagnetic fields. Starting from the generally covariant matrix equations, generalized via the Weyl-Fock-Ivanenko tetrad method, we employ explicit forms of the four main Γ^a matrices of dimension 16×16 within the corresponding first-order system. The analysis is carried out for spacetime metrics that permit the existence of nonrelativistic equations. To separate the large and small components of the complete wave function, three projective operators are constructed from the fourth-order minimal polynomial of the $\Gamma^0_{16\times 16}$ matrix. The explicit forms of these components are obtained, and the set of independent variables is identified; in particular, only four large components are independent. Following the standard procedure, we derive the nonrelativistic system of equations for a 4-component wave function. The resulting Hamiltonian includes contributions from the electromagnetic field and additional geometric terms expressed through the Ricci rotation coefficients, Ricci scalar R and Ricci tensor R_{ab}. We also isolate the term describing the interaction between the magnetic moment of the spin-3/2 particle and the external magnetic field. This interaction term is expressed via the spin matrices S_i and the components of the magnetic field vector $\mathbf{B}$.

Keywords: spin-3/2 particle, non-relativistic approximation, projective operators, electromagnetic fields, Riemannian space-time

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1. INTRODUCTION

Following the seminal investigations of Pauli–Fierz [1, 2] and Rarita–Schwinger [3], the theory of particles with spin $S = 3/2$ has been developed extensively in the scientific literature [4–14]. More recently [15–17], for a spin-3/2 particle in the presence of external magnetic and Coulomb fields, exact solutions have been obtained and the corresponding energy spectra determined. These results were derived from systems of 16 equations in polar and radial coordinates, obtained by separation of variables. However, to our knowledge, the explicit form of the corresponding nonrelativistic Hamiltonians has not been presented.

In this paper, we address this gap by deriving a Pauli-like equation for a spin-3/2 particle in the presence of both arbitrary electromagnetic fields and a general spacetime background that admits nonrelativistic quantum-mechanical equations – that is, we determine the explicit form of the corresponding nonrelativistic Hamiltonian. To verify the correctness and practical applicability of the derived Hamiltonian, we present its form for a particular case: a spin-3/2 particle in a uniform magnetic field, where the result agrees with [15, 16].

The general form of the nonrelativistic Hamiltonian can, in principle, be established explicitly for any external electromagnetic field and any spacetime background of the relevant structure. This approach can also be extended to spin-2 particles. In that case, the analysis becomes more complex, as one must begin with the system of 39 first-order equations formulated by Fedorov [18] and Regge [19]. Nevertheless, the overall logic of the derivation remains the same as for the spin-3/2 case.

The theoretical interest in the nonrelativistic description of higher-spin particles is reinforced by practical considerations, as spin-3/2 and spin-2 particles are experimentally observable as resonances.

2. THE BASIC EQUATION

The generally covariant equation for a spin-3/2 particle [1–4] takes the form

$$\gamma^5 \epsilon_\rho^{\ \sigma\alpha\beta}(x)\,\gamma_\sigma\left(iD_\alpha - \frac{1}{2}M\gamma_\alpha\right)\Psi_\beta = 0, \quad D_\alpha = \nabla_\alpha + \Gamma_\alpha + ieA_\alpha, \tag{1}$$

and can equivalently be written in the local tetrad form as

$$\gamma^5(\mu^{[ca]})_k^n\gamma_c\left[i(D_a)_n^l - M\gamma_a\delta_n^l\right]\Psi_l = 0, \quad \epsilon_k^{can} = (\mu^{[ca]})_k^n, \tag{2}$$

where the generalized derivatives are defined as

$$D_a = e_{(a)}^{\alpha}(\partial_\alpha + ieA_\alpha) + \frac{1}{2}\left(\sigma^{ps}\otimes I + I\otimes j^{ps}\right)\gamma_{[ps]a} = e_{(a)}^{\alpha}(\partial_\alpha + ieA_\alpha) + \Sigma_a. \tag{3}$$

3. LARGE AND SMALL NON-RELATIVISTIC COMPONENTS

The wave function can be represented as a matrix, where the first index A corresponds to the bispinor component and the second index (n) to the 4-vector component:

$$\Psi_{A(n)} = \begin{vmatrix} f_0 & f_1 & f_2 & f_3 \\ g_0 & g_1 & g_2 & g_3 \\ h_0 & h_1 & h_2 & h_3 \\ d_0 & d_1 & d_2 & d_3 \end{vmatrix}. \tag{4}$$

We will represent the complete Ψ as a 16-dimensional column vector.

It is convenient to write the main equation in the matrix form:

$$\left(Y_0 D_0 + Y^1 D_1 + Y^2 D_2 + Y^3 D_3 + iM\right)\Psi = 0. \tag{5}$$

Following the standard approach, the large and small components in the non-relativistic limit are determined by projective operators constructed from the matrix Y^0. This matrix Y_0 satisfies the minimal fourth-order equation:

$$Y_0^2\left(Y_0^2 - I_{16}\right) = 0. \tag{6}$$

Accordingly, there are three projective operators (given here in explicit form):

$$P_0 = I_{16} - Y_0^2, \quad P_1 = P_+ = +\frac{1}{2}Y_0^2(Y + I_{16}), \quad P_2 = P_- = -\frac{1}{2}Y_0^2(Y - I_{16}), \tag{7}$$

which possess the standard properties of orthogonality and completeness:

$$P_0 + P_+ + P_- = I_{16}, \quad P_0^2 = P_0, \quad P_1^2 = P_1, \quad P_2^2 = P_2. \tag{8}$$

We thus obtain three projective components, each containing a certain number of independent variables:

$$\Psi_0 = P_0\Psi, \quad \Psi_+ = \Psi_1 = P_1\Psi, \quad \Psi_- = \Psi_2 = P_2\Psi, \tag{9}$$

$$\Psi_0 = \begin{vmatrix} f_0 \\ g_0 \\ h_0 \\ d_0 \\ (f_1 + if_2 - g_3)/3 \\ (f_3 + g_1 - ig_2)/3 \\ (-d_3 + h_1 + ih_2)/3 \\ (d_1 - id_2 + h_3)/3 \\ (-if_1 + f_2 + ig_3)/3 \\ i(f_3 + g_1 - ig_2)/3 \\ (i(d_3 - h_1) + h_2)/3 \\ i(d_1 - id_2 + h_3)/3 \\ (f_3 + g_1 - ig_2)/3 \\ (-f_1 - if_2 + g_3)/3 \\ (d_1 - id_2 + h_3)/3 \\ (d_3 - h_1 - ih_2)/3 \end{vmatrix} = \begin{vmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \\ -iS_5 \\ iS_6 \\ -iS_7 \\ iS_8 \\ S_6 \\ -S_5 \\ S_8 \\ -S_7 \end{vmatrix}, \tag{10}$$

$$\Psi_- = \frac{1}{6}\begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -d_3 + 2f_1 - if_2 + g_3 - 2h_1 + ih_2 \\ -2d_1 - id_2 - f_3 + 2g_1 + ig_2 + h_3 \\ d_3 - 2f_1 + if_2 - g_3 + 2h_1 - ih_2 \\ 2d_1 + id_2 + f_3 - 2g_1 - ig_2 - h_3 \\ i(d_3 + f_1 - 2if_2 - g_3 - h_1 + 2ih_2) \\ i(d_1 + 2id_2 - f_3 - g_1 - 2ig_2 + h_3) \\ -i(d_3 + f_1 - 2if_2 - g_3 - h_1 + 2ih_2) \\ -i(d_1 + 2id_2 - f_3 - g_1 - 2ig_2 + h_3) \\ d_1 - id_2 + 2f_3 - g_1 + ig_2 - 2h_3 \\ -2d_3 + f_1 + if_2 + 2g_3 - h_1 - ih_2 \\ -d_1 + id_2 - 2f_3 + g_1 - ig_2 + 2h_3 \\ 2d_3 - f_1 - if_2 - 2g_3 + h_1 + ih_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P_1 \\ P_2 \\ -P_1 \\ -P_2 \\ P_3 \\ P_4 \\ -P_3 \\ -P_4 \\ P_5 \\ P_6 \\ -P_5 \\ -P_6 \end{vmatrix}, \tag{11}$$

$$\Psi_+ = \frac{1}{6}\begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ d_3 + 2f_1 - if_2 + g_3 + 2h_1 - ih_2 \\ 2d_1 + id_2 - f_3 + 2g_1 + ig_2 - h_3 \\ d_3 + 2f_1 - if_2 + g_3 + 2h_1 - ih_2 \\ 2d_1 + id_2 - f_3 + 2g_1 + ig_2 - h_3 \\ -i(d_3 - f_1 + 2if_2 + g_3 - h_1 + 2ih_2) \\ -i(d_1 + 2id_2 + f_3 + g_1 + 2ig_2 + h_3) \\ -i(d_3 - f_1 + 2if_2 + g_3 - h_1 + 2ih_2) \\ -i(d_1 + 2id_2 + f_3 + g_1 + 2ig_2 + h_3) \\ -d_1 + id_2 + 2f_3 - g_1 + ig_2 + 2h_3 \\ 2d_3 + f_1 + if_2 + 2g_3 + h_1 + ih_2 \\ -d_1 + id_2 + 2f_3 - g_1 + ig_2 + 2h_3 \\ 2d_3 + f_1 + if_2 + 2g_3 + h_1 + ih_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ L_1 \\ L_2 \\ L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_5 \\ L_6 \end{vmatrix}. \tag{12}$$

While performing the non-relativistic approximation, we treat Ψ_+ as the large component, and Ψ_- and Ψ_0 as small components: $P_i \ll L_i$; $S_i \ll L_i$. In total, we have the following 20 variables:

$$\Psi_+ : \{L_1, \ldots, L_6\}; \quad \Psi_0 : \{S_1, \ldots, S_8\}, \quad \Psi_- : \{P_1, \ldots, P_6\}. \tag{13}$$

There exist relations (constraints) between the large and small components, which make it possible to express some of the variables in terms of others:

$$iL_3 = L_6 - L_1, \quad iL_4 = L_5 + L_2, \tag{14}$$

$$P_1 + iP_3 - P_6 = 0, \quad P_2 - iP_4 + P_5 = 0. \tag{15}$$

Introducing new variables $y_1, \ldots, y_{12}$:

$$S_5 + P_1 = y_1, \quad S_6 + P_2 = y_2, \quad S_7 - P_1 = y_3, \quad S_8 - P_2 = y_4, \tag{16}$$

$$iS_5 + P_3 = y_5, \quad iS_6 + P_4 = y_6, \quad iS_7 - P_3 = y_7, \quad iS_8 - P_4 = y_8, \tag{17}$$

$$S_6 + P_5 = y_9, \quad -S_5 + P_6 = y_{10}, \quad S_8 - P_5 = y_{11}, \quad -S_7 - P_6 = y_{12}, \tag{18}$$

the four independent variables S_5, S_6, S_7, S_8 can be expressed in terms of the 12 auxiliary y-variables. This makes it possible to construct a closed system for the independent large components.

4. THE NON-RELATIVISTIC APPROXIMATION

The non-relativistic approximation is possible only in space–time models with the following structure:

$$dS^2 = (dx^0)^2 + g_{ij}(x)\,dx^i dx^j, \quad e_{(a)\alpha}(x) = \begin{vmatrix} 1 & 0 \\ 0 & e_{(i)k}(x) \end{vmatrix}. \tag{19}$$

For these space models, only four connections differ from zero:

$$\Sigma_0 = \frac{1}{2} J^{ik} e^m_{(i)} \left(\nabla_0 e_{(k)m}\right), \quad \Sigma_l = \frac{1}{2} J^{ik} e^m_{(i)} \left(\nabla_l e_{(k)m}\right), \tag{20}$$

and the contribution of three generators J^{01}, J^{02}, J^{03} is absent.

The basic matrix equation in the non-relativistic metric for a spin-3/2 particle has the form:

$$\left(Y^0 \bar{D}_0 + Y^1 \bar{D}_1 + Y^2 \bar{D}_2 + Y^3 \bar{D}_3 + iM\right)\Psi = 0, \tag{21}$$

where

$$\bar{D}_0 = (\partial_0 + ieA_0) + \left(\sigma^{23} \otimes I + I \otimes j^{23}\right)\gamma_{[23]0} + \left(\sigma^{31} \otimes I + I \otimes j^{31}\right)\gamma_{[31]0} + \left(\sigma^{12} \otimes I + I \otimes j^{12}\right)\gamma_{[12]0}, \tag{22}$$

$$\bar{D}_1 = e^k_{(1)}(\partial_k + ieA_k) + \left(\sigma^{23} \otimes I + I \otimes j^{23}\right)\gamma_{[23]1} + \left(\sigma^{31} \otimes I + I \otimes j^{31}\right)\gamma_{[31]1} + \left(\sigma^{12} \otimes I + I \otimes j^{12}\right)\gamma_{[12]1}, \tag{23}$$

$$\begin{aligned} \bar{D}_2 = e^k_{(2)}(\partial_k + ieA_k) + \left(\sigma^{23} \otimes I + I \otimes j^{23}\right)\gamma_{[23]2} \\ + \left(\sigma^{31} \otimes I + I \otimes j^{31}\right)\gamma_{[31]2} + \left(\sigma^{12} \otimes I + I \otimes j^{12}\right)\gamma_{[12]2}, \end{aligned} \tag{24}$$

$$\bar{D}_3 = (\partial_k + ieA_k) + \left(\sigma^{23} \otimes I + I \otimes j^{23}\right)\gamma_{[23]3} + \left(\sigma^{31} \otimes I + I \otimes j^{31}\right)\gamma_{[31]3} + \left(\sigma^{12} \otimes I + I \otimes j^{12}\right)\gamma_{[12]3}. \tag{25}$$

In this equation, the terms can be distinguished according to their physical meaning: some account for the dynamics in the electromagnetic field, while others correspond to geometric effects associated with the Ricci coefficients. For the 12 Ricci coefficients required in the subsequent derivations, it is convenient to introduce the following special notations:

$$\begin{aligned} G_{10} = \gamma_{230}, \quad G_{20} = \gamma_{310}, \quad G_{30} = \gamma_{120}, \quad G_{11} = \gamma_{231}, \quad G_{21} = \gamma_{311}, \quad G_{31} = \gamma_{121}, \\ G_{12} = \gamma_{232}, \quad G_{22} = \gamma_{312}, \quad G_{32} = \gamma_{122}, \quad G_{13} = \gamma_{233}, \quad G_{23} = \gamma_{313}, \quad G_{33} = \gamma_{123}. \end{aligned} \tag{26}$$

In the basic equation (21) one can distinguish two parts:

$$\left(Y^0 D_0 + Y^1 D_1 + Y^2 D_2 + Y^3 D_3 + iM\right)\Psi + \left(Q^0\Psi + Q^1\Psi + Q^2\Psi + Q^3\Psi\right) = 0, \tag{27}$$

where

$$D_0 = (\partial_0 + ieA_0), \quad D_1 = e^k_{(1)}(\partial_k + ieA_k), \quad D_2 = e^k_{(2)}(\partial_k + ieA_k), \quad \bar{D}_3 = (\partial_k + ieA_k), \tag{28}$$

$$Q^0\Psi = Y^0\left[\left(\sigma^{23}\Psi + \Psi\tilde{j}^{23}\right)G_{10} + \left(\sigma^{31}\Psi + \Psi\tilde{j}^{31}\right)G_{20} + \left(\sigma^{12}\Psi + \Psi\tilde{j}^{12}\right)G_{30}\right], \tag{29}$$

$$Q^1\Psi = Y^1\left[\left(\sigma^{23}\Psi + \Psi\tilde{j}^{23}\right)G_{11} + \left(\sigma^{31}\Psi + \Psi\tilde{j}^{31}\right)G_{21} + \left(\sigma^{12}\Psi + \Psi\tilde{j}^{12}\right)G_{31}\right], \tag{30}$$

$$Q^2\Psi = Y^2\left[\left(\sigma^{23}\Psi + \Psi\tilde{j}^{23}\right)G_{12} + \left(\sigma^{31}\Psi + \Psi\tilde{j}^{31}\right)G_{22} + \left(\sigma^{12}\Psi + \Psi\tilde{j}^{12}\right)G_{32}\right], \tag{31}$$

$$Q^3\Psi = Y^3\left[\left(\sigma^{23}\Psi + \Psi\tilde{j}^{23}\right)G_{13} + \left(\sigma^{31}\Psi + \Psi\tilde{j}^{31}\right)G_{23} + \left(\sigma^{12}\Psi + \Psi\tilde{j}^{12}\right)G_{33}\right]. \tag{32}$$

After performing the necessary calculations, and taking into account the decomposition of all components into large and small parts, we obtain 16 rather complicated equations; these will not be reproduced here.

It is known that, when performing the non-relativistic approximation, we assign the following orders of smallness to the relevant quantities:

$$L_{..} \sim 1, \quad S_{..}, \quad y_{..} \sim x, \quad \frac{D_j}{M} \sim x, \quad \frac{G_{ij}}{M} \sim x, \quad \frac{D_0}{M} \sim x^2, \quad \frac{G_{j0}}{M} \sim x^2. \tag{33}$$

In what follows, we will need only the equations of orders x and x^2. Taking this into account, we separate the system into equations of order x and equations of order x^2.

The equations of order x allow us to express the small components (of order x) through the derivatives D_0 and D_j acting on the large components L_1, L_2, L_5, L_6, as well as through the Ricci coefficients. Substituting these small components into the equations of order x^2, we obtain six equations for the four independent large variables L_1, L_2, L_5, L_6. These equations contain terms of the form $2MD_0L_A$, with $A = 1,2,5,6$. It can be shown that only four of these equations are independent. In what follows, we will work with this reduced set. Since they are still quite complicated, we temporarily remove all terms containing the Ricci rotation coefficients—effectively returning to the case of Cartesian coordinates in Minkowski space.

Further, we transform this system of four equations to a new set of variables:

$$\Psi_1 = L_1 + L_6, \quad \Psi_2 = L_2 - L_5, \quad \Psi_3 = 2L_1 - L_6, \quad \Psi_4 = 2L_2 + L_5. \tag{34}$$

With the following notations:

$$D_2D_3 - D_3D_2 = D_{23}, \quad D_3D_1 - D_1D_3 = D_{31}, \quad D_1D_2 - D_2D_3 = D_{12}, \tag{35}$$

$$\Delta = (D_1D_1 + D_2D_2 + D_3D_3), \quad D_0 = \partial_0 + ieA_0, \quad D_j = e^k_{(j)}(\partial_k + ieA_k), \tag{36}$$

they read

$$MiD_0\Psi_1 + \frac{1}{2}\Delta\Psi_1 - i\frac{1}{6}D_{12}\Psi_1 + \frac{1}{2}D_{31}\Psi_2 - \frac{i}{6}D_{23}\Psi_2 + \frac{i}{3}D_{12}\Psi_3 + \frac{i}{3}D_{23}\Psi_4 + \ldots = 0, \tag{37}$$

$$MiD_0\Psi_2 + \frac{1}{2}\Delta\Psi_2 + \frac{i}{6}D_{12}\Psi_2 - \frac{1}{2}D_{31}\Psi_1 - \frac{i}{6}D_{23}\Psi_1 + \frac{i}{3}D_{23}\Psi_3 - \frac{i}{3}D_{12}\Psi_4 + \ldots = 0, \tag{38}$$

$$MiD_0\Psi_3 + \frac{1}{2}\Delta\Psi_3 + \frac{i}{2}D_{12}\Psi_3 + \frac{1}{3}D_{31}\Psi_2 + \frac{i}{3}D_{23}\Psi_2 - \frac{1}{6}D_{31}\Psi_4 - \frac{i}{6}D_{23}\Psi_4 + \ldots = 0, \tag{39}$$

$$MiD_0\Psi_4 + \frac{1}{2}\Delta\Psi_4 - \frac{i}{2}D_{12}\Psi_4 - \frac{1}{3}D_{31}\Psi_1 + \frac{i}{3}D_{23}\Psi_1 + \frac{1}{6}D_{31}\Psi_3 - \frac{i}{6}D_{23}\Psi_3 + \ldots = 0, \tag{40}$$

where "..." denotes the omitted terms. This system may be written in the compact matrix form:

$$\Psi = \begin{vmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{vmatrix}, \quad iMD_0\Psi + \frac{1}{2}\Delta\Psi + (S_1D_{23} + S_2D_{31} + S_3D_{12})\Psi = 0, \tag{41}$$

$$S_1 = \begin{vmatrix} 0 & \frac{i}{2} & 0 & -i \\ \frac{i}{2} & 0 & -i & 0 \\ 0 & -i & 0 & \frac{i}{2} \\ -i & 0 & \frac{i}{2} & 0 \end{vmatrix}, \quad S_2 = \begin{vmatrix} 0 & -\frac{3}{2} & 0 & 0 \\ \frac{3}{2} & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ 1 & 0 & -\frac{1}{2} & 0 \end{vmatrix}, \quad S_3 = \begin{vmatrix} \frac{i}{2} & 0 & -i & 0 \\ 0 & -\frac{i}{2} & 0 & i \\ 0 & 0 & -\frac{3i}{2} & 0 \\ 0 & 0 & 0 & \frac{3i}{2} \end{vmatrix}, \tag{42}$$

These matrices obey the standard commutation relations: $S_iS_j - S_jS_i = \epsilon_{i,j,k}S_k$. By applying a suitable linear transformation, we introduce new spin matrices:

$$\bar{S}_1 = \begin{vmatrix} 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{3}{2} & 0 & -1 & 0 \\ 0 & -1 & 0 & -\frac{3}{2} \\ 0 & 0 & -\frac{1}{2} & 0 \end{vmatrix}, \quad \bar{S}_2 = \begin{vmatrix} 0 & -\frac{i}{2} & 0 & 0 \\ \frac{3i}{2} & 0 & -i & 0 \\ 0 & i & 0 & -\frac{3i}{2} \\ 0 & 0 & \frac{i}{2} & 0 \end{vmatrix}, \quad \bar{S}_3 = \begin{vmatrix} -\frac{3}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{3}{2} \end{vmatrix}. \tag{43}$$

Performing similar calculations but now retaining all terms involving the Ricci coefficients, we arrive at a more complicated system of four equations, which can be written in matrix form as

$$2M\,(D_0 + A_0)\Psi + (S_1D_{23} + S_2D_{31} + S_3D_{12})\Psi + (D_1A_1)\Psi + (A_1 + B_1)\,D_1\Psi + (D_2A_2)\Psi$$
$$+ (A_2 + B_2)\,D_2\Psi + (D_3A_3)\Psi + (A_3 + B_3)\,D_3\Psi + \Delta\Psi = 0. \quad (44)$$

The matrices A_0, A_1, A_2, A_3 and B_1, B_2, B_3 depend linearly on nine Ricci coefficients; the matrix Δ depends on products of Ricci coefficients $G_{..} \times G_{..}$.

It turns out that all these matrices can be decomposed into linear combinations of nine independent 4×4 basic elements (spin matrices and their products):

$$S_1 = t_1,\quad S_2 = t_2,\quad S_3 = t_3,\quad S_1^2 = t_4,\quad S_2^2 = t_5,\quad S_3^2 = t_6,\quad S_2S_3 = t_7,\quad S_3S_1 = t_8,\quad S_1S_2 = t_9, \quad (45)$$

and similar decompositions also hold for the matrices A_0 and Δ.

Thus, in the nonrelativistic Hamiltonian, additional geometrical terms arise that depend on the Ricci rotation coefficients G_{ab}, the Ricci scalar $R(x)$, and the tetrad components of the Ricci tensor:

$$R_{\alpha\beta}(x) \Rightarrow R_{ab}(x) = e^{\alpha}_{(a)}e^{\beta}_{(b)}R_{\alpha\beta}(x). \quad (46)$$

5. EXAMPLE: THE PARTICLE IN THE ELECTRIC AND MAGNETIC FIELDS

Using cylindrical coordinates $x^{\alpha} = (t, r, \phi, z)$, we consider the case of uniform electric and magnetic fields:

$$dS^2 = dt^2 - dr^2 - r^2d\phi^2 - dz^2,\quad e^{\alpha}_{(a)} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}, \quad (47)$$

$$0 \Rightarrow t,\quad 1 \Rightarrow r,\quad 2 \Rightarrow \phi,\quad 3 \Rightarrow z,\quad \gamma_{122} = \frac{1}{r}, \quad (48)$$

$$D_0 = \partial_0 + ieA_0,\quad D_1 = \partial_r + ieA_r,\quad D_2 = \frac{1}{r}(\partial_\phi + ieA_\phi),\quad D_3 = \partial_z + ieA_z,\quad \gamma_{122} = \frac{1}{r}. \quad (49)$$

Let us now discuss the case of uniform magnetic and electric fields along the z-axis:

$$D_0 = \partial_t + ieEz,\quad D_1 = \partial_r,\quad D_2 = \frac{1}{r}\left(\partial_\phi + ie\frac{Br^2}{2}\right),\quad D_3 = \partial_z, \quad (50)$$

$$A_0 = 0,\quad \Delta = \frac{\partial^2}{\partial r^2} + \left(\frac{1}{r}\partial_\phi + \frac{ieB}{2}r\right)^2 + \frac{\partial^2}{\partial z^2}, \quad (51)$$

$$D_{23} = 0,\quad D_{31} = 0,\quad D_{12} = -\frac{1}{r^2}\left(\partial_\phi - ie\frac{Br^2}{2}\right). \quad (52)$$

The basic Pauli-like equation becomes

$$i(\partial_t + ieEz)\,\bar{\Psi} = -\frac{1}{2M}\left[\frac{\partial^2}{\partial r^2} + \left(\frac{1}{r}\partial_\phi + \frac{ieB}{2}r\right)^2 + \frac{\partial^2}{\partial z^2}\right]\bar{\Psi} + \frac{i}{3M}\frac{1}{r^2}\left(\partial_\phi - ie\frac{Br^2}{2}\right)\bar{S}_3\bar{\Psi}$$
$$- \frac{i}{2M}\left[(\partial_r\bar{A}_1) + (\bar{A}_1 + \bar{B}_1)\,\partial_r + (\bar{A}_2 + \bar{B}_2)\frac{1}{r}\left(\partial_\phi + ie\frac{Br^2}{2}\right) + (\bar{A}_3 + \bar{B}_3)\,\partial_z + \bar{\Sigma}\right]\bar{\Psi} = 0. \quad (53)$$

The involved matrices are

$$(\partial_r \bar{A}_1) = \frac{i}{r^2}\begin{vmatrix} 9 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 9 \end{vmatrix}, \quad \bar{A}_1 + \bar{B}_1 = -\frac{6i}{r} I, \quad \bar{A}_2 + \bar{B}_2 = -\frac{8}{r}\begin{vmatrix} -3/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 3/2 \end{vmatrix}, \tag{54}$$

$$\bar{A}_3 + \bar{B}_3 = 0, \quad \bar{\Sigma} = \frac{3i}{r^2}\begin{vmatrix} 9 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 9 \end{vmatrix}, \quad \bar{S}_3 = \begin{vmatrix} -3/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 3/2 \end{vmatrix}. \tag{55}$$

This structure shows that, in the cylindrical basis, the non-relativistic equation consists of four uncoupled equations of similar form, so the Hamiltonian takes a particularly simple form.

5. CONCLUSIONS

The goal of the present paper was to develop a procedure for performing the non-relativistic approximation in the first-order 39-component theory for a spin-3/2 particle in curved space-time, in the presence of external electromagnetic fields.

To distinguish the large and small constituents in the complete wave function, we used three projective operators constructed on the basis of the minimal polynomial of fourth order for the matrix $\Gamma^0_{16\times16}$. The relevant large and small components were found in explicit form. Among these, we identified the independent variables; in particular, among the large components there are only four independent ones.

Following the standard procedure, we derived the non-relativistic system of equations for a 4-component wave function. The corresponding Hamiltonian depends on the electromagnetic field, while additional geometrical terms are determined by the Ricci rotation coefficients (these terms are expressed through the Ricci scalar R and the Ricci tensor R_{ab} in tetrad form). The term describing the interaction of the magnetic moment of the spin-3/2 particle with the external magnetic field was identified separately; this additional term was constructed using the spin matrices S_i and the components of the magnetic field **B**.

AUTHOR CONTRIBUTION

The authors equally contributed to this work in the conceptualization of the study, in the mathematical derivations, and in the writing of the manuscript. All authors have read and agreed to the published version of the manuscript.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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