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Non-relativistic Approximation for a Spin 3/2 Particle in Presence of Electromagnetic and Gravitational Fields

A.V. Ivashkevich* and V.M. Red'kov†
B. I. Stepanov Institute of Physics of NAS of Belarus, 68 Nezavisimosti Ave., 220072 Minsk, BELARUS

A.M. Ishkhanyan‡
Institute for Physical Research Ashtarak, 0204 Armenia

The goal of the present paper is investigation of the non-relativistic approximation in the first order 39-component theory for a spin 2 particle, in curved space-time, and in presence of external electromagnetic fields. We start with the generally covariant matrix equations generalized according to Weyl–Fock–Ivanenko tetrad method. We apply explicit expressions for four main matrices Γ^a with dimension 16×16 in the relevant first order system of equations, for space-time metrics allowing for existence of the non-relativistic equations.

For distinguishing the large and small constituents in the complete wave function, we use three projective operators constructed on the base of the minimal polynomial of the 4-th order for the matrix $\Gamma^0_{16\times 16}$. The relevant large and small components are found in explicit form. Among them we have found independent variables; in particular, among the large components there exist only four independent ones.

Acting in accordance with the known general procedure, we have derived the non-relativistic system of equations for a 4-component wave function; the relevant Hamiltonian depends on electromagnetic field and additional geometrical terms are determined by the Ricci rotation coefficients (these terms should be determined by Ricci scalar R and Ricci tensor R_{ab} in tetrad form. The terms describing interaction of the magnetic moment of the spin 3/2 particle with the external magnetic field is separated< this additional term is constructed with the use of the spin matrices S_i and the components of the magnetic field $\vec{B}$.

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1. Introduction. The basic equation

In the present paper, we will study the problem of non-relativistic approximation for a spin 3/2 particle [1, 2]; see also [3] –[24] in curved space-time.

We start with the known form of the basic equation for spin 3/2 particle [19], [24]:

$$\gamma^5 \epsilon_\rho{}^{\sigma\alpha\beta}(x)\gamma_\sigma\Big(iD_\alpha - \frac{1}{2}M\gamma_\alpha\Big)\Psi_\beta = 0, \quad D_\alpha = \nabla_\alpha + \Gamma_\alpha + ieA_\alpha; \tag{1}$$

where $M = mc/\hbar$ is a mass parameter. After transition in the wave function to the tetrad representation by vector index $\Psi_\beta = e^{(b)}_\beta \Psi_b$, this equation takes the form

$$\gamma^5 \epsilon_k{}^{can}\gamma_c \left[i(D_a)_n{}^l - \frac{1}{2}M\gamma_a\delta_n{}^l \right]\Psi_l = 0, \tag{2}$$

where generalized derivative are used

$$D_a = e^{\alpha}_{(a)}(\partial_\alpha + ieA_\alpha) + \frac{1}{2}(\sigma^{ps}\otimes I + I \otimes j^{ps})\,\gamma_{[ps]a} = e^{\alpha}_{(a)}(\partial_\alpha + ieA_\alpha) + \Sigma_a. \tag{3}$$

With the use of six matrices $\epsilon_k{}^{can} = (\mu^{[ca]})_k{}^n$, eq. (2) may be presented as follows

$$\gamma^5(\mu^{[ca]})_k{}^n\gamma_c\left[i(D_a)_n{}^l - M\gamma_a\delta_n{}^l\right]\Psi_l = 0, \tag{4}$$

*E-mail: ivashkevich.alina@yandex.by
†E-mail: v.redkov@ifanbel.bas-net.by
‡E-mail: aishkhanyan@gmail.com

The summing formula is

$$\mu^{[ca]}\gamma_c D_a$$

$$= \Big(\gamma^1\otimes\mu^{[01]}+\gamma^2\otimes\mu^{[02]}+\gamma^3\otimes\mu^{[03]}\Big)D_0\Psi+\Big(\gamma^0\otimes\mu^{[01]}+\gamma^2\otimes\mu^{[12]}-\gamma^3\otimes\mu^{[31]}\Big)D_1\Psi$$

$$+\Big(\gamma^0\otimes\mu^{[02]}+\gamma^3\otimes\mu^{[23]}-\gamma^1\otimes\mu^{[12]}\Big)D_2\Psi+\Big(\gamma^0\otimes\mu^{[03]}+\gamma^1\otimes\mu^{[31]}-\gamma^2\otimes\mu^{[23]}\Big)D_3\Psi\,.$$

whence we derive the detailed form of eq. (2):

$$\Big(\gamma^1\otimes\mu^{[01]}+\gamma^2\otimes\mu^{[02]}+\gamma^3\otimes\mu^{[03]}\Big)D_0\Psi+\Big(\gamma^0\otimes\mu^{[01]}+\gamma^2\otimes\mu^{[12]}-\gamma^3\otimes\mu^{[31]}\Big)D_1\Psi$$

$$+\Big(\gamma^0\otimes\mu^{[02]}+\gamma^3\otimes\mu^{[23]}-\gamma^1\otimes\mu^{[12]}\Big)D_2\Psi+\Big(\gamma^0\otimes\mu^{[03]}+\gamma^1\otimes\mu^{[31]}-\gamma^2\otimes\mu^{[23]}\Big)D_3\Psi$$

$$+iM\frac{1}{2}\Big\{s_{01}\otimes\mu^{[01]}+s_{02}\otimes\mu^{[02]}+s_{03}\otimes\mu^{[03]}+s_{23}\otimes\mu^{[23]}+s_{31}\otimes\mu^{[31]}+s_{12}\otimes\mu^{[12]}\Big\}\Psi=0\,, \tag{5}$$

where $s_{ab}=\gamma_a\gamma_b-\gamma_b\gamma_a$ (note the location of the indices). We need expression for matrices $\mu^{[ca]}$:

$$\mu^{[01]}=\begin{vmatrix}0&0&0&0\\0&0&0&0\\0&0&0&-1\\0&0&1&0\end{vmatrix},\mu^{[02]}=\begin{vmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&-1&0&0\end{vmatrix},\mu^{[03]}=\begin{vmatrix}0&0&0&0\\0&0&-1&0\\0&1&0&0\\0&0&0&0\end{vmatrix},$$

$$\mu^{[23]}=\begin{vmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{vmatrix},\mu^{[31]}=\begin{vmatrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{vmatrix},\mu^{[12]}=\begin{vmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{vmatrix};$$

and also explicit expressions for Dirac matrices (and generators for 4-vector and bispinor)

$$\gamma^0=\begin{vmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{vmatrix},\gamma^1=\begin{vmatrix}0&0&0&-1\\0&0&-1&0\\0&1&0&0\\1&0&0&0\end{vmatrix},\gamma^2=\begin{vmatrix}0&0&0&i\\0&0&-i&0\\0&-i&0&0\\i&0&0&0\end{vmatrix},\gamma^3=\begin{vmatrix}0&0&-1&0\\0&0&0&1\\1&0&0&0\\0&-1&0&0\end{vmatrix};$$

$$\sigma^{12}=\frac{1}{2}\begin{vmatrix}-i&0&0&0\\0&i&0&0\\0&0&-i&0\\0&0&0&i\end{vmatrix},\quad j^{12}=\begin{vmatrix}0&0&0&0\\0&0&-1&0\\0&1&0&0\\0&0&0&0\end{vmatrix},$$

$$s_{01}=-\begin{vmatrix}0&2&0&0\\2&0&0&0\\0&0&0&-2\\0&0&-2&0\end{vmatrix},\quad s_{02}=-\begin{vmatrix}0&-2i&0&0\\2i&0&0&0\\0&0&0&2i\\0&0&-2i&0\end{vmatrix},\quad s_{03}=-\begin{vmatrix}2&0&0&0\\0&-2&0&0\\0&0&-2&0\\0&0&0&2\end{vmatrix},$$

$$s_{23}=\begin{vmatrix}0&-2i&0&0\\-2i&0&0&0\\0&0&0&-2i\\0&0&-2i&0\end{vmatrix},\quad s_{31}=\begin{vmatrix}0&-2&0&0\\2&0&0&0\\0&0&0&-2\\0&0&2&0\end{vmatrix},\quad s_{12}=\begin{vmatrix}-2i&0&0&0\\0&2i&0&0\\0&0&-2i&0\\0&0&0&2i\end{vmatrix}.$$

Equation (5) may be written in a short form

$$\Big(\Gamma^0D_0+\Gamma^0D_0+\Gamma^0D_0+\Gamma^0D_0+iM\Gamma\Big)\Psi=0, \tag{6}$$

where

$$D_0 = e^{\alpha}_{(0)}(\partial_\alpha + ieA_\alpha) + \frac{1}{2}(\sigma^{ps} \otimes I + I \otimes j^{ps})\gamma_{[ps]0},$$
$$D_1 = e^{\alpha}_{(1)}(\partial_\alpha + ieA_\alpha) + \frac{1}{2}(\sigma^{ps} \otimes I + I \otimes j^{ps})\gamma_{[ps]1},$$
$$D_2 = e^{\alpha}_{(2)}(\partial_\alpha + ieA_\alpha) + \frac{1}{2}(\sigma^{ps} \otimes I + I \otimes j^{ps})\gamma_{[ps]2},$$
$$D_3 = e^{\alpha}_{(3)}(\partial_\alpha + ieA_\alpha) + \frac{1}{2}(\sigma^{ps} \otimes I + I \otimes j^{ps})\gamma_{[ps]3};$$

$$\Gamma^0 = \Big(\gamma^1 \otimes \mu^{[01]} + \gamma^2 \otimes \mu^{[02]} + \gamma^3 \otimes \mu^{[03]}\Big), \Gamma^1 = \Big(\gamma^0 \otimes \mu^{[01]} + \gamma^2 \otimes \mu^{[12]} - \gamma^3 \otimes \mu^{[31]}\Big),$$
$$\Gamma^2 = \Big(\gamma^0 \otimes \mu^{[02]} + \gamma^3 \otimes \mu^{[23]} - \gamma^1 \otimes \mu^{[12]}\Big), \Gamma^3 = \Big(\gamma^0 \otimes \mu^{[03]} + \gamma^1 \otimes \mu^{[31]} - \gamma^2 \otimes \mu^{[23]}\Big),$$
$$\Gamma = \frac{1}{2}\Big\{s_{01} \otimes \mu^{[01]} + s_{02} \otimes \mu^{[02]} + s_{03} \otimes \mu^{[03]} + s_{23} \otimes \mu^{[23]} + s_{31} \otimes \mu^{[31]} + s_{12} \otimes \mu^{[12]}\Big\}.$$

2. Large and small components of the wave function

The non-relativistic approximation in wave equations (independently on the value of a particle spin) is possible only in space-time models with the following structure [19])

$$dS^2 = (dx^0)^2 + g_{ij}(x)dx^i dx^j, \quad e_{(a)\alpha}(x) = \begin{vmatrix} 1 & 0 \\ 0 & e_{(i)k}(x) \end{vmatrix}, \tag{7}$$

in this case, expressions for connection simplify

$$\Sigma_0 = \frac{1}{2}\, J^{ik} e^{m}_{(i)}\, (\nabla_0 e_{(k)m}), \quad \Sigma_l = \frac{1}{2}\, J^{ik} e^{m}_{(i)}\, (\nabla_l e_{(k)m})\,, \tag{8}$$

the contribution of generators J^{0k} vanishes identically. The wave function of the particle may be presented as a matrix (the first index is bispinor one, and the second is 4-vector one)

$$\Psi_{A(n)} = \Phi_{A(n)} = \begin{vmatrix} f_0 & f_1 & f_2 & f_3 \\ g_0 & g_1 & g_2 & g_3 \\ h_0 & h_1 & h_2 & h_3 \\ d_0 & d_1 & d_2 & d_3 \end{vmatrix}. \tag{9}$$

It is convenient to multiply Eq. (6) by Γ^{-1}, then we have

$$\Big(Y^0 D_0 + Y^1 D_1 + Y^2 D_2 + Y^3 D_3 + iM\Big)\Psi = 0. \tag{10}$$

let us find 16-dimensional presentation for the matrix Γ^0; to this end we consider the action of the matrix Γ^0 on the wave function

$$\Gamma^0\Psi = \gamma^1\Psi\widetilde{\mu}^{[01]} + \gamma^2\Psi\widetilde{\mu}^{[02]} + \gamma^3\Psi\widetilde{\mu}^{[03]}$$

$$= \gamma^1 \begin{vmatrix} f_0 & f_1 & f_2 & f_3 \\ g_0 & g_1 & g_2 & g_3 \\ h_0 & h_1 & h_2 & h_3 \\ d_0 & d_1 & d_2 & d_3 \end{vmatrix} \widetilde{\mu}^{[01]} + \gamma^2 \begin{vmatrix} f_0 & f_1 & f_2 & f_3 \\ g_0 & g_1 & g_2 & g_3 \\ h_0 & h_1 & h_2 & h_3 \\ d_0 & d_1 & d_2 & d_3 \end{vmatrix} \widetilde{\mu}^{[02]} + \gamma^3 \begin{vmatrix} f_0 & f_1 & f_2 & f_3 \\ g_0 & g_1 & g_2 & g_3 \\ h_0 & h_1 & h_2 & h_3 \\ d_0 & d_1 & d_2 & d_3 \end{vmatrix} \widetilde{\mu}^{[03]}$$

$$= \begin{vmatrix} 0 & id_3 + h_2 & d_3 - h_1 & -id_1 - d_2 \\ 0 & -d_2 - ih_3 & d_1 + h_3 & ih_1 - h_2 \\ 0 & -f_2 - ig_3 & f_1 - g_3 & ig_1 + g_2 \\ 0 & if_3 + g_2 & -f_3 - g_1 & f_2 - if_1 \end{vmatrix};$$

so in 16-dimensional form we have

$$
\Gamma^0\Psi = \left|\begin{array}{cccccccccccccccc}
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & i \\
. & . & . & . & . & . & . & . & . & . & . & -1 & . & . & -i & . \\
. & . & . & . & . & . & . & . & -1 & . & . & . & . & -i & . & . \\
. & . & . & . & . & . & . & . & . & 1 & . & . & i & . & . & . \\
. & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & 1 \\
. & . & . & . & . & . & . & 1 & . & . & . & . & . & . & 1 & . \\
. & . & . & . & 1 & . & . & . & . & . & . & . & . & -1 & . & . \\
. & . & . & . & . & -1 & . & . & . & . & . & . & -1 & . & . & . \\
. & . & . & . & . & . & . & -i & . & . & . & -1 & . & . & . & . \\
. & . & . & . & . & . & i & . & . & . & -1 & . & . & . & . & . \\
. & . & . & . & . & i & . & . & . & 1 & . & . & . & . & . & . \\
. & . & . & . & -i & . & . & . & 1 & . & . & . & . & . & . & .
\end{array}\right| \left|\begin{array}{c} f_0 \\ g_0 \\ h_0 \\ d_0 \\ f_1 \\ g_1 \\ h_1 \\ d_1 \\ f_2 \\ g_2 \\ h_2 \\ d_2 \\ f_3 \\ g_3 \\ h_3 \\ d_3 \end{array}\right| .
$$

Similarly, we find expressions for matrices Γ^i, Γ:

$$
\Gamma^1\Psi = \left|\begin{array}{cccccccccccccccc}
. & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & i \\
. & . & . & . & . & . & . & . & . & . & . & -1 & . & . & -i & . \\
. & . & . & . & . & . & . & . & -1 & . & . & . & . & -i & . & . \\
. & . & . & . & . & . & . & . & . & 1 & . & . & i & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . & . & . & . & . & . & -1 & . \\
. & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . & -1 \\
-1 & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . \\
. & 1 & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . \\
. & . & . & i & . & . & . & . & . & . & 1 & . & . & . & . & . \\
. & . & -i & . & . & . & . & . & . & . & . & 1 & . & . & . & . \\
. & -i & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . \\
i & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & .
\end{array}\right| \left|\begin{array}{c} f_0 \\ g_0 \\ h_0 \\ d_0 \\ f_1 \\ g_1 \\ h_1 \\ d_1 \\ f_2 \\ g_2 \\ h_2 \\ d_2 \\ f_3 \\ g_3 \\ h_3 \\ d_3 \end{array}\right| ;
$$

$$
\Gamma^2\Psi = \left|\begin{array}{cccccccccccccccc}
. & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & 1 \\
. & . & . & . & . & . & . & 1 & . & . & . & . & . & . & 1 & . \\
. & . & . & . & 1 & . & . & . & . & . & . & . & . & -1 & . & . \\
. & . & . & . & . & -1 & . & . & . & . & . & . & -1 & . & . & . \\
. & . & -1 & . & . & . & . & . & . & . & . & . & . & . & 1 & . \\
. & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & 1 \\
1 & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . \\
. & -1 & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & 1 & . & . & -1 & . & . & . & . & . & . & . & . & . \\
. & . & 1 & . & . & . & . & -1 & . & . & . & . & . & . & . & . \\
. & -1 & . & . & -1 & . & . & . & . & . & . & . & . & . & . & . \\
-1 & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . & .
\end{array}\right| \left|\begin{array}{c} f_0 \\ g_0 \\ h_0 \\ d_0 \\ f_1 \\ g_1 \\ h_1 \\ d_1 \\ f_2 \\ g_2 \\ h_2 \\ d_2 \\ f_3 \\ g_3 \\ h_3 \\ d_3 \end{array}\right| ;
$$

$$\Gamma^3\Psi = \left|\begin{array}{cccccccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -i & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & i & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & i & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -i & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -i & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & i & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & i & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
-i & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -1 & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}\right| \left|\begin{array}{c} f_0 \\ g_0 \\ h_0 \\ d_0 \\ f_1 \\ g_1 \\ h_1 \\ d_1 \\ f_2 \\ g_2 \\ h_2 \\ d_2 \\ f_3 \\ g_3 \\ h_3 \\ d_3 \end{array}\right|;$$

$$\Gamma = \frac{1}{2}\left|\begin{array}{cccccccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & -2i & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & -2i & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -2i & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot & 2i & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -2i & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & -2i & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -2i & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot & 2i \\
\cdot & -2i & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot & 2i & \cdot & \cdot \\
-2i & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & -2i & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -2i & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & \cdot & \cdot & -2i \\
\cdot & \cdot & -2i & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & 2i & \cdot \\
\cdot & -2 & \cdot & \cdot & -2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2 & \cdot & \cdot \\
2 & \cdot & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -2 & \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -2 \\
\cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -2 & \cdot \\
-2i & \cdot & \cdot & \cdot & \cdot & -2i & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 2i & \cdot & \cdot & 2i & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -2i & \cdot & \cdot & \cdot & \cdot & 2i & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 2i & \cdot & \cdot & -2i & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}\right|;$$

Further we find $Y^0 = \Gamma^{-1}\Gamma^0, Y^i = \Gamma^{-1}\Gamma^i$:

In accordance with general approach, the large and small components in non-relativistic limit are to be determined by projective operators constructed through the matrix Y^0. This matrix $Y^0 = Y_0$ obeys the minimal 4th order equation $Y_0^2(Y_0^2 - 1) = 0$. So, there exist three projective operators

$$P_0 = 1 - Y_0^2, \quad P_1 = P_+ = +\frac{1}{2}Y_0^2(Y+1), \quad P_2 = P_- = -\frac{1}{2}Y_0^2(Y-1), \tag{11}$$

with the needed properties

$$P_0 + P_+ + P_- = 1, \quad P_0^2 = P_0, \quad P_1^2 = P_1, \quad P_2^2 = P_2.$$

Explicitly they read

$$P_0 = \left|\begin{array}{cccccccccccccccc}
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot & \frac{i}{3} & \cdot & \cdot & \cdot & \cdot & -\frac{1}{3} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot & -\frac{i}{3} & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot & \frac{i}{3} & \cdot & \cdot & \cdot & \cdot & -\frac{1}{3} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot & -\frac{i}{3} & \cdot & \cdot & \frac{1}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & -\frac{i}{3} & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot & \cdot & \frac{i}{3} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{i}{3} & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \frac{i}{3} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{i}{3} & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot & \cdot & \frac{i}{3} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{i}{3} & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \frac{i}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot & -\frac{i}{3} & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -\frac{1}{3} & \cdot & \cdot & \cdot & -\frac{i}{3} & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \cdot & \cdot & -\frac{i}{3} & \cdot & \cdot & \frac{1}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{3} & \cdot & \cdot & \cdot & -\frac{i}{3} & \cdot & \cdot & \cdot & \cdot & \frac{1}{3}
\end{array}\right|,$$

$$
P_1 = \left|\begin{array}{cccccccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \frac{1}{3} & \cdot & -\frac{i}{6} & \cdot & -\frac{i}{6} & \cdot & \cdot & \frac{1}{6} & \cdot & \frac{1}{6} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \frac{1}{3} & \cdot & \frac{i}{6} & \cdot & \frac{i}{6} & -\frac{1}{6} & \cdot & -\frac{1}{6} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \frac{1}{3} & \cdot & -\frac{i}{6} & \cdot & -\frac{i}{6} & \cdot & \cdot & \frac{1}{6} & \cdot & \frac{1}{6} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & \frac{1}{3} & \cdot & \frac{i}{6} & \cdot & \frac{i}{6} & -\frac{1}{6} & \cdot & -\frac{1}{6} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \frac{i}{6} & \cdot & \frac{i}{6} & \cdot & \frac{1}{3} & \cdot & \frac{1}{3} & \cdot & \cdot & -\frac{i}{6} & \cdot & -\frac{i}{6} \\
\cdot & \cdot & \cdot & \cdot & \cdot & -\frac{i}{6} & \cdot & -\frac{i}{6} & \cdot & \frac{1}{3} & \cdot & \frac{1}{3} & -\frac{i}{6} & \cdot & -\frac{i}{6} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \frac{i}{6} & \cdot & \frac{i}{6} & \cdot & \frac{1}{3} & \cdot & \frac{1}{3} & \cdot & \cdot & -\frac{i}{6} & \cdot & -\frac{i}{6} \\
\cdot & \cdot & \cdot & \cdot & \cdot & -\frac{i}{6} & \cdot & -\frac{i}{6} & \cdot & \frac{1}{3} & \cdot & \frac{1}{3} & -\frac{i}{6} & \cdot & -\frac{i}{6} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{6} & \cdot & -\frac{1}{6} & \cdot & \frac{i}{6} & \cdot & \frac{i}{6} & \frac{1}{3} & \cdot & \frac{1}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \frac{1}{6} & \cdot & \frac{1}{6} & \cdot & \frac{i}{6} & \cdot & \frac{i}{6} & \cdot & \cdot & \frac{1}{3} & \cdot & \frac{1}{3} \\
\cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{6} & \cdot & -\frac{1}{6} & \cdot & \frac{i}{6} & \cdot & \frac{i}{6} & \frac{1}{3} & \cdot & \frac{1}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \frac{1}{6} & \cdot & \frac{1}{6} & \cdot & \frac{i}{6} & \cdot & \frac{i}{6} & \cdot & \cdot & \frac{1}{3} & \cdot & \frac{1}{3}
\end{array}\right|,
$$

$$
P_2 = \left|\begin{array}{cccccccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & -\frac{1}{3} & \cdot & -\frac{i}{6} & \cdot & \frac{i}{6} & \cdot & \cdot & \frac{1}{6} & \cdot & -\frac{1}{6} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{3} & \cdot & -\frac{1}{3} & \cdot & \frac{i}{6} & \cdot & -\frac{i}{6} & -\frac{1}{6} & \cdot & \frac{1}{6} & \cdot \\
\cdot & \cdot & \cdot & \cdot & -\frac{1}{3} & \cdot & \frac{1}{3} & \cdot & \frac{i}{6} & \cdot & -\frac{i}{6} & \cdot & \cdot & -\frac{1}{6} & \cdot & \frac{1}{6} \\
\cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{3} & \cdot & \frac{1}{3} & \cdot & -\frac{i}{6} & \cdot & \frac{i}{6} & \frac{1}{6} & \cdot & -\frac{1}{6} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \frac{i}{6} & \cdot & -\frac{i}{6} & \cdot & \frac{1}{3} & \cdot & -\frac{1}{3} & \cdot & \cdot & -\frac{i}{6} & \cdot & \frac{i}{6} \\
\cdot & \cdot & \cdot & \cdot & \cdot & -\frac{i}{6} & \cdot & \frac{i}{6} & \cdot & \frac{1}{3} & \cdot & -\frac{1}{3} & -\frac{i}{6} & \cdot & \frac{i}{6} & \cdot \\
\cdot & \cdot & \cdot & \cdot & -\frac{i}{6} & \cdot & \frac{i}{6} & \cdot & -\frac{1}{3} & \cdot & \frac{1}{3} & \cdot & \cdot & \frac{i}{6} & \cdot & -\frac{i}{6} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{i}{6} & \cdot & -\frac{i}{6} & \cdot & -\frac{1}{3} & \cdot & \frac{1}{3} & \frac{i}{6} & \cdot & -\frac{i}{6} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{6} & \cdot & \frac{1}{6} & \cdot & \frac{i}{6} & \cdot & -\frac{i}{6} & \frac{1}{3} & \cdot & -\frac{1}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \frac{1}{6} & \cdot & -\frac{1}{6} & \cdot & \frac{i}{6} & \cdot & -\frac{i}{6} & \cdot & \cdot & \frac{1}{3} & \cdot & -\frac{1}{3} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{6} & \cdot & -\frac{1}{6} & \cdot & -\frac{i}{6} & \cdot & \frac{i}{6} & -\frac{1}{3} & \cdot & \frac{1}{3} & \cdot \\
\cdot & \cdot & \cdot & \cdot & -\frac{1}{6} & \cdot & \frac{1}{6} & \cdot & -\frac{i}{6} & \cdot & \frac{i}{6} & \cdot & \cdot & -\frac{1}{3} & \cdot & \frac{1}{3}
\end{array}\right|.
$$

Further we find three projective constituents (in each we can see a number of independent variables):

$$
\Psi_0 = P_0\Psi\,, \quad \Psi_+ = \Psi_1 = P_1\Psi, \quad \Psi_- = \Psi_2 = P_2\Psi;
$$

$$
\Psi_0 = \left|\begin{array}{c}
f_0 \\ g_0 \\ h_0 \\ d_0 \\[4pt]
\frac{1}{3}\,(f_1 + if_2 - g_3) \\ \frac{1}{3}\,(f_3 + g_1 - ig_2) \\ \frac{1}{3}\,(-d_3 + h_1 + ih_2) \\ \frac{1}{3}\,(d_1 - id_2 + h_3) \\[4pt]
\frac{1}{3}\,(-if_1 + f_2 + ig_3) \\ \frac{1}{3}i\,(f_3 + g_1 - ig_2) \\ \frac{1}{3}\,(i\,(d_3 - h_1) + h_2) \\ \frac{1}{3}i\,(d_1 - id_2 + h_3) \\[4pt]
\frac{1}{3}\,(f_3 + g_1 - ig_2) \\ \frac{1}{3}\,(-f_1 - if_2 + g_3) \\ \frac{1}{3}\,(d_1 - id_2 + h_3) \\ \frac{1}{3}\,(d_3 - h_1 - ih_2)
\end{array}\right|
=
\left|\begin{array}{c}
S^0{}_1 \\ S^0{}_2 \\ S^0{}_3 \\ S^0{}_4 \\[4pt]
S^1{}_1 \\ S^1{}_2 \\ S^1{}_3 \\ S^1{}_4 \\[4pt]
S^2{}_1 \\ S^2{}_2 \\ S^2{}_3 \\ S^2{}_4 \\[4pt]
S^3{}_1 \\ S^3{}_2 \\ S^3{}_3 \\ S^3{}_4
\end{array}\right|
=
\left|\begin{array}{c}
S^0{}_1 \\ S^0{}_2 \\ S^0{}_3 \\ S^0{}_4 \\[4pt]
S^1{}_1 \\ S^1{}_2 \\ S^1{}_3 \\ S^1{}_4 \\[4pt]
-iS^1{}_1 \\ iS^1{}_2 \\ -iS^1{}_3 \\ iS^1{}_4 \\[4pt]
S^1{}_2 \\ -S^1{}_1 \\ S^1{}_4 \\ -S^1{}_3
\end{array}\right|
=
\left|\begin{array}{c}
S_1 \\ S_2 \\ S_3 \\ S_4 \\[4pt]
S_5 \\ S_6 \\ S_7 \\ S_8 \\[4pt]
-iS_5 \\ iS_6 \\ -iS_7 \\ iS_8 \\[4pt]
S_6 \\ -S_5 \\ S_8 \\ -S_7
\end{array}\right|,
$$

$$
\Psi_+ = \Psi_1 = \left| \begin{array}{c}
0 \\ 0 \\ 0 \\ 0 \\
\frac{1}{6}(d_3 + 2f_1 - if_2 + g_3 + 2h_1 - ih_2) \\
\frac{1}{6}(2d_1 + id_2 - f_3 + 2g_1 + ig_2 - h_3) \\
\frac{1}{6}(d_3 + 2f_1 - if_2 + g_3 + 2h_1 - ih_2) \\
\frac{1}{6}(2d_1 + id_2 - f_3 + 2g_1 + ig_2 - h_3) \\
-\frac{1}{6}i(d_3 - f_1 + 2if_2 + g_3 - h_1 + 2ih_2) \\
-\frac{1}{6}i(d_1 + 2id_2 + f_3 + g_1 + 2ig_2 + h_3) \\
-\frac{1}{6}i(d_3 - f_1 + 2if_2 + g_3 - h_1 + 2ih_2) \\
-\frac{1}{6}i(d_1 + 2id_2 + f_3 + g_1 + 2ig_2 + h_3) \\
\frac{1}{6}(-d_1 + id_2 + 2f_3 - g_1 + ig_2 + 2h_3) \\
\frac{1}{6}(2d_3 + f_1 + if_2 + 2g_3 + h_1 + ih_2) \\
\frac{1}{6}(-d_1 + id_2 + 2f_3 - g_1 + ig_2 + 2h_3) \\
\frac{1}{6}(2d_3 + f_1 + if_2 + 2g_3 + h_1 + ih_2)
\end{array} \right| = \left| \begin{array}{c}
0 \\ 0 \\ 0 \\ 0 \\
{L^1}_1 \\ {L^1}_2 \\ {L^1}_3 \\ {L^1}_4 \\
{L^2}_1 \\ {L^2}_2 \\ {L^2}_3 \\ {L^2}_4 \\
{L^3}_1 \\ {L^3}_2 \\ {L^3}_3 \\ {L^3}_4
\end{array} \right| = \left| \begin{array}{c}
0 \\ 0 \\ 0 \\ 0 \\
{L^1}_1 \\ {L^1}_2 \\ {L^1}_1 \\ {L^1}_2 \\
{L^2}_1 \\ {L^2}_2 \\ {L^2}_1 \\ {L^2}_2 \\
{L^3}_1 \\ {L^3}_2 \\ {L^3}_1 \\ {L^3}_2
\end{array} \right| = \left| \begin{array}{c}
0 \\ 0 \\ 0 \\ 0 \\
L_1 \\ L_2 \\ L_1 \\ L_2 \\
L_3 \\ L_4 \\ L_3 \\ L_4 \\
L_5 \\ L_6 \\ L_5 \\ L_6
\end{array} \right| ,
$$

$$
\Psi_- = \Psi_2 = \left| \begin{array}{c}
0 \\ 0 \\ 0 \\ 0 \\
\frac{1}{6}(-d_3 + 2f_1 - if_2 + g_3 - 2h_1 + ih_2) \\
\frac{1}{6}(-2d_1 - id_2 - f_3 + 2g_1 + ig_2 + h_3) \\
\frac{1}{6}(d_3 - 2f_1 + if_2 - g_3 + 2h_1 - ih_2) \\
\frac{1}{6}(2d_1 + id_2 + f_3 - 2g_1 - ig_2 - h_3) \\
\frac{1}{6}i(d_3 + f_1 - 2if_2 - g_3 - h_1 + 2ih_2) \\
\frac{1}{6}i(d_1 + 2id_2 - f_3 - g_1 - 2ig_2 + h_3) \\
-\frac{1}{6}i(d_3 + f_1 - 2if_2 - g_3 - h_1 + 2ih_2) \\
-\frac{1}{6}i(d_1 + 2id_2 - f_3 - g_1 - 2ig_2 + h_3) \\
\frac{1}{6}(d_1 - id_2 + 2f_3 - g_1 + ig_2 - 2h_3) \\
\frac{1}{6}(-2d_3 + f_1 + if_2 + 2g_3 - h_1 - ih_2) \\
\frac{1}{6}(-d_1 + id_2 - 2f_3 + g_1 - ig_2 + 2h_3) \\
\frac{1}{6}(2d_3 - f_1 - if_2 - 2g_3 + h_1 + ih_2)
\end{array} \right| = \left| \begin{array}{c}
0 \\ 0 \\ 0 \\ 0 \\
{P^1}_1 \\ {P^1}_2 \\ {P^1}_3 \\ {P^1}_4 \\
{P^2}_1 \\ {P^2}_2 \\ {P^2}_3 \\ {P^2}_4 \\
{P^3}_1 \\ {P^3}_2 \\ {P^3}_3 \\ {P^3}_4
\end{array} \right| = \left| \begin{array}{c}
0 \\ 0 \\ 0 \\ 0 \\
{P^1}_1 \\ {P^1}_2 \\ -{P^1}_1 \\ -{P^1}_2 \\
{P^2}_1 \\ {P^2}_2 \\ -{P^2}_1 \\ -{P^2}_2 \\
{P^3}_1 \\ {P^3}_2 \\ -{P^3}_1 \\ -{P^3}_2
\end{array} \right| = \left| \begin{array}{c}
0 \\ 0 \\ 0 \\ 0 \\
P_1 \\ P_2 \\ -P_1 \\ -P_2 \\
P_3 \\ P_4 \\ -P_3 \\ -P_4 \\
P_5 \\ P_6 \\ -P_5 \\ -P_6
\end{array} \right| .
$$

While performing the non-relativistic approximation, we should consider Ψ_+ as large component, and Ψ_-, Ψ_0 as small ones:

$$
P_i << L_i, \qquad S_i << L_i\,; \tag{12}
$$

in total we have the following 20 variables:

$$
\Psi_+, \ \{L_1, ..., L_6\}; \quad \Psi_0, \ \{S_1, ..., S_8\}; \quad \Psi_-, \ \{P_1, ..., P_6\}. \tag{13}
$$

3. Constraints for large and small components

Let us consider equations, defining the large variables $L_1, ..., L_6$; we preserve only six independent ones:

$$
\begin{array}{ll}
1 & L_1 = \dfrac{1}{6}(d_3 + 2f_1 - if_2 + g_3 + 2h_1 - ih_2) \\
5 & L_3 = -\dfrac{1}{6}i(d_3 - f_1 + 2if_2 + g_3 - h_1 + 2ih_2) \\
10 & L_6 = \dfrac{1}{6}(2d_3 + f_1 + if_2 + 2g_3 + h_1 + ih_2)
\end{array} \quad \Longrightarrow \quad L_1 + iL_3 - L_6 = 0; \tag{14}
$$

$$
\begin{array}{ll}
2 & L_2 = \dfrac{1}{6}(2d_1 + id_2 - f_3 + 2g_1 + ig_2 - h_3) \\
6 & L_4 = -\dfrac{1}{6}i(d_1 + 2id_2 + f_3 + g_1 + 2ig_2 + h_3) \\
9 & L_5 = \dfrac{1}{6}(-d_1 + id_2 + 2f_3 - g_1 + ig_2 + 2h_3)
\end{array} \quad \Longrightarrow \quad L_2 - iL_4 + L_5 = 0. \tag{15}
$$

Therefore, exist only four independent variables; for definiteness we eliminate L_3 and L_4:

$$iL_3 = L_6 - L_1, \quad iL_4 = L_5 + L_2. \tag{16}$$

Let us consider the constituents of Ψ_-:

$$\Psi_- = \left| \begin{array}{c} \frac{1}{6}(-d_3 + 2f_1 - if_2 + g_3 - 2h_1 + ih_2) \\ \frac{1}{6}(-2d_1 - id_2 - f_3 + 2g_1 + ig_2 + h_3) \\ \frac{1}{6}(d_3 - 2f_1 + if_2 - g_3 + 2h_1 - ih_2) \\ \frac{1}{6}(2d_1 + id_2 + f_3 - 2g_1 - ig_2 - h_3) \\ \\ \frac{1}{6}i(d_3 + f_1 - 2if_2 - g_3 - h_1 + 2ih_2) \\ \frac{1}{6}i(d_1 + 2id_2 - f_3 - g_1 - 2ig_2 + h_3) \\ -\frac{1}{6}i(d_3 + f_1 - 2if_2 - g_3 - h_1 + 2ih_2) \\ -\frac{1}{6}i(d_1 + 2id_2 - f_3 - g_1 - 2ig_2 + h_3) \\ \\ \frac{1}{6}(d_1 - id_2 + 2f_3 - g_1 + ig_2 - 2h_3) \\ \frac{1}{6}(-2d_3 + f_1 + if_2 + 2g_3 - h_1 - ih_2) \\ \frac{1}{6}(-d_1 + id_2 - 2f_3 + g_1 - ig_2 + 2h_3) \\ \frac{1}{6}(2d_3 - f_1 - if_2 - 2g_3 + h_1 + ih_2) \end{array} \right| = \left| \begin{array}{c} +P_1 \\ +P_2 \\ -P_1 \\ -P_2 \\ \\ +P_3 \\ +P_4 \\ -P_3 \\ -P_4 \\ \\ +P_5 \\ +P_6 \\ -P_5 \\ -P_6 \end{array} \right| \tag{17}$$

We readily find two constraints

$$(A) \quad P_1 + iP_3 - P_6 = 0, \qquad (B) \quad P_2 - iP_4 + P_5 = 0; \tag{18}$$

thew will be needed below. Now let us consider the sum

$$\Psi_0 + \Psi_- = \left| \begin{array}{c} S_5 + P_1 \\ S_6 + P_2 \\ S_7 - P_1 \\ S_8 - P_2 \\ \\ iS_5 + P_3 \\ iS_6 + P_4 \\ iS_7 - P_3 \\ iS_8 - P_4 \\ \\ S_6 + P_5 \\ -S_5 + P_6 \\ S_8 - P_5 \\ -S_7 - P_6 \end{array} \right| = \left| \begin{array}{c} +y_1 \\ +y_2 \\ +y_3 \\ +y_4 \\ \\ +y_5 \\ +y_6 \\ +y_7 \\ +y_8 \\ \\ +y_9 \\ +y_{10} \\ +y_{11} \\ +y_{12} \end{array} \right|, \tag{19}$$

where we introduce new notations $y_1, ..., y_{12}$:

$$\begin{gathered} S_5 + P_1 = y_1, \quad S_6 + P_2 = y_2, \quad S_7 - P_1 = y_3, \quad S_8 - P_2 = y_4, \\ iS_5 + P_3 = y_5, \quad iS_6 + P_4 = y_6, \quad iS_7 - P_3 = y_7, \quad iS_8 - P_4 = y_8, \\ S_6 + P_5 = y_9, \quad -S_5 + P_6 = y_{10}, \quad S_8 - P_5 = y_{11}, \quad -S_7 - P_6 = y_{12}. \end{gathered} \tag{20}$$

so that

$$\Psi = \left| \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 + L_1 + P_1 \\ S_6 + L_2 + P_2 \\ S_7 + L_1 - P_1 \\ S_8 + L_2 - P_2 \\ iS_5 + L_3 + P_3 \\ iS_6 + L_4 + P_4 \\ iS_7 + L_3 - P_3 \\ iS_8 + L_4 - P_4 \\ S_6 + L_5 + P_5 \\ -S_5 + L_6 + P_6 \\ S_8 + L_5 - P_5 \\ -S_7 + L_6 - P_6 \end{array} \right| = \left| \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ L_1 + y_1 \\ L_2 + y_2 \\ L_1 + y_3 \\ L_2 + y_4 \\ L_3 + y_5 \\ L_4 + y_6 \\ L_3 + y_7 \\ L_4 + y_8 \\ L_5 + y_9 \\ L_6 + y_{10} \\ L_5 + y_{11} \\ L_6 + y_{12} \end{array} \right| \tag{$*$}$$

We can combine the above variables as follows

$$\begin{gathered} y_1 + y_3 = S_5 + S_7, \quad y_2 + y_4 = S_6 + S_8, \quad y_5 + y_7 = i(S_5 + S_7), \\ y_6 + y_8 = i(S_6 + S_8), \quad y_9 + y_{11} = S_6 + S_8, \quad y_{10} + y_{12} = -(S_5 + S_7); \end{gathered} \tag{21}$$

the we get

$$\begin{aligned} 1)\quad & y_1 - y_3 = S_5 - S_7 + 2P_1, \\ 2)\quad & y_2 - y_4 = S_6 - S_8 + 2P_2, \\ 3)\quad & y_5 - y_7 = i(S_5 - S_7) + 2P_3, \\ 4)\quad & y_6 - y_8 = i(S_6 - S_8) + 2P_4, \\ 5)\quad & y_9 - y_{11} = S_6 - S_8 + 2P_5, \\ 6)\quad & y_{10} - y_{12} = -(S_5 - S_7) + 2P_6. \end{aligned} \tag{22}$$

From the system (21) we derive six identities (from which only four ones are independent)

$$\begin{aligned} &(y_1 + y_3) + (y_{10} + y_{12}) = 0, \quad (y_1 + y_3) + i(y_5 + y_7) = 0, \quad (y_{10} + y_{12}) = i(y_5 + y_7); \\ &(y_2 + y_4) - (y_9 + y_{11}) = 0, \quad (y_2 + y_4) + i(y_6 + y_8) = 0, \quad (y_9 + y_{11}) = -i(y_6 + y_8). \end{aligned} \tag{23}$$

Relations (22)

$$P_1 + iP_3 - P_6 = 0, \qquad P_2 - iP_4 + P_5 = 0;$$

taking into account eqs. (18), we combine as follows

$$1) + i \cdot 3) - 6), \qquad 2) - i \cdot 4) + 5).$$

In this way, we obtain

$$\begin{aligned} S_5 - S_7 &= (y_1 - y_3) + i(y_5 - y_7) - (y_{10} - y_{12}), \\ 3(S_6 - S_8) &= (y_2 - y_4) - i(y_6 - y_8) + (y_9 - y_{11}). \end{aligned} \tag{24}$$

In turn, from (21), it follows

$$y_1 + y_3 = S_5 + S_7, \quad y_2 + y_4 = S_6 + S_8.$$

Therefore, the four variables S_5, S_6, S_7, S_8 may be expressed through the y-variables.

4. The non-relativistic approximation

The non-relativistic approximation is possible only for space-time models with the following structure

$$dS^2 = (dx^0)^2 + g_{ij}(x)dx^i dx^j \ , \quad e_{(a)\alpha}(x) = \left| \begin{matrix} 1 & 0 \\ 0 & e_{(i)k}(x) \end{matrix} \right| . \tag{25}$$

In this case, the components of the connection have the structure

$$\Sigma_0 = \frac{1}{2}\, J^{ik} e^m_{(i)}\, (\nabla_0 e_{(k)m}) \ , \qquad \Sigma_l = \frac{1}{2}\, J^{ik} e^m_{(i)}\, (\nabla_l e_{(k)m}) \ , \tag{26}$$

it should be noted that any contributions in the connections due to generators J^{0k} vanish identically. So the generally covariant matrix equations for a spin 3/2 particle has the form

$$\Big(Y^0 \bar{D}_0 + Y^1 \bar{D}_1 + Y^2 \bar{D}_2 + \Gamma^3 \bar{D}_3 + iM \Big) \Psi = 0, \tag{27}$$

where

$$\begin{aligned} \bar{D}_0 &= (\partial_0 + ieA_0) + (\sigma^{23} \otimes I + I \otimes j^{23})\gamma_{[23]0} + (\sigma^{31} \otimes I + I \otimes j^{31})\gamma_{[31]0} + (\sigma^{12} \otimes I + I \otimes j^{12})\gamma_{[12]0}, \\ \bar{D}_1 &= e^k_{(1)}(\partial_k + ieA_k) + (\sigma^{23} \otimes I + I \otimes j^{23})\gamma_{[23]1} + (\sigma^{31} \otimes I + I \otimes j^{31})\gamma_{[31]1} + (\sigma^{12} \otimes I + I \otimes j^{12})\gamma_{[12]1}, \\ \bar{D}_2 &= e^k_{(2)}(\partial_k + ieA_k) + (\sigma^{23} \otimes I + I \otimes j^{23})\gamma_{[23]2} + (\sigma^{31} \otimes I + I \otimes j^{31})\gamma_{[31]2} + (\sigma^{12} \otimes I + I \otimes j^{12})\gamma_{[12]2}, \\ \bar{D}_3 &= (\partial_k + ieA_k) + (\sigma^{23} \otimes I + I \otimes j^{23})\gamma_{[23]3} + (\sigma^{31} \otimes I + I \otimes j^{31})\gamma_{[31]3} + (\sigma^{12} \otimes I + I \otimes j^{12})\gamma_{[12]3}. \end{aligned} \tag{28}$$

In diagonal metrics, expressions for derivatives (28) simplify. The terms in eq. (27) may be presented as

$$
\begin{aligned}
Y^0 \bar{D}_0 \Psi &= Y^0 \Big[D_0 \Psi + (\sigma^{23}\Psi + \Psi \tilde{j}^{23}) G_{10} + (\sigma^{31}\Psi + \Psi \tilde{j}^{31}) G_{20} + (\sigma^{12}\Psi + \Psi \tilde{j}^{12}) G_{30} \Big],\\
Y^1 \bar{D}_1 \Psi &= Y^1 \Big[D_1 \Psi + (\sigma^{23}\Psi + \Psi \tilde{j}^{23}) G_{11} + (\sigma^{31}\Psi + \Psi \tilde{j}^{31}) G_{21} + (\sigma^{12}\Psi + \Psi \tilde{j}^{12}) G_{31} \Big],\\
Y^2 \bar{D}_2 \Psi &= Y^2 \Big[D_2 \Psi + (\sigma^{23}\Psi + \Psi \tilde{j}^{23}) G_{12} + (\sigma^{31}\Psi + \Psi \tilde{j}^{31}) G_{22} + (\sigma^{12}\Psi + \Psi \tilde{j}^{12}) G_{32} \Big],\\
Y^3 \bar{D}_3 \Psi &= Y^3 \Big[D_3 \Psi + (\sigma^{23}\Psi + \Psi \tilde{j}^{23}) G_{13} + (\sigma^{31}\Psi + \Psi \tilde{j}^{31}) G_{23} + (\sigma^{12}\Psi + \Psi \tilde{j}^{12}) G_{33} \Big];
\end{aligned}
\tag{29}
$$

where we have used the shortening notations for the Ricci rotation coefficients

$$
\begin{aligned}
&G_{10} = \gamma_{230}, \quad G_{20} = \gamma_{310}, \quad G_{30} = \gamma_{120},\\
&G_{11} = \gamma_{231}, \quad G_{21} = \gamma_{311}, \quad G_{31} = \gamma_{121},\\
&G_{12} = \gamma_{232}, \quad G_{22} = \gamma_{312}, \quad G_{32} = \gamma_{122},\\
&G_{13} = \gamma_{233}, \quad G_{23} = \gamma_{313}, \quad G_{33} = \gamma_{123}.
\end{aligned}
\tag{30}
$$

In equation

$$
\Big(Y^0 \bar{D}_0 + Y^1 \bar{D}_1 + Y^2 \bar{D}_2 + Y^3 \bar{D}_3 + iM \Big) \Psi = 0,
$$

one can distinguish two parts

$$
\Big(Y^0 D_0 + Y^1 D_1 + Y^2 D_2 + Y^3 D_3 + iM \Big) \Psi + \Big(Q^0 \Psi + Q^1 \Psi + Q^2 \Psi + Q^3 \Psi \Big) = 0,
\tag{31}
$$

where $D_0 = (\partial_0 + ieA_0)$, $D_1 = e^k_{(1)} (\partial_k + ieA_k)$, $D_2 = e^k_{(2)} (\partial_k + ieA_k)$, $\quad \bar{D}_3 = (\partial_k + ieA_k)$, and

$$
\begin{aligned}
Q^0 \Psi &= Y^0 \Big[(\sigma^{23}\Psi + \Psi \tilde{j}^{23}) G_{10} + (\sigma^{31}\Psi + \Psi \tilde{j}^{31}) G_{20} + (\sigma^{12}\Psi + \Psi \tilde{j}^{12}) G_{30} \Big],\\
Q^1 \Psi &= Y^1 \Big[(\sigma^{23}\Psi + \Psi \tilde{j}^{23}) G_{11} + (\sigma^{31}\Psi + \Psi \tilde{j}^{31}) G_{21} + (\sigma^{12}\Psi + \Psi \tilde{j}^{12}) G_{31} \Big],\\
Q^2 \Psi &= Y^2 \Big[(\sigma^{23}\Psi + \Psi \tilde{j}^{23}) G_{12} + (\sigma^{31}\Psi + \Psi \tilde{j}^{31}) G_{22} + (\sigma^{12}\Psi + \Psi \tilde{j}^{12}) G_{32} \Big],\\
Q^3 \Psi &= Y^3 \Big[(\sigma^{23}\Psi + \Psi \tilde{j}^{23}) G_{13} + (\sigma^{31}\Psi + \Psi \tilde{j}^{31}) G_{23} + (\sigma^{12}\Psi + \Psi \tilde{j}^{12}) G_{33} \Big].
\end{aligned}
\tag{32}
$$

We need expressions for three generators

$$
j^{12} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}, \quad
j^{31} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix}, \quad
j^{23} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{vmatrix},
$$

After performing the needed calculations, we derive 16 equations with respect to large $L_{..}$ and small $y_{..}$ components (equations are collected in pairs)

1 $$\frac{2}{3} D_0 (y_4 - i y_8 + y_{11}) + \frac{1}{3} D_1 (L_1 - 2S_4 - i y_7 + y_{12}) + \frac{1}{3} i D_2 (L_1 - L_6 + 2S_4 + y_3 - y_{12}) +$$
$$+ \frac{1}{3} D_3 (L_5 - 2S_3 - y_4 + i y_8) + iMS_1 +$$
$$+ G_{33}(-iL_5 + i(2S_3 + y_4) + y_8) + G_{12}(-L_2 - 3L_5 + 2S_3 + y_4 - 2iy_8 - y_{11}) + G_{11}(-iL_2 + 2iS_3 + y_8 + iy_{11}) +$$
$$+ iG_{22}(L_2 + L_5 + 2S_3 + y_4 + y_{11}) - 2iG_{30}(y_4 - iy_8 + y_{11}) + G_{21}(-L_2 + 2L_5 - 2S_3 - 2y_4 + iy_8 + y_{11}) +$$
$$+ iG_{13}(2L_1 - 3L_6 + 2S_4 + y_3 - iy_7 - 2y_{12}) + G_{23}(-2L_1 - L_6 + 2S_4 - y_3 + iy_7 - 2y_{12}) - iG_{31}(3L_1 - 2L_6 + 2S_4 + 2y_3 - iy_7 - y_{12}) -$$
$$- 2G_{20}(y_3 + iy_7 - y_{12}) + 2G_{10}(-iy_3 + y_7 + iy_{12}) + G_{32}(3L_1 - L_6 - 2S_4 + y_3 - 2iy_7 + y_{12}) = 0,$$

3
$$- \frac{2}{3} D_0 (y_2 - i y_6 + y_9) + \frac{1}{3} D_1 (L_1 + 2S_2 - i y_5 + y_{10}) + \frac{1}{3} i D_2 (L_1 - L_6 - 2S_2 + y_1 - y_{10}) +$$
$$+ \frac{1}{3} D_3 (L_5 + 2S_1 - y_2 + i y_6) + iMS_3 +$$
$$G_{33}(y_6 - i(L_5 + 2S_1 - y_2)) + G_{12}(-L_2 - 3L_5 - 2S_1 + y_2 - 2iy_6 - y_9) + G_{11}(-iL_2 - 2iS_1 + y_6 + iy_9) +$$
$$+ iG_{22}(L_2 + L_5 - 2S_1 + y_2 + y_9) + 2iG_{30}(y_2 - iy_6 + y_9) + G_{21}(-L_2 + 2L_5 + 2S_1 - 2y_2 + iy_6 + y_9) +$$
$$+ iG_{13}(2L_1 - 3L_6 - 2S_2 + y_1 - iy_5 - 2y_{10}) + G_{23}(-2L_1 - L_6 - 2S_2 - y_1 + iy_5 - 2y_{10}) - iG_{31}(3L_1 - 2L_6 - 2S_2 + 2y_1 - iy_5 - y_{10}) +$$

$$+2iG_{10}(y_1+iy_5-y_{10})+2G_{20}(y_1+iy_5-y_{10})+G_{32}(3L_1-L_6+2S_2+y_1-2iy_5+y_{10})=0,$$

2

$$\frac{2}{3}D_0(y_3+iy_7-y_{12})+\frac{1}{3}D_1(L_2-2S_3+iy_8-y_{11})-\frac{1}{3}iD_2(L_2+L_5+2S_3+y_4+y_{11})+$$
$$+\frac{1}{3}D_3(L_6+2S_4+y_3+iy_7)+iMS_2+$$
$$+iG_{33}(L_6+2S_4+y_3+iy_7)+G_{32}(3L_2+L_5-2S_3+y_4+2iy_8-y_{11})-2iG_{10}(y_4-iy_8+y_{11})+$$
$$+2G_{20}(y_4-iy_8+y_{11})+iG_{31}(3L_2+2L_5+2S_3+2y_4+iy_8+y_{11})+G_{23}(-2L_2+L_5+2S_3-y_4-iy_8+2y_{11})-$$
$$-iG_{13}(2L_2+3L_5+2S_3+y_4+iy_8+2y_{11})+iG_{22}(L_1-L_6+2S_4+y_3-y_{12})+2iG_{30}(y_3+iy_7-y_{12})+$$
$$+G_{12}(L_1-3L_6-2S_4-y_3-2iy_7-y_{12})-iG_{11}(L_1-2S_4-iy_7+y_{12})+G_{21}(L_1+2L_6+2S_4+2y_3+iy_7+y_{12})=0,$$

4

$$-\frac{2}{3}D_0(y_1+iy_5-y_{10})+\frac{1}{3}D_1(L_2+2S_1+iy_6-y_9)-\frac{1}{3}iD_2(L_2+L_5-2S_1+y_2+y_9)+$$
$$+\frac{1}{3}D_3(L_6-2S_2+y_1+iy_5)+iMS_4+,$$
$$+iG_{33}(L_6-2S_2+y_1+iy_5)+G_{32}(3L_2+L_5+2S_1+y_2+2iy_6-y_9)+2iG_{10}(y_2-iy_6+y_9)-$$
$$-2G_{20}(y_2-iy_6+y_9)+iG_{31}(3L_2+2L_5-2S_1+2y_2+iy_6+y_9)+G_{23}(-2L_2+L_5-2S_1-y_2-iy_6+2y_9)-$$
$$-iG_{13}(2L_2+3L_5-2S_1+y_2+iy_6+2y_9)+iG_{22}(L_1-L_6-2S_2+y_1-y_{10})+G_{12}(L_1-3L_6+2S_2-y_1-2iy_5-y_{10})+$$
$$+2G_{30}(-iy_1+y_5+iy_{10})-iG_{11}(L_1+2S_2-iy_5+y_{10})+G_{21}(L_1+2L_6-2S_2+2y_1+iy_5+y_{10})=0,$$

5

$$\frac{1}{3}D_0(3L_1+2y_3-iy_7+y_{12})-\frac{2}{3}D_1(L_2+S_3+iy_8-y_{11})+\frac{1}{3}iD_2(2L_2-L_5+S_3+2y_4-y_{11})+$$
$$+\frac{1}{3}D_3(-3L_1+L_6-S_4-2y_3+iy_7)+iM(L_1+y_1)+$$
$$+G_{33}(9iL_1-i(5L_6+S_4-4y_3)+5y_7)+G_{23}(L_2-5L_5-S_3+2y_4-iy_8-4y_{11})+iG_{13}(L_2-3L_5+S_3+2y_4-iy_8-2y_{11})+$$
$$+G_{32}(-6L_2-5L_5+S_3-2y_4-4iy_8-y_{11})+G_{10}(-3iL_2-2iy_4+y_8+iy_{11})-2iG_{31}3L_2+2L_5-S_3+2y_4+iy_8+y_{11})+$$
$$+G_{20}(-3L_2+6L_5-4y_4+iy_8+5y_{11})+G_{12}(4L_1-3L_6+S_4+2y_3-2iy_7-y_{12})-iG_{30}(9L_1-6L_6+4y_3-5iy_7-y_{12})+$$
$$+2iG_{11}(L_1+S_4-iy_7+y_{12})-2G_{21}(L_1+2L_6-S_4+2y_3+iy_7+y_{12})+iG_{22}(4L_1+5L_6-S_4+4y_3+5y_{12}),$$

7

$$\frac{1}{3}D_0(3L_1+2y_1-iy_5+y_{10})-\frac{2}{3}D_1(-L_2+S_1-iy_6+y_9)+\frac{1}{3}iD_2(-2L_2+L_5+S_1-2y_2+y_9)+$$
$$+\frac{1}{3}D_3(3L_1-L_6-S_2+2y_1-iy_5)+iM(L_1+y_3)+$$
$$G_{33}(-i(9L_1-5L_6+S_2+4y_1)-5y_5)-iG_{13}(L_2-3L_5-S_1+2y_2-iy_6-2y_9)+G_{10}(-3iL_2-2iy_2+y_6+iy_9)+$$
$$+2iG_{31}(3L_2+2L_5+S_1+2y_2+iy_6+y_9)+G_{32}(6L_2+5L_5+S_1+2y_2+4iy_6+y_9)+G_{23}(-L_2+5L_5-S_1-2y_2+iy_6+4y_9)+$$
$$+G_{20}(-3L_2+6L_5-4y_2+iy_6+5y_9)-iG_{30}(9L_1-6L_6+4y_1-5iy_5-y_{10})-2iG_{11}(L_1-S_2-iy_5+y_{10})+$$
$$+2G_{21}(L_1+2L_6+S_2+2y_1+iy_5+y_{10})+G_{12}(-4L_1+3L_6+S_2-2y_1+2iy_5+y_{10})-iG_{22}4L_1+5L_6+S_2+4y_1+5y_{10})=0,$$

6

$$\frac{1}{3}D_0(3L_2+2y_4+iy_8-y_{11})-\frac{2}{3}D_1(L_1+S_4-iy_7+y_{12})-\frac{1}{3}iD_2(2L_1+L_6+S_4+2y_3+y_{12})+$$
$$+\frac{1}{3}D_3(3L_2+L_5+S_3+2y_4+iy_8)+iM(L_2+y_2)+$$
$$iG_{33}(9L_2+5L_5-S_3+4y_4+5iy_8)+iG_{22}(4L_2-5L_5-S_3+4y_4-5y_{11})+2G_{21}(L_2-2L_5-S_3+2y_4-iy_8-y_{11})+$$
$$+2iG_{11}(L_2+S_3+iy_8-y_{11})+G_{12}(-4L_2-3L_5-S_3-2y_4-2iy_8-y_{11})+iG_{30}(9L_2+6L_5+4y_4+5iy_8+y_{11})+$$
$$+2iG_{31}(3L_1-2L_6-S_4+2y_3-iy_7-y_{12})-iG_{10}(3L_1+2y_3-iy_7+y_{12})+G_{32}(-6L_1+5L_6+S_4-2y_3+4iy_7+y_{12})-$$
$$-iG_{13}(L_1+3L_6+S_4+2y_3+iy_7+2y_{12})+G_{23}(L_1+5L_6-S_4+2y_3+iy_7+4y_{12})+G_{20}(3L_1+6L_6+4y_3+iy_7+5y_{12}),$$

8

$$\frac{1}{3}D_0(3L_2+2y_2+iy_6-y_9)+\frac{2}{3}D_1(L_1-S_2-iy_5+y_{10})+\frac{1}{3}iD_2(2L_1+L_6-S_2+2y_1+y_{10})+$$
$$+\frac{1}{3}D_3(-3L_2-L_5+S_1-2y_2-iy_6)+iM(L_2+y_4)+$$
$$+G_{33}(5y_6-i(9L_2+5L_5+S_1+4y_2))-iG_{22}(4L_2-5L_5+S_1+4y_2-5y_9)-2G_{21}(L_2-2L_5+S_1+2y_2-iy_6-y_9)-$$
$$-2iG_{11}(L_2-S_1+iy_6-y_9)+G_{12}(4L_2+3L_5-S_1+2y_2+2iy_6+y_9)+iG_{30}(9L_2+6L_5+4y_2+5iy_6+y_9)+$$
$$+G_{23}(-L_1-5L_6-S_2-2y_1-iy_5-4y_{10})-2iG_{31}(3L_1-2L_6+S_2+2y_1-iy_5-y_{10})+G_{32}(6L_1-5L_6+S_2+2y_1-4iy_5-y_{10})-$$
$$-iG_{10}(3L_1+2y_1-iy_5+y_{10})+iG_{13}(L_1+3L_6-S_2+2y_1+iy_5+2y_{10})+G_{20}(3L_1+6L_6+4y_1+iy_5+5y_{10}),$$

9

$$\frac{1}{3}iD_0(3L_1-3L_6+y_3-2iy_7-y_{12})+\frac{1}{3}iD_1(2L_2+3L_5-S_3+2iy_8+y_{11})+\frac{2}{3}D_2(L_2+L_5-S_3+y_4+y_{11})+$$
$$+\frac{1}{3}iD_3(-3L_1+2L_6+S_4-y_3+2iy_7)+M(-L_1+L_6+iy_5)+$$
$$+G_{33}(-9L_1+4L_6-S_4-5y_3+4iy_7)+G_{10}(-3L_2-9L_5+y_4-4iy_8-5y_{11})+2iG_{32}(3L_2+L_5+S_3+y_4+2iy_8-y_{11})+$$
$$+G_{31}(-6L_2-L_5-S_3-4y_4-2iy_8+y_{11})+iG_{20}(3L_2+3L_5+y_4+2iy_8+y_{11})-iG_{23}(L_2+4L_5-S_3-y_4+2iy_8+2y_{11})+$$
$$+G_{13}(L_2+6L_5+S_3-y_4+2iy_8+4y_{11})+2iG_{12}(L_1-3L_6+S_4-y_3-2iy_7-y_{12})+2G_{22}(-L_1+L_6+S_4-y_3+y_{12})-$$
$$-iG_{21}(4L_1-L_6-S_4+2y_3-2iy_7+y_{12})+G_{30}(9L_1-3L_6+5y_3-4iy_7+y_{12})+G_{11}(-4L_1+9L_6-S_4+4iy_7+5y_{12}),$$

11

$$\frac{1}{3}iD_0(3L_1-3L_6+y_1-2iy_5-y_{10})-\frac{1}{3}iD_1(2L_2+3L_5+S_1+2iy_6+y_9)-\frac{2}{3}D_2(L_2+L_5+S_1+y_2+y_9)+$$
$$+\frac{1}{3}iD_3(3L_1-2L_6+S_2+y_1-2iy_5)+M(-L_1+L_6+iy_7)+$$
$$+G_{33}(9L_1-4L_6-S_2+5y_1-4iy_5)+G_{10}(-3L_2-9L_5+y_2-4iy_6-5y_9)+G_{13}(-L_2-6L_5+S_1+y_2-2iy_6-4y_9)-$$
$$-2iG_{32}(3L_2+L_5-S_1+y_2+2iy_6-y_9)+G_{31}(6L_2+L_5-S_1+4y_2+2iy_6-y_9)+iG_{20}(3L_2+3L_5+y_2+2iy_6+y_9)+$$
$$+iG_{23}(L_2+4L_5+S_1-y_2+2iy_6+2y_9)+G_{11}(4L_1-9L_6-S_2-4iy_5-5y_{10})+2G_{22}(L_1-L_6+S_2+y_1-y_{10})-$$
$$-2iG_{12}(L_1-3L_6-S_2-y_1-2iy_5-y_{10})+iG_{21}(4L_1-L_6+S_2+2y_1-2iy_5+y_{10})+G_{30}(9L_1-3L_6+5y_1-4iy_5+y_{10})=0,$$

10

$$-\frac{1}{3}iD_0(3L_2+3L_5+y_4+2iy_8+y_{11})+\frac{1}{3}iD_1(-2L_1+3L_6+S_4+2iy_7+y_{12})-\frac{2}{3}D_2(-L_1+L_6+S_4-y_3+y_{12})-$$
$$-\frac{1}{3}iD_3(3L_2+2L_5-S_3+y_4+2iy_8)+M(L_2+L_5+iy_6)+$$
$$+G_{33}(9L_2+4L_5+S_3+5y_4+4iy_8)-iG_{21}(4L_2+L_5-S_3+2y_4+2iy_8-y_{11})+G_{30}(9L_2+3L_5+5y_4+4iy_8-y_{11})+$$
$$+2G_{22}(L_2+L_5-S_3+y_4+y_{11})+2iG_{12}(L_2+3L_5+S_3-y_4+2iy_8+y_{11})+G_{11}(4L_2+9L_5+S_3+4iy_8+5y_{11})+$$
$$+G_{10}3L_1-9L_6-y_3-4iy_7-5y_{12})+G_{13}(L_1-6L_6+S_4-y_3-2iy_7-4y_{12})+iG_{23}(L_1-4L_6-S_4-y_3-2iy_7-2y_{12})+$$
$$+iG_{20}(3L_1-3L_6+y_3-2iy_7-y_{12})+G_{31}(-6L_1+L_6-S_4-4y_3+2iy_7-y_{12})-2iG_{32}(3L_1-L_6+S_4+y_3-2iy_7+y_{12}),$$

12

$$-\frac{1}{3}iD_0(3L_2+3L_5+y_2+2iy_6+y_9)+\frac{1}{3}iD_1(2L_1-3L_6+S_2-2iy_5-y_{10})-\frac{2}{3}D_2(L_1-L_6+S_2+y_1-y_{10})+$$
$$+\frac{1}{3}iD_3(3L_2+2L_5+S_1+y_2+2iy_6)+M(L_2+L_5+iy_8)+$$
$$+G_{33}(-9L_2-4L_5+S_1-5y_2-4iy_6)+G_{11}-4L_2-9L_5+S_1-4iy_6-5y_9)+iG_{21}(4L_2+L_5+S_1+2y_2+2iy_6-y_9)+$$
$$+G_{30}(9L_2+3L_5+5y_2+4iy_6-y_9)-2G_{22}(L_2+L_5+S_1+y_2+y_9)-2iG_{12}(L_2+3L_5-S_1-y_2+2iy_6+y_9)+$$
$$+G_{10}(3L_1-9L_6-y_1-4iy_5-5y_{10})-iG_{23}(L_1-4L_6+S_2-y_1-2iy_5-2y_{10})+iG_{20}(3L_1-3L_6+y_1-2iy_5-y_{10})+$$
$$+2iG_{32}(3L_1-L_6-S_2+y_1-2iy_5+y_{10})+G_{31}(6L_1-L_6-S_2+4y_1-2iy_5+y_{10})+G_{13}(-L_1+6L_6+S_2+y_1+2iy_5+4y_{10}),$$

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$$\frac{1}{3}D_0(3L_5-y_4+iy_8+2y_{11})+\frac{1}{3}D_1(L_1-3L_6+S_4-iy_7-2y_{12})+\frac{1}{3}iD_2(L_1+2L_6-S_4+y_3+2y_{12})-$$
$$-\frac{2}{3}D_3(L_5+S_3-y_4+iy_8)+iM(L_5+y_9)+$$
$$+2iG_{33}(L_5+S_3-y_4+iy_8)-iG_{22}(5L_2-4L_5+S_3+5y_4-4y_{11})+G_{21}(5L_2-L_5+S_3+4y_4+iy_8-2y_{11})+$$
$$+G_{30}(-3iL_5+iy_4+y_8-2iy_{11})+G_{12}(5L_2+6L_5-S_3+y_4+4iy_8+2y_{11})+iG_{11}(5L_2+9L_5-S_3+5iy_8+4y_{11})+$$
$$+G_{20}(-6L_1-3L_6-5y_3+iy_7-4y_{12})+iG_{10}(6L_1-9L_6+y_3-5iy_7-4y_{12})-2iG_{13}(2L_1-3L_6-S_4+y_3-iy_7-2y_{12})+$$
$$+G_{32}(3L_1-4L_6+S_4+y_3-2iy_7-2y_{12})+2G_{23}(2L_1+L_6+S_4+y_3-iy_7+2y_{12})-iG_{31}3L_1+L_6-S_4+2y_3-iy_7+2y_{12}),$$

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$$\frac{1}{3}D_0(3L_5-y_2+iy_6+2y_9)+\frac{1}{3}D_1(-L_1+3L_6+S_2+iy_5+2y_{10})-\frac{1}{3}iD_2(L_1+2L_6+S_2+y_1+2y_{10})-$$
$$-\frac{2}{3}D_3(-L_5+S_1+y_2-iy_6)+iM(L_5+y_{11})+$$
$$+G_{33}(-2iL_5+2i(S_1+y_2)+2y_6)+iG_{22}(5L_2-4L_5-S_1+5y_2-4y_9)+G_{12}(-5L_2-6L_5-S_1-y_2-4iy_6-2y_9)+$$
$$+G_{30}(-3iL_5+iy_2+y_6-2iy_9)+G_{21}(-5L_2+L_5+S_1-4y_2-iy_6+2y_9)-iG_{11}(5L_2+9L_5+S_1+5iy_6+4y_9)+$$
$$+G_{20}(-6L_1-3L_6-5y_1+iy_5-4y_{10})+iG_{10}(6L_1-9L_6+y_1-5iy_5-4y_{10})+2iG_{13}(2L_1-3L_6+S_2+y_1-iy_5-2y_{10})-$$

$$-2G_{23}(2L_1+L_6-S_2+y_1-iy_5+2y_{10})+iG_{31}(3L_1+L_6+S_2+2y_1-iy_5+2y_{10})+G_{32}(-3L_1+4L_6+S_2-y_1+2iy_5+2y_{10}),$$

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$$\frac{1}{3}D_0(3L_6+y_3+iy_7+2y_{12})+\frac{1}{3}D_1(-L_2-3L_5-S_3-iy_8-2y_{11})+\frac{1}{3}iD_2(L_2-2L_5-S_3+y_4-2y_{11})+$$
$$+\frac{2}{3}D_3(L_6-S_4+y_3+iy_7)+iM(L_6+y_{10})+$$
$$+2iG_{33}(L_6-S_4+y_3+iy_7)-2G_{23}(2L_2-L_5+S_3+y_4+iy_8-2y_{11})+G_{32}(-3L_2-4L_5-S_3-y_4-2iy_8-2y_{11})-$$
$$-2iG_{13}(2L_2+3L_5-S_3+y_4+iy_8+2y_{11})+G_{20}(-6L_2+3L_5-5y_4-iy_8+4y_{11})-iG_{10}(6L_2+9L_5+y_4+5iy_8+4y_{11})+$$
$$+G_{31}i(L_5+S_3-2y_4-iy_8+2y_{11})-3iL_2)+G_{12}(5L_1-6L_6-S_4+y_3-4iy_7-2y_{12})+G_{21}(5L_1+L_6+S_4+4y_3-iy_7+2y_{12})+$$
$$+iG_{30}(3L_6+y_3+iy_7+2y_{12})+iG_{22}(5L_1+4L_6+S_4+5y_3+4y_{12})+G_{11}(i(9L_6+S_4+5iy_7+4y_{12})-5iL_1),$$

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$$\frac{1}{3}D_0(3L_6+y_1+iy_5+2y_{10})+\frac{1}{3}D_1(L_2+3L_5-S_1+iy_6+2y_9)-\frac{1}{3}iD_2(L_2-2L_5+S_1+y_2-2y_9)-$$
$$-\frac{2}{3}D_3(L_6+S_2+y_1+iy_5)+iM(L_6+y_{12})+$$
$$+G_{33}(2y_5-2i(L_6+S_2+y_1))+2G_{23}(2L_2-L_5-S_1+y_2+iy_6-2y_9)+iG_{31}(3L_2-L_5+S_1+2y_2+iy_6-2y_9)+$$
$$+2iG_{13}(2L_2+3L_5+S_1+y_2+iy_6+2y_9)+G_{32}(3L_2+4L_5-S_1+y_2+2iy_6+2y_9)+G_{20}(-6L_2+3L_5-5y_2-iy_6+4y_9)-$$
$$-iG_{10}(6L_2+9L_5+y_2+5iy_6+4y_9)+iG_{11}(5L_1-9L_6+S_2-5iy_5-4y_{10})+G_{21}(-5L_1-L_6+S_2-4y_1+iy_5-2y_{10})+$$
$$+iG_{30}3L_6+y_1+iy_5+2y_{10})+G_{12}(-5L_1+6L_6-S_2-y_1+4iy_5+2y_{10})-iG_{22}(5L_1+4L_6-S_2+5y_1+4y_{10}).$$

In each pair, we sum and subtract equations; besides, we separate the rest energy by formal change

$$D_0 \Rightarrow (-iM+D_0).$$

In this way, we obtain

$1+3$

$$-\frac{2}{3}\left(y_2-y_4-iy_6+iy_8+y_9-y_{11}\right)D_0+\frac{1}{3}D_1\left(2L_1+2S_2-2S_4-iy_5-iy_7+y_{10}+y_{12}\right)+$$
$$+\frac{1}{3}iD_2\left(2L_1-2L_6-2S_2+2S_4+y_1+y_3-y_{10}-y_{12}\right)+\frac{1}{3}D_3\left(2L_5+2S_1-2S_3-y_2-y_4+i\left(y_6+y_8\right)\right)+$$
$$+M\left(i\left(S_1+S_3\right)+\frac{2}{3}i\left(y_2-y_4-iy_6+iy_8+y_9-y_{11}\right)\right)+$$
$$+G_{11}(-2iL_2-2iS_1+2iS_3+y_6+y_8+i(y_9+y_{11}))$$
$$+iG_{22}(2L_2+2L_5-2S_1+2S_3+y_2+y_4+y_9+y_{11})$$
$$+G_{33}(-2iL_5-2iS_1+2iS_3+iy_2+iy_4+y_6+y_8)$$
$$+2iG_{10}(y_1-y_3+iy_5-iy_7-y_{10}+y_{12})+2G_{20}(y_1-y_3+iy_5-iy_7-y_{10}+y_{12})+2iG_{30}(y_2-y_4-iy_6+iy_8+y_9-y_{11})$$
$$+G_{12}(-2L_2-6L_5-2S_1+2S_3+y_2+y_4-2iy_6-2iy_8-y_9-y_{11})$$
$$+G_{21}(-2L_2+4L_5+2S_1-2S_3-2y_2-2y_4+iy_6+iy_8+y_9+y_{11})$$
$$+iG_{13}(4L_1-6L_6-2S_2+2S_4+y_1+y_3-i(y_5+y_7)-2y_{10}-2y_{12})$$
$$-iG_{31}(6L_1-4L_6-2S_2+2S_4+2y_1+2y_3-i(y_5+y_7)-y_{10}-y_{12})$$
$$+G_{23}(-4L_1-2L_6-2S_2+2S_4-y_1-y_3+i(y_5+y_7+2i(y_{10}+y_{12})))$$
$$+G_{32}(6L_1-2L_6+2S_2-2S_4+y_1+y_3-2iy_5-2iy_7+y_{10}+y_{12})=0,$$

$1-3$

$$\frac{2}{3}\left(y_2+y_4-iy_6-iy_8+y_9+y_{11}\right)D_0+\frac{1}{3}D_1\left(-2S_2-2S_4+iy_5-iy_7-y_{10}+y_{12}\right)+$$
$$+\frac{1}{3}iD_2\left(2S_2+2S_4-y_1+y_3+y_{10}-y_{12}\right)+\frac{1}{3}D_3\left(-2S_1-2S_3+y_2-y_4-iy_6+iy_8\right)+$$
$$+M\left(i\left(S_1-S_3\right)-\frac{2}{3}i\left(y_2+y_4-iy_6-iy_8+y_9+y_{11}\right)\right)+$$
$$+G_{11}(2iS_1+2iS_3-y_6+y_8-iy_9+iy_{11})+iG_{22}(2S_1+2S_3-y_2+y_4-y_9+y_{11})+G_{33}(y_8+i(2S_1+2S_3-y_2+y_4+iy_6))$$
$$-2iG_{10}(y_1+y_3+i(y_5+y_7+i(y_{10}+y_{12})))-2G_{20}(y_1+y_3+i(y_5+y_7+i(y_{10}+y_{12})))-2iG_{30}(y_2+y_4-iy_6-iy_8+y_9+y_{11})$$
$$+G_{12}(2S_1+2S_3-y_2+y_4+2iy_6-2iy_8+y_9-y_{11})+G_{21}(-2S_1-2S_3+2y_2-2y_4-iy_6+iy_8-y_9+y_{11})$$
$$+iG_{13}(2S_2+2S_4-y_1+y_3+iy_5-iy_7+2y_{10}-2y_{12})-iG_{31}(2S_2+2S_4-2y_1+2y_3+iy_5-iy_7+y_{10}-y_{12})$$
$$+G_{23}(2S_2+2S_4+y_1-y_3-iy_5+iy_7+2y_{10}-2y_{12})+G_{32}(-2S_2-2S_4-y_1+y_3+2iy_5-2iy_7-y_{10}+y_{12})=0,$$

$2+4$

$$-\frac{2}{3}\left(y_1-y_3+iy_5-iy_7-y_{10}+y_{12}\right)D_0+\frac{1}{3}D_1\left(2L_2+2S_1-2S_3+i\left(y_6+y_8+i\left(y_9+y_{11}\right)\right)\right)-$$
$$-\frac{1}{3}iD_2\left(2L_2+2L_5-2S_1+2S_3+y_2+y_4+y_9+y_{11}\right)+\frac{1}{3}D_3\left(2L_6-2S_2+2S_4+y_1+y_3+i\left(y_5+y_7\right)\right)+$$
$$+M\left(i\left(S_2+S_4\right)+\frac{2}{3}i\left(y_1-y_3+iy_5-iy_7-y_{10}+y_{12}\right)\right)$$
$$-iG_{11}(2L_1+2S_2-2S_4-iy_5-iy_7+y_{10}+y_{12})+iG_{22}(2L_1-2L_6-2S_2+2S_4+y_1+y_3-y_{10}-y_{12})$$
$$+iG_{33}(2L_6-2S_2+2S_4+y_1+y_3+i(y_5+y_7))$$
$$+2iG_{10}(y_2-y_4-iy_6+iy_8+y_9-y_{11})+G_{20}(2(y_4-iy_8+y_{11})-2(y_2-iy_6+y_9))-2iG_{30}(y_1-y_3+iy_5-iy_7-y_{10}+y_{12})$$
$$+G_{12}(2L_1-6L_6+2S_2-2S_4-y_1-y_3-2iy_5-2iy_7-y_{10}-y_{12})$$
$$+G_{21}(2L_1+4L_6-2S_2+2S_4+2y_1+2y_3+iy_5+iy_7+y_{10}+y_{12})$$
$$-iG_{13}(4L_2+6L_5-2S_1+2S_3+y_2+y_4+iy_6+iy_8+2(y_9+y_{11}))$$
$$+iG_{31}(6L_2+4L_5-2S_1+2S_3+2y_2+2y_4+iy_6+iy_8+y_9+y_{11})$$
$$+G_{23}(-4L_2+2L_5-2S_1+2S_3-y_2-y_4-i(y_6+y_8+2i(y_9+y_{11})))$$
$$+G_{32}(6L_2+2L_5+2S_1-2S_3+y_2+y_4+2iy_6+2iy_8-y_9-y_{11})=0,$$

$2-4$

$$\frac{2}{3}\left(y_1+y_3+i\left(y_5+y_7+i\left(y_{10}+y_{12}\right)\right)\right)D_0+\frac{1}{3}D_1\left(-2S_1-2S_3-iy_6+iy_8+y_9-y_{11}\right)-$$
$$-\frac{1}{3}iD_2\left(2S_1+2S_3-y_2+y_4-y_9+y_{11}\right)+\frac{1}{3}D_3\left(2S_2+2S_4-y_1+y_3-iy_5+iy_7\right)+$$
$$+M\left(i\left(S_2-S_4\right)-\frac{2}{3}i\left(y_1+y_3+i\left(y_5+y_7+i\left(y_{10}+y_{12}\right)\right)\right)\right)$$
$$+G_{11}(2iS_2+2iS_4+y_5-y_7+i(y_{10}-y_{12}))+iG_{22}(2S_2+2S_4-y_1+y_3+y_{10}-y_{12})+G_{33}(2iS_2+2iS_4-iy_1+iy_3+y_5-y_7)$$
$$-2iG_{10}(y_2+y_4-iy_6-iy_8+y_9+y_{11})+2G_{20}(y_2+y_4-iy_6-iy_8+y_9+y_{11})+2iG_{30}(y_1+y_3+i(y_5+y_7+i(y_{10}+y_{12})))$$
$$+G_{12}(-2S_2-2S_4+y_1-y_3+2iy_5-2iy_7+y_{10}-y_{12})+G_{21}(2S_2+2S_4-2y_1+2y_3-iy_5+iy_7-y_{10}+y_{12})$$
$$-iG_{13}(2S_1+2S_3-y_2+y_4-iy_6+iy_8-2y_9+2y_{11})+iG_{31}(2S_1+2S_3-2y_2+2y_4-iy_6+iy_8-y_9+y_{11})$$
$$+G_{23}(2S_1+2S_3+y_2-y_4+iy_6-iy_8-2y_9+2y_{11})+G_{32}(-2S_1-2S_3-y_2+y_4-2iy_6+2iy_8+y_9-y_{11})=0,$$

$5+7$

$$\frac{1}{3}D_0\left(6L_1+2y_1+2y_3-iy_5-iy_7+y_{10}+y_{12}\right)-\frac{2}{3}D_1\left(S_1+S_3-iy_6+iy_8+y_9-y_{11}\right)+$$
$$+\frac{1}{3}iD_2\left(S_1+S_3-2y_2+2y_4+y_9-y_{11}\right)+\frac{1}{3}D_3\left(-S_2-S_4+2y_1-2y_3-iy_5+iy_7\right)+$$
$$+\frac{1}{3}iM\left(y_1+y_3+i\left(y_5+y_7+i\left(y_{10}+y_{12}\right)\right)\right)$$
$$+2iG_{11}(S_2+S_4+iy_5-iy_7-y_{10}+y_{12})-iG_{22}(S_2+S_4+4y_1-4y_3+5y_{10}-5y_{12})+G_{33}(-i(S_2+S_4+4y_1-4y_3)-5y_5+5y_7)$$
$$+G_{10}(-6iL_2-2iy_2-2iy_4+y_6+y_8+i(y_9+y_{11}))+G_{20}(-6L_2+12L_5-4y_2-4y_4+iy_6+iy_8+5(y_9+y_{11}))-$$
$$-iG_{30}(18L_1-12L_6+4y_1+4y_3-5iy_5-5iy_7-y_{10}-y_{12})$$
$$+G_{12}(S_2+S_4-2y_1+2y_3+2iy_5-2iy_7+y_{10}-y_{12})+2G_{21}(S_2+S_4+2y_1-2y_3+iy_5-iy_7+y_{10}-y_{12})$$
$$+iG_{13}(S_1+S_3-2y_2+2y_4+iy_6-iy_8+2y_9-2y_{11})+2iG_{31}(S_1+S_3+2y_2-2y_4+iy_6-iy_8+y_9-y_{11})+$$
$$+G_{23}(-S_1-S_3-2y_2+2y_4+iy_6-iy_8+4y_9-4y_{11})+G_{32}(S_1+S_3+2y_2-2y_4+4iy_6-4iy_8+y_9-y_{11})=0,$$

$5-7$

$$\frac{1}{3}\left(-2y_1+2y_3+iy_5-iy_7-y_{10}+y_{12}\right)D_0-\frac{2}{3}D_1\left(2L_2-S_1+S_3+i\left(y_6+y_8+i\left(y_9+y_{11}\right)\right)\right)+$$
$$+\frac{1}{3}iD_2\left(4L_2-2L_5-S_1+S_3+2y_2+2y_4-y_9-y_{11}\right)+\frac{1}{3}D_3\left(-6L_1+2L_6+S_2-S_4-2y_1-2y_3+i\left(y_5+y_7\right)\right)+$$
$$+\frac{1}{3}iM\left(5y_1-5y_3-iy_5+iy_7+y_{10}-y_{12}\right)$$
$$+2iG_{11}(2L_1-S_2+S_4-iy_5-iy_7+y_{10}+y_{12})+iG_{22}(8L_1+10L_6+S_2-S_4+4y_1+4y_3+5(y_{10}+y_{12}))$$
$$+iG_{33}(18L_1-10L_6+S_2-S_4+4y_1+4y_3-5i(y_5+y_7))$$
$$+G_{10}(2iy_2-2iy_4-y_6+y_8-iy_9+iy_{11})+G_{20}(4y_2-4y_4-iy_6+iy_8-5y_9+5y_{11})+iG_{30}(4y_1-4y_3-5iy_5+5iy_7-y_{10}+y_{12})$$
$$+G_{12}(8L_1-6L_6-S_2+S_4+2y_1+2y_3-2iy_5-2iy_7-y_{10}-y_{12})-2G_{21}(2L_1+4L_6+S_2-S_4+2y_1+2y_3+iy_5+iy_7+y_{10}+y_{12})$$
$$+iG_{13}(2L_2-6L_5-S_1+S_3+2y_2+2y_4-i(y_6+y_8)-2y_9-2y_{11})$$
$$-2iG_{31}(6L_2+4L_5+S_1-S_3+2y_2+2y_4+iy_6+iy_8+y_9+y_{11})$$

$$+G_{23}(2L_2-10L_5+S_1-S_3+2y_2+2y_4-i(y_6+y_8)-4y_9-4y_{11})$$
$$+G_{32}(-12L_2-10L_5-S_1+S_3-2y_2-2y_4-4iy_6-4iy_8-y_9-y_{11})=0,$$

$6+8$

$$\frac{1}{3}D_0\left(6L_2+2y_2+2y_4+i\left(y_6+y_8+i\left(y_9+y_{11}\right)\right)\right)-\frac{2}{3}D_1\left(S_2+S_4+iy_5-iy_7-y_{10}+y_{12}\right)-$$
$$-\frac{1}{3}iD_2\left(S_2+S_4-2y_1+2y_3-y_{10}+y_{12}\right)+\frac{1}{3}D_3\left(S_1+S_3-2y_2+2y_4-iy_6+iy_8\right)+$$
$$+\frac{1}{3}iM\left(y_2+y_4-iy_6-iy_8+y_9+y_{11}\right)$$
$$+2iG_{11}(S_1+S_3-iy_6+iy_8+y_9-y_{11})-iG_{22}(S_1+S_3+4y_2-4y_4-5y_9+5y_{11})-iG_{33}(S_1+S_3+4y_2-4y_4+5i(y_6-y_8))$$
$$-iG_{10}(6L_1+2y_1+2y_3-iy_5-iy_7+y_{10}+y_{12})+G_{20}(6L_1+12L_6+4y_1+4y_3+iy_5+iy_7+5(y_{10}+y_{12}))$$
$$+iG_{30}(18L_2+12L_5+4y_2+4y_4+5iy_6+5iy_8+y_9+y_{11})$$
$$+G_{12}(-S_1-S_3+2y_2-2y_4+2iy_6-2iy_8+y_9-y_{11})-2G_{21}(S_1+S_3+2y_2-2y_4-iy_6+iy_8-y_9+y_{11})$$
$$-iG_{13}(S_2+S_4-2y_1+2y_3-iy_5+iy_7-2y_{10}+2y_{12})-2iG_{31}(S_2+S_4+2y_1-2y_3-iy_5+iy_7-y_{10}+y_{12})$$
$$+G_{23}(-S_2-S_4-2y_1+2y_3-iy_5+iy_7-4y_{10}+4y_{12})+G_{32}(S_2+S_4+2y_1-2y_3-4iy_5+4iy_7-y_{10}+y_{12})=0,$$

$6-8$

$$\frac{1}{3}\left(-2y_2+2y_4-iy_6+iy_8+y_9-y_{11}\right)D_0-\frac{2}{3}D_1\left(2L_1-S_2+S_4-iy_5-iy_7+y_{10}+y_{12}\right)-$$
$$-\frac{1}{3}iD_2\left(4L_1+2L_6-S_2+S_4+2y_1+2y_3+y_{10}+y_{12}\right)+\frac{1}{3}D_3\left(6L_2+2L_5-S_1+S_3+2y_2+2y_4+i\left(y_6+y_8\right)\right)+$$
$$+\frac{1}{3}iM\left(5y_2-5y_4+iy_6-iy_8-y_9+y_{11}\right)$$
$$+2iG_{11}(2L_2-S_1+S_3+i(y_6+y_8+i(y_9+y_{11})))+iG_{22}(8L_2-10L_5+S_1-S_3+4y_2+4y_4-5(y_9+y_{11}))$$
$$+iG_{33}(18L_2+10L_5+S_1-S_3+4y_2+4y_4+5i(y_6+y_8))$$
$$G_{10}(2iy_1-2iy_3+y_5-y_7+i(y_{10}-y_{12}))+G_{20}(-4y_1+4y_3-iy_5+iy_7-5y_{10}+5y_{12})-iG_{30}(4y_2-4y_4+5iy_6-5iy_8+y_9-y_{11})$$
$$+G_{12}(-8L_2-6L_5+S_1-S_3-2y_2-2y_4-2iy_6-2iy_8-y_9-y_{11})+2G_{21}(2L_2-4L_5+S_1-S_3+2y_2+2y_4-i(y_6+y_8)-y_9-y_{11})$$
$$-iG_{13}(2L_1+6L_6-S_2+S_4+2y_1+2y_3+iy_5+iy_7+2(y_{10}+y_{12}))$$
$$+2iG_{31}(6L_1-4L_6+S_2-S_4+2y_1+2y_3-i(y_5+y_7)-y_{10}-y_{12})$$
$$+G_{23}(2L_1+10L_6+S_2-S_4+2y_1+2y_3+iy_5+iy_7+4(y_{10}+y_{12}))$$
$$+G_{32}(-12L_1+10L_6-S_2+S_4-2y_1-2y_3+4iy_5+4iy_7+y_{10}+y_{12})=0,$$

$9+11$

$$\frac{1}{3}iD_0\left(6L_1-6L_6+y_1+y_3-2iy_5-2iy_7-y_{10}-y_{12}\right)-\frac{1}{3}iD_1\left(S_1+S_3+2iy_6-2iy_8+y_9-y_{11}\right)-$$
$$-\frac{2}{3}D_2\left(S_1+S_3+y_2-y_4+y_9-y_{11}\right)+\frac{1}{3}iD_3\left(S_2+S_4+y_1-y_3-2iy_5+2iy_7\right)+$$
$$+\frac{1}{3}M\left(y_1+y_3+i\left(y_5+y_7+i\left(y_{10}+y_{12}\right)\right)\right)$$
$$+G_{11}(-S_2-S_4-4iy_5+4iy_7-5y_{10}+5y_{12})+2G_{22}(S_2+S_4+y_1-y_3-y_{10}+y_{12})+G_{33}(-S_2-S_4+5y_1-5y_3-4iy_5+4iy_7)$$
$$+G_{10}(-6L_2-18L_5+y_2+y_4-4iy_6-4iy_8-5(y_9+y_{11}))+iG_{20}(6L_2+6L_5+y_2+y_4+2iy_6+2iy_8+y_9+y_{11})$$
$$+G_{30}(18L_1-6L_6+5y_1+5y_3-4iy_5-4iy_7+y_{10}+y_{12})$$
$$+2iG_{12}(S_2+S_4+y_1-y_3+2iy_5-2iy_7+y_{10}-y_{12})+iG_{21}(S_2+S_4+2y_1-2y_3-2iy_5+2iy_7+y_{10}-y_{12})$$
$$+G_{13}(S_1+S_3+y_2-y_4-2iy_6+2iy_8-4y_9+4y_{11})+G_{31}(-S_1-S_3+4y_2-4y_4+2iy_6-2iy_8-y_9+y_{11})+$$
$$+iG_{23}(S_1+S_3-y_2+y_4+2iy_6-2iy_8+2y_9-2y_{11})+2iG_{32}(S_1+S_3-y_2+y_4-2iy_6+2iy_8+y_9-y_{11})=0,$$

$9-11$

$$-\frac{1}{3}i\left(y_1-y_3-2iy_5+2iy_7-y_{10}+y_{12}\right)D_0+\frac{1}{3}iD_1\left(4L_2+6L_5+S_1-S_3+2iy_6+2iy_8+y_9+y_{11}\right)+$$
$$+\frac{2}{3}D_2\left(2L_2+2L_5+S_1-S_3+y_2+y_4+y_9+y_{11}\right)-\frac{1}{3}iD_3\left(6L_1-4L_6+S_2-S_4+y_1+y_3-2i\left(y_5+y_7\right)\right)+$$
$$+\frac{1}{3}M\left(-y_1+y_3+5iy_5-5iy_7+y_{10}-y_{12}\right)$$
$$+G_{11}(-8L_1+18L_6+S_2-S_4+4iy_5+4iy_7+5(y_{10}+y_{12}))-2G_{22}(2L_1-2L_6+S_2-S_4+y_1+y_3-y_{10}-y_{12})$$
$$+G_{33}(-18L_1+8L_6+S_2-S_4-5y_1-5y_3+4i(y_5+y_7))+$$
$$+G_{10}(-y_2+y_4+4iy_6-4iy_8+5y_9-5y_{11})-iG_{20}(y_2-y_4+2iy_6-2iy_8+y_9-y_{11})+G_{30}(-5y_1+5y_3+4iy_5-4iy_7-y_{10}+y_{12})$$

$$+2iG_{12}(2L_1-6L_6-S_2+S_4-y_1-y_3-2iy_5-2iy_7-y_{10}-y_{12})-iG_{21}(8L_1-2L_6+S_2-S_4+2y_1+2y_3-2iy_5-2iy_7+y_{10}+y_{12})$$
$$+G_{13}(2L_2+12L_5-S_1+S_3-y_2-y_4+2iy_6+2iy_8+4(y_9+y_{11}))$$
$$+G_{31}(-12L_2-2L_5+S_1-S_3-4y_2-4y_4-2iy_6-2iy_8+y_9+y_{11})$$
$$-iG_{23}(2L_2+8L_5+S_1-S_3-y_2-y_4+2(iy_6+iy_8+y_9+y_{11}))$$
$$+2iG_{32}(6L_2+2L_5-S_1+S_3+y_2+y_4+2iy_6+2iy_8-y_9-y_{11})=0,$$

$10+12$

$$-\frac{1}{3}iD_0\left(6L_2+6L_5+y_2+y_4+2iy_6+2iy_8+y_9+y_{11}\right)+\frac{1}{3}iD_1\left(S_2+S_4-2iy_5+2iy_7-y_{10}+y_{12}\right)-$$
$$-\frac{2}{3}D_2\left(S_2+S_4+y_1-y_3-y_{10}+y_{12}\right)+\frac{1}{3}iD_3\left(S_1+S_3+y_2-y_4+2i\left(y_6-y_8\right)\right)+$$
$$+\frac{1}{3}M\left(-y_2-y_4+i\left(y_6+y_8+i\left(y_9+y_{11}\right)\right)\right)$$
$$+G_{11}(S_1+S_3-4iy_6+4iy_8-5y_9+5y_{11})-2G_{22}(S_1+S_3+y_2-y_4+y_9-y_{11})+G_{33}(S_1+S_3-5y_2+5y_4-4iy_6+4iy_8)$$
$$+G_{10}(6L_1-18L_6-y_1-y_3-4iy_5-4iy_7-5(y_{10}+y_{12}))+iG_{20}(6L_1-6L_6+y_1+y_3-2iy_5-2iy_7-y_{10}-y_{12})$$
$$+G_{30}(18L_2+6L_5+5y_2+5y_4+4iy_6+4iy_8-y_9-y_{11})$$
$$+2iG_{12}(S_1+S_3+y_2-y_4-2iy_6+2iy_8-y_9+y_{11})+iG_{21}(S_1+S_3+2y_2-2y_4+2iy_6-2iy_8-y_9+y_{11})$$
$$+G_{13}(S_2+S_4+y_1-y_3+2iy_5-2iy_7+4y_{10}-4y_{12})+G_{31}(-S_2-S_4+4y_1-4y_3-2iy_5+2iy_7+y_{10}-y_{12})$$
$$-iG_{23}(S_2+S_4-y_1+y_3-2iy_5+2iy_7-2y_{10}+2y_{12})-2iG_{32}(S_2+S_4-y_1+y_3+2iy_5-2iy_7-y_{10}+y_{12})=0,$$

$10-12$

$$\frac{1}{3}i\left(y_2-y_4+2iy_6-2iy_8+y_9-y_{11}\right)D_0-\frac{1}{3}iD_1\left(4L_1-6L_6+S_2-S_4-2iy_5-2iy_7-y_{10}-y_{12}\right)+$$
$$+\frac{2}{3}D_2\left(2L_1-2L_6+S_2-S_4+y_1+y_3-y_{10}-y_{12}\right)-\frac{1}{3}iD_3\left(6L_2+4L_5+S_1-S_3+y_2+y_4+2i\left(y_6+y_8\right)\right)+$$
$$+\frac{1}{3}M\left(y_2-y_4+5iy_6-5iy_8+y_9-y_{11}\right)$$
$$+G_{11}(8L_2+18L_5-S_1+S_3+4iy_6+4iy_8+5(y_9+y_{11}))+2G_{22}(2L_2+2L_5+S_1-S_3+y_2+y_4+y_9+y_{11})$$
$$+G_{33}(18L_2+8L_5-S_1+S_3+5y_2+5y_4+4i(y_6+y_8))$$
$$+G_{10}(y_1-y_3+4iy_5-4iy_7+5y_{10}-5y_{12})-iG_{20}(y_1-y_3-2iy_5+2iy_7-y_{10}+y_{12})+G_{30}(-5y_2+5y_4-4iy_6+4iy_8+y_9-y_{11})$$
$$+2iG_{12}(2L_2+6L_5-S_1+S_3-y_2-y_4+2iy_6+2iy_8+y_9+y_{11})$$
$$-iG_{21}(8L_2+2L_5+S_1-S_3+2y_2+2y_4+2iy_6+2iy_8-y_9-y_{11})$$
$$+G_{13}(2L_1-12L_6-S_2+S_4-y_1-y_3-2i(y_5+y_7)-4y_{10}-4y_{12})$$
$$+G_{31}(-12L_1+2L_6+S_2-S_4-4y_1-4y_3+2iy_5+2iy_7-y_{10}-y_{12})$$
$$+iG_{23}(2L_1-8L_6+S_2-S_4-y_1-y_3-2i(y_5+y_7)-2y_{10}-2y_{12})$$
$$-2iG_{32}(6L_1-2L_6-S_2+S_4+y_1+y_3-2iy_5-2iy_7+y_{10}+y_{12})=0,$$

$13+15$

$$\frac{1}{3}D_0\left(6L_5-y_2-y_4+iy_6+iy_8+2\left(y_9+y_{11}\right)\right)+\frac{1}{3}D_1\left(S_2+S_4+iy_5-iy_7+2y_{10}-2y_{12}\right)-$$
$$-\frac{1}{3}iD_2\left(S_2+S_4+y_1-y_3+2y_{10}-2y_{12}\right)-\frac{2}{3}D_3\left(S_1+S_3+y_2-y_4-iy_6+iy_8\right)+$$
$$+\frac{1}{3}iM\left(y_2+y_4-iy_6-iy_8+y_9+y_{11}\right)$$
$$-iG_{11}(S_1+S_3+5iy_6-5iy_8+4y_9-4y_{11})-iG_{22}(S_1+S_3-5y_2+5y_4+4y_9-4y_{11})+2iG_{33}(S_1+S_3+y_2-y_4-iy_6+iy_8)$$
$$+iG_{10}(12L_1-18L_6+y_1+y_3-5iy_5-5iy_7-4(y_{10}+y_{12}))+G_{20}(-12L_1-6L_6-5y_1-5y_3+i(y_5+y_7+4i(y_{10}+y_{12})))$$
$$+G_{30}(-6iL_5+iy_2+iy_4+y_6+y_8-2i(y_9+y_{11}))$$
$$+G_{12}(-S_1-S_3-y_2+y_4-4iy_6+4iy_8-2y_9+2y_{11})+G_{21}(S_1+S_3-4y_2+4y_4-iy_6+iy_8+2y_9-2y_{11})$$
$$+2iG_{13}(S_2+S_4+y_1-y_3-iy_5+iy_7-2y_{10}+2y_{12})+iG_{31}(S_2+S_4+2y_1-2y_3-iy_5+iy_7+2y_{10}-2y_{12})$$
$$+2G_{23}(S_2+S_4-y_1+y_3+iy_5-iy_7-2y_{10}+2y_{12})+G_{32}(S_2+S_4-y_1+y_3+2iy_5-2iy_7+2y_{10}-2y_{12})=0,$$

$13-15$

$$\frac{1}{3}\left(y_2-y_4-iy_6+iy_8-2y_9+2y_{11}\right)D_0+\frac{1}{3}D_1\left(2L_1-6L_6-S_2+S_4-i\left(y_5+y_7\right)-2y_{10}-2y_{12}\right)+$$
$$+\frac{1}{3}iD_2\left(2L_1+4L_6+S_2-S_4+y_1+y_3+2\left(y_{10}+y_{12}\right)\right)+\frac{2}{3}D_3\left(-2L_5+S_1-S_3+y_2+y_4-i\left(y_6+y_8\right)\right)-$$

$$-\frac{1}{3}iM\left(y_2-y_4-iy_6+iy_8-5y_9+5y_{11}\right)$$
$$+iG_{11}(10L_2+18L_5+S_1-S_3+5iy_6+5iy_8+4(y_9+y_{11}))-iG_{22}(10L_2-8L_5-S_1+S_3+5y_2+5y_4-4(y_9+y_{11}))$$
$$+2iG_{33}(2L_5-S_1+S_3-y_2-y_4+i(y_6+y_8))$$
$$-iG_{10}(y_1-y_3-5iy_5+5iy_7-4y_{10}+4y_{12})+G_{20}(5y_1-5y_3-iy_5+iy_7+4y_{10}-4y_{12})$$
$$+G_{30}(-iy_2+iy_4-y_6+y_8+2i(y_9-y_{11}))$$
$$+G_{12}(10L_2+12L_5+S_1-S_3+y_2+y_4+2(2iy_6+2iy_8+y_9+y_{11}))$$
$$+G_{21}(10L_2-2L_5-S_1+S_3+4y_2+4y_4+i(y_6+y_8+2i(y_9+y_{11})))$$
$$-2iG_{13}(4L_1-6L_6+S_2-S_4+y_1+y_3-i(y_5+y_7)-2y_{10}-2y_{12})$$
$$-iG_{31}(6L_1+2L_6+S_2-S_4+2y_1+2y_3-i(y_5+y_7+2i(y_{10}+y_{12})))$$
$$+2G_{23}(4L_1+2L_6-S_2+S_4+y_1+y_3-i(y_5+y_7+2i(y_{10}+y_{12})))$$
$$+G_{32}(6L_1-8L_6-S_2+S_4+y_1+y_3-2i(y_5+y_7)-2y_{10}-2y_{12})=0,$$

$14+16$

$$\frac{1}{3}D_0\left(6L_6+y_1+y_3+iy_5+iy_7+2\left(y_{10}+y_{12}\right)\right)+\frac{1}{3}D_1\left(-S_1-S_3+iy_6-iy_8+2y_9-2y_{11}\right)-$$
$$-\frac{1}{3}iD_2\left(S_1+S_3+y_2-y_4-2y_9+2y_{11}\right)-\frac{2}{3}D_3\left(S_2+S_4+y_1-y_3+i\left(y_5-y_7\right)\right)-$$
$$-\frac{1}{3}iM\left(y_1+y_3+i\left(y_5+y_7+i\left(y_{10}+y_{12}\right)\right)\right)$$
$$+iG_{11}(S_2+S_4-5iy_5+5iy_7-4y_{10}+4y_{12})+iG_{22}(S_2+S_4-5y_1+5y_3-4y_{10}+4y_{12})-2iG_{33}(S_2+S_4+y_1-y_3+i(y_5-y_7))$$
$$-iG_{10}(12L_2+18L_5+y_2+y_4+5iy_6+5iy_8+4(y_9+y_{11}))+G_{20}(-12L_2+6L_5-5y_2-5y_4-i(y_6+y_8+4i(y_9+y_{11})))$$
$$+iG_{30}(6L_6+y_1+y_3+iy_5+iy_7+2(y_{10}+y_{12}))$$
$$+G_{12}(-S_2-S_4-y_1+y_3+4iy_5-4iy_7+2y_{10}-2y_{12})+G_{21}(S_2+S_4-4y_1+4y_3+iy_5-iy_7-2y_{10}+2y_{12})+$$
$$+2iG_{13}(S_1+S_3+y_2-y_4+iy_6-iy_8+2y_9-2y_{11})+iG_{31}(S_1+S_3+2y_2-2y_4+iy_6-iy_8-2y_9+2y_{11})$$
$$-2G_{23}(S_1+S_3-y_2+y_4-iy_6+iy_8+2y_9-2y_{11})+G_{32}(-S_1-S_3+y_2-y_4+2iy_6-2iy_8+2y_9-2y_{11})=0,$$

$14-16$

$$\frac{1}{3}\left(-y_1+y_3-iy_5+iy_7-2y_{10}+2y_{12}\right)D_0+\frac{1}{3}D_1\left(-2L_2-6L_5+S_1-S_3-i\left(y_6+y_8\right)-2y_9-2y_{11}\right)+$$
$$+\frac{1}{3}iD_2\left(2L_2-4L_5+S_1-S_3+y_2+y_4-2\left(y_9+y_{11}\right)\right)+\frac{2}{3}D_3\left(2L_6+S_2-S_4+y_1+y_3+i\left(y_5+y_7\right)\right)+$$
$$+\frac{1}{3}iM\left(y_1-y_3+iy_5-iy_7+5y_{10}-5y_{12}\right)$$
$$-iG_{11}(10L_1-18L_6+S_2-S_4-5iy_5-5iy_7-4(y_{10}+y_{12}))+iG_{22}(10L_1+8L_6-S_2+S_4+5y_1+5y_3+4(y_{10}+y_{12}))$$
$$+2iG_{33}(2L_6+S_2-S_4+y_1+y_3+i(y_5+y_7))$$
$$+iG_{10}(y_2-y_4+5iy_6-5iy_8+4y_9-4y_{11})+G_{20}(5y_2-5y_4+iy_6-iy_8-4y_9+4y_{11})+G_{30}(-iy_1+iy_3+y_5-y_7-2iy_{10}+2iy_{12})$$
$$+G_{12}(10L_1-12L_6+S_2-S_4+y_1+y_3-4iy_5-4iy_7-2(y_{10}+y_{12}))$$
$$+G_{21}(10L_1+2L_6-S_2+S_4+4y_1+4y_3-i(y_5+y_7+2i(y_{10}+y_{12})))$$
$$-2iG_{13}(4L_2+6L_5+S_1-S_3+y_2+y_4+iy_6+iy_8+2(y_9+y_{11}))$$
$$-iG_{31}(6L_2-2L_5+S_1-S_3+2y_2+2y_4+i(y_6+y_8+2i(y_9+y_{11})))$$
$$-2G_{23}(4L_2-2L_5-S_1+S_3+y_2+y_4+i(y_6+y_8+2i(y_9+y_{11})))$$
$$+G_{32}(-6L_2-8L_5+S_1-S_3-y_2-y_4-2i(y_6+y_8)-2y_9-2y_{11})=0.$$

When performing the non-relativistic approximation, we should assume the following smallness orders for the involved quantities

$$L_{..}\sim 1,\quad S_{..}\sim x,\qquad y_{..}\sim x,\quad \frac{D_j}{M}\sim x,\quad \frac{G_{ij}}{M}\sim x,\quad \frac{D_0}{M}\sim x^2,\quad \frac{G_{j0}}{M}\sim x^2; \tag{33}$$

in fact, we will need only the orders x and x^2.

We divide equations of order x in two group; the first group is

$$1\quad \frac{2}{3}D_1L_1-2iG_{11}L_2-2iG_{33}L_5+\frac{2}{3}D_3L_5+2iG_{22}(L_2+L_5)-2G_{12}(L_2+3L_5)$$
$$+G_{21}(4L_5-2L_2)+2iG_{13}(2L_1-3L_6)+G_{32}(6L_1-2L_6)$$
$$+\frac{2}{3}iD_2(L_1-L_6)+G_{31}(4iL_6-6iL_1)-2G_{23}(2L_1+L_6)+\frac{1}{3}iM(3S_1+3S_3+2(y_2-y_4-iy_6+iy_8+y_9-y_{11}))=0,$$

$$2 \quad \frac{1}{3}iM(3S_1 - 3S_3 - 2(y_2 + y_4 - iy_6 - iy_8 + y_9 + y_{11})) = 0,$$

$$3 \quad -2iG_{11}L_1 + \frac{2}{3}D_1L_2 - \frac{2}{3}iD_2(L_2+L_5) + 2G_{32}(3L_2+L_5) + G_{23}(2L_5-4L_2) + 2iG_{31}(3L_2+2L_5) - 2iG_{13}(2L_2+3L_5)$$
$$+2G_{12}(L_1 - 3L_6) + 2iG_{22}(L_1 - L_6) + 2iG_{33}L_6 + \frac{2}{3}D_3L_6 + 2G_{21}(L_1 + 2L_6)$$
$$+\frac{1}{3}iM(3S_2 + 3S_4 + 2(y_1 - y_3 + iy_5 - iy_7 - y_{10} + y_{12})) = 0,$$

$$4 \quad \frac{1}{3}iM(3S_2 - 3S_4 - 2(y_1 + y_3 + i(y_5 + y_7 + i(y_{10} + y_{12})))) = 0,$$

$$5 \quad \frac{1}{3}iM(y_1 + y_3 + i(y_5 + y_7 + i(y_{10} + y_{12}))) = 0,$$

$$6 \quad 4iG_{11}L_1 - \frac{4}{3}D_1L_2 + 2G_{23}(L_2 - 5L_5) + 2iG_{13}(L_2 - 3L_5) + \frac{2}{3}iD_2(2L_2 - L_5) - 4iG_{31}(3L_2 + 2L_5) - 2G_{32}(6L_2 + 5L_5)$$
$$+G_{12}(8L_1 - 6L_6) + 2iG_{33}(9L_1 - 5L_6) + D_3(\frac{2L_6}{3} - 2L_1) - 4G_{21}(L_1 + 2L_6) + 2iG_{22}(4L_1 + 5L_6)$$
$$+\frac{1}{3}iM(5y_1 - 5y_3 - iy_5 + iy_7 + y_{10} - y_{12}) = 0,$$

$$7 \quad \frac{1}{3}iM(y_2 + y_4 - iy_6 - iy_8 + y_9 + y_{11}) = 0,$$

$$8 \quad -\frac{4}{3}D_1L_1 + 4iG_{11}L_2 + G_{12}(-8L_2 - 6L_5) + 2iG_{22}(4L_2 - 5L_5) + 4G_{21}(L_2 - 2L_5) + \frac{2}{3}D_3(3L_2 + L_5) + 2iG_{33}(9L_2 + 5L_5)$$
$$+4iG_{31}(3L_1 - 2L_6) - \frac{2}{3}iD_2(2L_1 + L_6) - 2iG_{13}(L_1 + 3L_6) + 2G_{23}(L_1 + 5L_6) +$$
$$+G_{32}(10L_6 - 12L_1) + \frac{1}{3}iM(5y_2 - 5y_4 + iy_6 - iy_8 - y_9 + y_{11}) = 0,$$

$$9 \quad \frac{1}{3}M(y_1 + y_3 + i(y_5 + y_7 + i(y_{10} + y_{12}))) = 0,$$

$$10 \quad \frac{4}{3}D_2(L_2 + L_5) + 4iG_{32}(3L_2 + L_5) - 2G_{31}(6L_2 + L_5) + \frac{2}{3}iD_1(2L_2 + 3L_5)$$
$$-2iG_{23}(L_2 + 4L_5) + 2G_{13}(L_2 + 6L_5) + 4iG_{12}(L_1 - 3L_6)$$
$$+D_3(\frac{4iL_6}{3} - 2iL_1) + G_{21}(2iL_6 - 8iL_1) + G_{22}(4L_6 - 4L_1) + G_{33}(8L_6 - 18L_1) + G_{11}(18L_6 - 8L_1)$$
$$+\frac{1}{3}M(-y_1 + y_3 + 5iy_5 - 5iy_7 + y_{10} - y_{12}) = 0,$$

$$11 \quad \frac{1}{3}M(-y_2 - y_4 + i(y_6 + y_8 + i(y_9 + y_{11}))) = 0,$$

$$12 \quad 4G_{22}(L_2 + L_5) - 2iG_{21}(4L_2 + L_5) - \frac{2}{3}iD_3(3L_2 + 2L_5) + 4iG_{12}(L_2 + 3L_5) + G_{33}(18L_2 + 8L_5)$$
$$+G_{11}(8L_2 + 18L_5) + 2G_{13}(L_1 - 6L_6) + 2iG_{23}(L_1 - 4L_6) + \frac{4}{3}D_2(L_1 - L_6) - 4iG_{32}(3L_1 - L_6) + D_1(2iL_6 - \frac{4iL_1}{3})$$
$$+2G_{31}(L_6 - 6L_1) + \frac{1}{3}M(y_2 - y_4 + 5iy_6 - 5iy_8 + y_9 - y_{11}) = 0,$$

$$13 \quad \frac{1}{3}iM(y_2 + y_4 - iy_6 - iy_8 + y_9 + y_{11}) = 0,$$

$$14 \quad -2iG_{22}(5L_2 - 4L_5) + G_{21}(10L_2 - 2L_5) + 4iG_{33}L_5 - \frac{4}{3}D_3L_5 + 2G_{12}(5L_2 + 6L_5) + 2iG_{11}(5L_2 + 9L_5) + G_{32}(6L_1 - 8L_6)$$
$$+\frac{2}{3}D_1(L_1 - 3L_6) - 4iG_{13}(2L_1 - 3L_6) + 4G_{23}(2L_1 + L_6) - 2iG_{31}(3L_1 + L_6) + \frac{2}{3}iD_2(L_1 + 2L_6)$$
$$-\frac{1}{3}iM(y_2 - y_4 - iy_6 + iy_8 - 5y_9 + 5y_{11}) = 0,$$

$$15 \quad -\frac{1}{3}iM(y_1 + y_3 + i(y_5 + y_7 + i(y_{10} + y_{12}))) = 0,$$

$$16 \quad G_{32}(-6L_2 - 8L_5) + \frac{2}{3}iD_2(L_2 - 2L_5) + G_{31}(2iL_5 - 6iL_2) - \frac{2}{3}D_1(L_2 + 3L_5) - 4iG_{13}(2L_2 + 3L_5)$$
$$+G_{23}(4L_5 - 8L_2) - 2iG_{11}(5L_1 - 9L_6)$$
$$+2G_{12}(5L_1 - 6L_6) + 4iG_{33}L_6 + \frac{4}{3}D_3L_6 + 2G_{21}(5L_1 + L_6) + 2iG_{22}(5L_1 + 4L_6)$$
$$+\frac{1}{3}iM(y_1 - y_3 + iy_5 - iy_7 + 5y_{10} - 5y_{12}) = 0.$$

Remaining 8 equations of the order x are rather complex; with their help we can express 8 small variables through the large ones and four free small variables y_1, y_2, y_5, y_6):

$$S_1 = \frac{1}{M}\Big[iD_1L_1 + 3G_{11}L_2 - 3iG_{21}(L_2 - 2L_5) + 3G_{33}L_5 + iD_3L_5 - 3G_{22}(L_2 + L_5)$$
$$-3iG_{12}(L_2 + 3L_5) + G_{31}(9L_1 - 6L_6)$$
$$+3iG_{32}(3L_1 - L_6) + D_2(L_6 - L_1) - 3iG_{23}(2L_1 + L_6) + G_{13}(9L_6 - 6L_1)\Big],$$

$$S_3 = \frac{1}{M}\Big[iD_1L_1 + 3G_{11}L_2 - 3iG_{21}(L_2 - 2L_5) + 3G_{33}L_5 + iD_3L_5 - 3G_{22}(L_2 + L_5) - 3iG_{12}(L_2 + 3L_5)$$
$$+G_{31}(9L_1 - 6L_6) + 3iG_{32}(3L_1 - L_6) + D_2(L_6 - L_1) - 3iG_{23}(2L_1 + L_6) + G_{13}(9L_6 - 6L_1)\Big],$$

$$S_2 = \frac{1}{M}\Big[3G_{11}L_1 + iD_1L_2 + G_{31}(-9L_2 - 6L_5) + G_{23}(3iL_5 - 6iL_2) + D_2(L_2 + L_5) + 3iG_{32}(3L_2 + L_5)$$
$$+G_{13}(6L_2 + 9L_5) + 3iG_{12}(L_1 - 3L_6) - 3G_{33}L_6 + iD_3L_6 + 3iG_{21}(L_1 + 2L_6) + G_{22}(3L_6 - 3L_1)\Big],$$

$$S_4 = \frac{1}{M}\Big[3G_{11}L_1 + iD_1L_2 + G_{31}(-9L_2 - 6L_5) + G_{23}(3iL_5 - 6iL_2) + D_2(L_2 + L_5) + 3iG_{32}(3L_2 + L_5)$$
$$+G_{13}(6L_2 + 9L_5) + 3iG_{12}(L_1 - 3L_6) - 3G_{33}L_6 + iD_3L_6 + 3iG_{21}(L_1 + 2L_6) + G_{22}(3L_6 - 3L_1)\Big],$$

$$y_3 = \frac{1}{M}\Big[3G_{11}L_1 - 3iG_{12}L_1 + iD_3L_1 + 3G_{13}L_2 + iD_1L_2 + D_2L_2 + G_{31}(-9L_2 - 6L_5) - 3iG_{23}(L_2 - 2L_5)$$
$$+3iG_{32}(3L_2 + 2L_5) + G_{33}(9L_1 - 6L_6) + 3iG_{21}(L_1 + 2L_6) + 3G_{22}(L_1 + 2L_6)\Big] + y_1,$$

$$y_4 = \frac{1}{M}\Big[-3G_{13}L_1 + iD_1L_1 - D_2L_1 + 3G_{11}L_2 + 3iG_{12}L_2 - iD_3L_2 - 3iG_{21}(L_2 - 2L_5) + 3G_{22}(L_2 - 2L_5)$$
$$+G_{33}(9L_2 + 6L_5) + G_{31}(9L_1 - 6L_6) + 3iG_{32}(3L_1 - 2L_6) - 3iG_{23}(L_1 + 2L_6)\Big] + y_2,$$

$$y_7 = \frac{1}{M}\Big[-3G_{23}(L_2 + L_5) + D_1(L_2 + L_5) - iD_2(L_2 + L_5) + 3iG_{31}(3L_2 + L_5) + 3G_{32}(3L_2 + L_5) - 3iG_{13}(L_2 + 3L_5)$$
$$+3iG_{11}(L_1 - 3L_6) + 3G_{12}(L_1 - 3L_6) + 3iG_{22}(L_1 - L_6) + 3iG_{33}(3L_1 - L_6) + D_3(L_6 - L_1) + G_{21}(3L_6 - 3L_1)\Big] + y_5,$$

$$y_8 = \frac{1}{M}\Big[D_3(-L_2 - L_5) - 3G_{21}(L_2 + L_5) - 3iG_{22}(L_2 + L_5) - 3iG_{33}(3L_2 + L_5) - 3iG_{11}(L_2 + 3L_5) + 3G_{12}(L_2 + 3L_5)$$
$$-3iG_{13}(L_1 - 3L_6) + 3G_{23}(L_1 - L_6) - iD_2(L_1 - L_6) + 3iG_{31}(3L_1 - L_6) + D_1(L_6 - L_1) + G_{32}(3L_6 - 9L_1)\Big] + y_6,$$

$$y_9 = \frac{1}{M}\Big[-iD_1L_1 - 3G_{11}L_2 + 3iG_{21}(L_2 - 2L_5) - 3G_{33}L_5 - iD_3L_5 + 3G_{22}(L_2 + L_5) + 3iG_{12}(L_2 + 3L_5)$$
$$+G_{13}(6L_1 - 9L_6) + D_2(L_1 - L_6) + G_{32}(3iL_6 - 9iL_1) + 3iG_{23}(2L_1 + L_6) + G_{31}(6L_6 - 9L_1)\Big] - y_2 + iy_6,$$

$$y_{10} = \frac{1}{M}\Big[3G_{11}L_1 + iD_1L_2 + G_{31}(-9L_2 - 6L_5) + G_{23}(3iL_5 - 6iL_2) + D_2(L_2 + L_5) + 3iG_{32}(3L_2 + L_5)$$
$$+G_{13}(6L_2 + 9L_5) + 3iG_{12}(L_1 - 3L_6) - 3G_{33}L_6 + iD_3L_6 + 3iG_{21}(L_1 + 2L_6) + G_{22}(3L_6 - 3L_1)\Big] + y_1 + iy_5,$$

$$y_{11} = \frac{1}{M}\Big[D_2L_1 - 3iG_{12}L_2 - 3iG_{21}(L_2 + L_5) + 3G_{11}(L_2 + 3L_5) + G_{22}(6L_5 - 3L_2) - iD_1(L_1 - L_6)$$
$$+G_{32}(6iL_6 - 9iL_1) + G_{31}(3L_6 - 9L_1)\Big] - y_2 + iy_6,$$

$$y_{12} = \frac{1}{M}\Big[-3iG_{12}L_1 + D_2L_2 + iD_1(L_2 + L_5) - 3G_{31}(3L_2 + L_5) + 3iG_{32}(3L_2 + 2L_5)$$
$$-3iG_{21}(L_1 - L_6) + 3G_{22}(L_1 + 2L_6) + G_{11}(9L_6 - 3L_1)\Big] + y_1 + iy_5.$$

Let us write down equations of the smallness order x^2:

$1'$

$$G_{33}(-2iS_1 + 2iS_3 + iy_2 + iy_4 + y_6 + y_8) + \frac{1}{3}D_3(2S_1 - 2S_3 - y_2 - y_4 + i(y_6 + y_8)) - iG_{22}(2S_1 - 2S_3 - y_2 - y_4 - y_9 - y_{11})$$
$$+G_{12}(-2S_1 + 2S_3 + y_2 + y_4 - 2iy_6 - 2iy_8 - y_9 - y_{11}) + G_{21}(2S_1 - 2S_3 - 2y_2 - 2y_4 + iy_6 + iy_8 + y_9 + y_{11})$$
$$+G_{11}(-2iS_1 + 2iS_3 + y_6 + y_8 + i(y_9 + y_{11})) - \frac{1}{3}iD_2(2S_2 - 2S_4 - y_1 - y_3 + y_{10} + y_{12}) + \frac{1}{3}D_1(2S_2 - 2S_4 - iy_5 - iy_7 + y_{10} + y_{12})$$
$$+iG_{31}(2S_2 - 2S_4 - 2y_1 - 2y_3 + iy_5 + iy_7 + y_{10} + y_{12}) + G_{32}(2S_2 - 2S_4 + y_1 + y_3 - 2iy_5 - 2iy_7 + y_{10} + y_{12})$$
$$+G_{13}(-2iS_2 + 2iS_4 + iy_1 + iy_3 + y_5 + y_7 - 2i(y_{10} + y_{12})) + G_{23}(-2S_2 + 2S_4 - y_1 - y_3 + i(y_5 + y_7 + 2i(y_{10} + y_{12}))) = 0,$$

$2'$

$$\frac{1}{3}D_3(-2S_1-2S_3+y_2-y_4-iy_6+iy_8)+G_{33}(i(2S_1+2S_3-y_2+y_4+iy_6)+y_8)+G_{12}(2S_1+2S_3-y_2+y_4+2iy_6-2iy_8+y_9-y_{11})$$
$$+G_{11}(2iS_1+2iS_3-y_6+y_8-iy_9+iy_{11})+iG_{22}(2S_1+2S_3-y_2+y_4-y_9+y_{11})+G_{21}(-2S_1-2S_3+2y_2-2y_4-iy_6+iy_8-y_9+y_{11})$$
$$+iG_{13}(2S_2+2S_4-y_1+y_3+iy_5-iy_7+2y_{10}-2y_{12})+G_{23}(2S_2+2S_4+y_1-y_3-iy_5+iy_7+2y_{10}-2y_{12})$$
$$+\frac{1}{3}iD_2(2S_2+2S_4-y_1+y_3+y_{10}-y_{12})-iG_{31}(2S_2+2S_4-2y_1+2y_3+iy_5-iy_7+y_{10}-y_{12})$$
$$+\frac{1}{3}D_1(-2S_2-2S_4+iy_5-iy_7-y_{10}+y_{12})+G_{32}(-2S_2-2S_4-y_1+y_3+2iy_5-2iy_7-y_{10}+y_{12})=0,$$

$3'$

$$\frac{1}{3}D_3(-2S_2+2S_4+y_1+y_3+i(y_5+y_7))+G_{33}(i(2S_4+y_1+y_3+i(y_5+y_7))-2iS_2)+\frac{1}{3}iD_2(2S_1-2S_3-y_2-y_4-y_9-y_{11})$$
$$+G_{32}(2S_1-2S_3+y_2+y_4+2iy_6+2iy_8-y_9-y_{11})+G_{31}(i(2S_3+2y_2+2y_4+iy_6+iy_8+y_9+y_{11})-2iS_1)$$
$$+\frac{1}{3}D_1(2S_1-2S_3+i(y_6+y_8+i(y_9+y_{11})))+G_{23}(-2S_1+2S_3-y_2-y_4-i(y_6+y_8+2i(y_9+y_{11})))$$
$$+G_{13}(2iS_1-i(2S_3+y_2+y_4+iy_6+iy_8+2(y_9+y_{11})))+G_{12}(2S_2-2S_4-y_1-y_3-2iy_5-2iy_7-y_{10}-y_{12})-$$
$$-iG_{22}(2S_2-2S_4-y_1-y_3+y_{10}+y_{12})-iG_{11}(2S_2-2S_4-iy_5-iy_7+y_{10}+y_{12})$$
$$+G_{21}(-2S_2+2S_4+2y_1+2y_3+iy_5+iy_7+y_{10}+y_{12})=0,$$

$4'$

$$G_{33}(2iS_2+2iS_4-iy_1+iy_3+y_5-y_7)+\frac{1}{3}D_3(2S_2+2S_4-y_1+y_3-iy_5+iy_7)+\frac{1}{3}D_1(-2S_1-2S_3-iy_6+iy_8+y_9-y_{11})$$
$$+G_{32}(-2S_1-2S_3-y_2+y_4-2iy_6+2iy_8+y_9-y_{11})-\frac{1}{3}iD_2(2S_1+2S_3-y_2+y_4-y_9+y_{11})$$
$$+iG_{31}(2S_1+2S_3-2y_2+2y_4-iy_6+iy_8-y_9+y_{11})+G_{23}(2S_1+2S_3+y_2-y_4+iy_6-iy_8-2y_9+2y_{11})$$
$$-iG_{13}(2S_1+2S_3-y_2+y_4-iy_6+iy_8-2y_9+2y_{11})+G_{11}(2iS_2+2iS_4+y_5-y_7+i(y_{10}-y_{12}))$$
$$+iG_{22}(2S_2+2S_4-y_1+y_3+y_{10}-y_{12})+G_{12}(-2S_2-2S_4+y_1-y_3+2iy_5-2iy_7+y_{10}-y_{12})$$
$$+G_{21}(2S_2+2S_4-2y_1+2y_3-iy_5+iy_7-y_{10}+y_{12})=0,$$

$5'$

$$2D_0L_1-6iG_{10}L_2-6G_{20}(L_2-2L_5)-6iG_{30}(3L_1-2L_6)+\frac{1}{3}D_3(-S_2-S_4+2y_1-2y_3-iy_5+iy_7)$$
$$+G_{33}(-i(S_2+S_4+4y_1-4y_3)-5y_5+5y_7)+G_{23}(-S_1-S_3-2y_2+2y_4+iy_6-iy_8+4y_9-4y_{11})$$
$$+iG_{13}(S_1+S_3-2y_2+2y_4+iy_6-iy_8+2y_9-2y_{11})+\frac{1}{3}iD_2(S_1+S_3-2y_2+2y_4+y_9-y_{11})$$
$$+2iG_{31}(S_1+S_3+2y_2-2y_4+iy_6-iy_8+y_9-y_{11})-\frac{2}{3}D_1(S_1+S_3-iy_6+iy_8+y_9-y_{11})$$
$$+G_{32}(S_1+S_3+2y_2-2y_4+4iy_6-4iy_8+y_9-y_{11})-iG_{22}(S_2+S_4+4y_1-4y_3+5y_{10}-5y_{12})$$
$$+2G_{21}(S_2+S_4+2y_1-2y_3+iy_5-iy_7+y_{10}-y_{12})+G_{12}(S_2+S_4-2y_1+2y_3+2iy_5-2iy_7+y_{10}-y_{12})$$
$$+2iG_{11}(S_2+S_4+iy_5-iy_7-y_{10}+y_{12})=0,$$

$6'$

$$\frac{1}{3}D_3(S_2-S_4-2y_1-2y_3+i(y_5+y_7))+G_{33}(iS_2-iS_4+4iy_1+4iy_3+5(y_5+y_7))$$
$$+G_{23}(S_1-S_3+2y_2+2y_4-i(y_6+y_8)-4y_9-4y_{11})$$
$$+G_{32}(-S_1+S_3-2y_2-2y_4-4iy_6-4iy_8-y_9-y_{11})-\frac{1}{3}iD_2(S_1-S_3-2y_2-2y_4+y_9+y_{11})$$
$$+\frac{2}{3}D_1(S_1-S_3-iy_6-iy_8+y_9+y_{11})-2iG_{31}(S_1-S_3+2y_2+2y_4+iy_6+iy_8+y_9+y_{11})$$
$$+G_{13}(-iS_1+iS_3+2iy_2+2iy_4+y_6+y_8-2i(y_9+y_{11}))+G_{12}(-S_2+S_4+2y_1+2y_3-2iy_5-2iy_7-y_{10}-y_{12})-$$
$$-2G_{21}(S_2-S_4+2y_1+2y_3+iy_5+iy_7+y_{10}+y_{12})+2G_{11}(-iS_2+iS_4+y_5+y_7+i(y_{10}+y_{12}))$$
$$+iG_{22}(S_2-S_4+4y_1+4y_3+5(y_{10}+y_{12}))=0,$$

$7'$

$$-6iG_{10}L_1+2D_0L_2+6iG_{30}(3L_2+2L_5)+6G_{20}(L_1+2L_6)-iG_{33}(S_1+S_3+4y_2-4y_4+5i(y_6-y_8))$$
$$+\frac{1}{3}D_3(S_1+S_3-2y_2+2y_4-iy_6+iy_8)+2iG_{11}(S_1+S_3-iy_6+iy_8+y_9-y_{11})$$
$$+G_{12}(-S_1-S_3+2y_2-2y_4+2iy_6-2iy_8+y_9-y_{11})-2G_{21}(S_1+S_3+2y_2-2y_4-iy_6+iy_8-y_9+y_{11})$$

$$-iG_{22}(S_1+S_3+4y_2-4y_4-5y_9+5y_{11})-\frac{1}{3}iD_2(S_2+S_4-2y_1+2y_3-y_{10}+y_{12})$$
$$-\frac{2}{3}D_1(S_2+S_4+iy_5-iy_7-y_{10}+y_{12})-2iG_{31}(S_2+S_4+2y_1-2y_3-iy_5+iy_7-y_{10}+y_{12})$$
$$+G_{32}(S_2+S_4+2y_1-2y_3-4iy_5+4iy_7-y_{10}+y_{12})-iG_{13}(S_2+S_4-2y_1+2y_3-iy_5+iy_7-2y_{10}+2y_{12})$$
$$+G_{23}(-S_2-S_4-2y_1+2y_3-iy_5+iy_7-4y_{10}+4y_{12})=0,$$

$8'$

$$\frac{1}{3}D_3(-S_1+S_3+2y_2+2y_4+i(y_6+y_8))+iG_{33}(S_1-S_3+4y_2+4y_4+5i(y_6+y_8))$$
$$+G_{12}(S_1-S_3-2y_2-2y_4-2iy_6-2iy_8-y_9-y_{11})$$
$$+2G_{21}(S_1-S_3+2y_2+2y_4-i(y_6+y_8)-y_9-y_{11})-2iG_{11}(S_1-S_3-iy_6-iy_8+y_9+y_{11})$$
$$+iG_{22}(S_1-S_3+4y_2+4y_4-5(y_9+y_{11}))+\frac{1}{3}iD_2(S_2-S_4-2y_1-2y_3-y_{10}-y_{12})$$
$$+2iG_{31}(S_2-S_4+2y_1+2y_3-i(y_5+y_7)-y_{10}-y_{12})+G_{32}(-S_2+S_4-2y_1-2y_3+4iy_5+4iy_7+y_{10}+y_{12})$$
$$+G_{23}(S_2-S_4+2y_1+2y_3+iy_5+iy_7+4(y_{10}+y_{12}))+\frac{2}{3}D_1(S_2-S_4+i(y_5+y_7+i(y_{10}+y_{12})))$$
$$+G_{13}(iS_2-i(S_4+2y_1+2y_3+iy_5+iy_7+2(y_{10}+y_{12})))=0,$$

$9'$

$$6iG_{20}(L_2+L_5)-6G_{10}(L_2+3L_5)+G_{30}(18L_1-6L_6)+2iD_0(L_1-L_6)+\frac{1}{3}iD_3(S_2+S_4+y_1-y_3-2iy_5+2iy_7)$$
$$+G_{33}(-S_2-S_4+5y_1-5y_3-4iy_5+4iy_7)+iG_{23}(S_1+S_3-y_2+y_4+2iy_6-2iy_8+2y_9-2y_{11})$$
$$-\frac{2}{3}D_2(S_1+S_3+y_2-y_4+y_9-y_{11})-\frac{1}{3}iD_1(S_1+S_3+2iy_6-2iy_8+y_9-y_{11})$$
$$+2iG_{32}(S_1+S_3-y_2+y_4-2iy_6+2iy_8+y_9-y_{11})+G_{31}(-S_1-S_3+4y_2-4y_4+2iy_6-2iy_8-y_9+y_{11})$$
$$+G_{13}(S_1+S_3+y_2-y_4-2iy_6+2iy_8-4y_9+4y_{11})+2iG_{12}(S_2+S_4+y_1-y_3+2iy_5-2iy_7+y_{10}-y_{12})$$
$$+iG_{21}(S_2+S_4+2y_1-2y_3-2iy_5+2iy_7+y_{10}-y_{12})+2G_{22}(S_2+S_4+y_1-y_3-y_{10}+y_{12})$$
$$+G_{11}(-S_2-S_4-4iy_5+4iy_7-5y_{10}+5y_{12})=0,$$

$10'$

$$-\frac{1}{3}iD_3(S_2-S_4+y_1+y_3-2i(y_5+y_7))+G_{33}(S_2-S_4-5y_1-5y_3+4i(y_5+y_7))+\frac{2}{3}D_2(S_1-S_3+y_2+y_4+y_9+y_{11})$$
$$+G_{31}(S_1-S_3-4y_2-4y_4-2iy_6-2iy_8+y_9+y_{11})-2iG_{32}(S_1-S_3-y_2-y_4-2iy_6-2iy_8+y_9+y_{11})$$
$$+\frac{1}{3}iD_1(S_1-S_3+2iy_6+2iy_8+y_9+y_{11})+G_{13}(-S_1+S_3-y_2-y_4+2iy_6+2iy_8+4(y_9+y_{11}))$$
$$-iG_{23}(S_1-S_3-y_2-y_4+2(iy_6+iy_8+y_9+y_{11}))-2G_{22}(S_2-S_4+y_1+y_3-y_{10}-y_{12})$$
$$-iG_{21}(S_2-S_4+2y_1+2y_3-2iy_5-2iy_7+y_{10}+y_{12})$$
$$-2iG_{12}(S_2-S_4+y_1+y_3+2iy_5+2iy_7+y_{10}+y_{12})+G_{11}(S_2-S_4+4iy_5+4iy_7+5(y_{10}+y_{12}))=0,$$

$11'$

$$-2iD_0(L_2+L_5)+6G_{30}(3L_2+L_5)+6G_{10}(L_1-3L_6)+6iG_{20}(L_1-L_6)+\frac{1}{3}iD_3(S_1+S_3+y_2-y_4+2i(y_6-y_8))$$
$$+G_{33}(S_1+S_3-5y_2+5y_4-4iy_6+4iy_8)-2G_{22}(S_1+S_3+y_2-y_4+y_9-y_{11})$$
$$+iG_{21}(S_1+S_3+2y_2-2y_4+2iy_6-2iy_8-y_9+y_{11})+2iG_{12}(S_1+S_3+y_2-y_4-2iy_6+2iy_8-y_9+y_{11})$$
$$+G_{11}(S_1+S_3-4iy_6+4iy_8-5y_9+5y_{11})+G_{13}(S_2+S_4+y_1-y_3+2iy_5-2iy_7+4y_{10}-4y_{12})$$
$$+G_{31}(-S_2-S_4+4y_1-4y_3-2iy_5+2iy_7+y_{10}-y_{12})-\frac{2}{3}D_2(S_2+S_4+y_1-y_3-y_{10}+y_{12})$$
$$-2iG_{32}(S_2+S_4-y_1+y_3+2iy_5-2iy_7-y_{10}+y_{12})+\frac{1}{3}iD_1(S_2+S_4-2iy_5+2iy_7-y_{10}+y_{12})-$$
$$-iG_{23}(S_2+S_4-y_1+y_3-2iy_5+2iy_7-2y_{10}+2y_{12})=0,$$

$12'$

$$-\frac{1}{3}iD_3(S_1-S_3+y_2+y_4+2i(y_6+y_8))+G_{33}(-S_1+S_3+5y_2+5y_4+4i(y_6+y_8))$$
$$-2iG_{12}(S_1-S_3+y_2+y_4-2iy_6-2iy_8-y_9-y_{11})$$
$$-iG_{21}(S_1-S_3+2y_2+2y_4+2iy_6+2iy_8-y_9-y_{11})+2G_{22}(S_1-S_3+y_2+y_4+y_9+y_{11})$$
$$+G_{11}(-S_1+S_3+4iy_6+4iy_8+5(y_9+y_{11}))+G_{13}(-S_2+S_4-y_1-y_3-2i(y_5+y_7)-4y_{10}-4y_{12})$$
$$+iG_{23}(S_2-S_4-y_1-y_3-2i(y_5+y_7)-2y_{10}-2y_{12})+\frac{2}{3}D_2(S_2-S_4+y_1+y_3-y_{10}-y_{12})$$
$$-\frac{1}{3}iD_1(S_2-S_4-2iy_5-2iy_7-y_{10}-y_{12})+G_{31}(S_2-S_4-4y_1-4y_3+2iy_5+2iy_7-y_{10}-y_{12})$$

$$+2iG_{32}(S_2 - S_4 - y_1 - y_3 + 2iy_5 + 2iy_7 - y_{10} - y_{12}) = 0,$$

$13'$

$$-6iG_{30}L_5 + 2D_0L_5 + 6iG_{10}(2L_1 - 3L_6) - 6G_{20}(2L_1 + L_6) + 2iG_{33}(S_1 + S_3 + y_2 - y_4 - iy_6 + iy_8)$$
$$-\frac{2}{3}D_3(S_1 + S_3 + y_2 - y_4 - iy_6 + iy_8)$$
$$-iG_{22}(S_1 + S_3 - 5y_2 + 5y_4 + 4y_9 - 4y_{11}) - iG_{11}(S_1 + S_3 + 5iy_6 - 5iy_8 + 4y_9 - 4y_{11})$$
$$+G_{21}(S_1 + S_3 - 4y_2 + 4y_4 - iy_6 + iy_8 + 2y_9 - 2y_{11}) + G_{12}(-S_1 - S_3 - y_2 + y_4 - 4iy_6 + 4iy_8 - 2y_9 + 2y_{11})$$
$$-\frac{1}{3}iD_2(S_2 + S_4 + y_1 - y_3 + 2y_{10} - 2y_{12}) + \frac{1}{3}D_1(S_2 + S_4 + iy_5 - iy_7 + 2y_{10} - 2y_{12})$$
$$+iG_{31}(S_2 + S_4 + 2y_1 - 2y_3 - iy_5 + iy_7 + 2y_{10} - 2y_{12}) + G_{32}(S_2 + S_4 - y_1 + y_3 + 2iy_5 - 2iy_7 + 2y_{10} - 2y_{12})$$
$$+2G_{23}(S_2 + S_4 - y_1 + y_3 + iy_5 - iy_7 - 2y_{10} + 2y_{12}) + 2iG_{13}(S_2 + S_4 + y_1 - y_3 - iy_5 + iy_7 - 2y_{10} + 2y_{12}) = 0,$$

$14'$

$$G_{33}(-2i(S_1 - S_3 + y_2 + y_4) - 2y_6 - 2y_8) + \frac{2}{3}D_3(S_1 - S_3 + y_2 + y_4 - i(y_6 + y_8)) + iG_{22}(S_1 - S_3 - 5y_2 - 5y_4 + 4(y_9 + y_{11}))$$
$$+iG_{11}(S_1 - S_3 + 5iy_6 + 5iy_8 + 4(y_9 + y_{11})) + G_{12}(S_1 - S_3 + y_2 + y_4 + 2(2iy_6 + 2iy_8 + y_9 + y_{11}))$$
$$+G_{21}(-S_1 + S_3 + 4y_2 + 4y_4 + i(y_6 + y_8 + 2i(y_9 + y_{11}))) + \frac{1}{3}D_1(-S_2 + S_4 - i(y_5 + y_7) - 2y_{10} - 2y_{12})$$
$$-2iG_{13}(S_2 - S_4 + y_1 + y_3 - i(y_5 + y_7) - 2y_{10} - 2y_{12}) + G_{32}(-S_2 + S_4 + y_1 + y_3 - 2i(y_5 + y_7) - 2y_{10} - 2y_{12})$$
$$+\frac{1}{3}iD_2(S_2 - S_4 + y_1 + y_3 + 2(y_{10} + y_{12})) + 2G_{23}(-S_2 + S_4 + y_1 + y_3 - i(y_5 + y_7 + 2i(y_{10} + y_{12})))$$
$$-iG_{31}(S_2 - S_4 + 2y_1 + 2y_3 - i(y_5 + y_7 + 2i(y_{10} + y_{12}))) = 0,$$

$15'$

$$6G_{20}(L_5 - 2L_2) - 6iG_{10}(2L_2 + 3L_5) + 6iG_{30}L_6 + 2D_0L_6 - 2iG_{33}(S_2 + S_4 + y_1 - y_3 + i(y_5 - y_7))$$
$$-\frac{2}{3}D_3(S_2 + S_4 + y_1 - y_3 + i(y_5 - y_7))$$
$$+\frac{1}{3}D_1(-S_1 - S_3 + iy_6 - iy_8 + 2y_9 - 2y_{11}) + 2iG_{13}(S_1 + S_3 + y_2 - y_4 + iy_6 - iy_8 + 2y_9 - 2y_{11})$$
$$-2G_{23}(S_1 + S_3 - y_2 + y_4 - iy_6 + iy_8 + 2y_9 - 2y_{11}) + G_{32}(-S_1 - S_3 + y_2 - y_4 + 2iy_6 - 2iy_8 + 2y_9 - 2y_{11})$$
$$-\frac{1}{3}iD_2(S_1 + S_3 + y_2 - y_4 - 2y_9 + 2y_{11}) + iG_{31}(S_1 + S_3 + 2y_2 - 2y_4 + iy_6 - iy_8 - 2y_9 + 2y_{11})$$
$$+G_{12}(-S_2 - S_4 - y_1 + y_3 + 4iy_5 - 4iy_7 + 2y_{10} - 2y_{12}) + G_{21}(S_2 + S_4 - 4y_1 + 4y_3 + iy_5 - iy_7 - 2y_{10} + 2y_{12})$$
$$+iG_{22}(S_2 + S_4 - 5y_1 + 5y_3 - 4y_{10} + 4y_{12}) + iG_{11}(S_2 + S_4 - 5iy_5 + 5iy_7 - 4y_{10} + 4y_{12}) = 0,$$

$16'$

$$2iG_{33}(S_2 - S_4 + y_1 + y_3 + i(y_5 + y_7)) + \frac{2}{3}D_3(S_2 - S_4 + y_1 + y_3 + i(y_5 + y_7)) + \frac{1}{3}D_1(S_1 - S_3 - i(y_6 + y_8) - 2y_9 - 2y_{11})$$
$$+G_{32}(S_1 - S_3 - y_2 - y_4 - 2i(y_6 + y_8) - 2y_9 - 2y_{11})$$
$$+\frac{1}{3}iD_2(S_1 - S_3 + y_2 + y_4 - 2(y_9 + y_{11})) - 2iG_{13}(S_1 - S_3 + y_2 + y_4 + iy_6 + iy_8 + 2(y_9 + y_{11}))$$
$$+2G_{23}(S_1 - S_3 - y_2 - y_4 - i(y_6 + y_8 + 2i(y_9 + y_{11}))) - iG_{31}(S_1 - S_3 + 2y_2 + 2y_4 + i(y_6 + y_8 + 2i(y_9 + y_{11})))$$
$$-iG_{22}(S_2 - S_4 - 5y_1 - 5y_3 - 4(y_{10} + y_{12})) + G_{12}(S_2 - S_4 + y_1 + y_3 - 4iy_5 - 4iy_7 - 2(y_{10} + y_{12}))$$
$$+G_{11}(-iS_2 + iS_4 - 5y_5 - 5y_7 + 4i(y_{10} + y_{12})) + G_{21}(-S_2 + S_4 + 4y_1 + 4y_3 - i(y_5 + y_7 + 2i(y_{10} + y_{12}))) = 0.$$

Now we should substitute expressions for small variables of order x in equations of order x^2; to follow the correct locations of multipliers, we re-designate all $G_{..}$ (which t should be located at the left) by symbols $J_{..}$; and all $D_{.}$, (which should be located at the left) by symbols $Q_{.}$).

In this way, we derive the following equations (we preserve only equations, containing D_0; and introduce the new numeration):

$5 \Longrightarrow 1)$

$$2Q_0L_1 - 6iJ_{10}L_2 - 6J_{20}\Big(L_2 - 2L_5\Big) - 6iJ_{30}\Big(3L_1 - 2L_6\Big)$$
$$+\frac{1}{M}\Big[\frac{2}{3}Q_1\Big((3G_{13} + 3iG_{23} - 9G_{31} - 9iG_{32} - iD_1 + D_2)L_1 - 3(G_{11} + iG_{12} - iG_{21} + G_{22} + 3G_{33})L_2 + iD_3L_2$$
$$+6(-iG_{21} + G_{22} - G_{33})L_5 + 6(iG_{23} + G_{31} + iG_{32})L_6\Big)$$
$$+2iJ_{11}\Big((3(G_{11} - iG_{12} + iG_{21} + G_{22} + 3G_{33}) + iD_3)L_1 + (3G_{13} - 3iG_{23} - 9G_{31} + 9iG_{32} + iD_1 + D_2)L_2$$

$$+6i(G_{23}+iG_{31}+G_{32})L_5+6(iG_{21}+G_{22}-G_{33})L_6\Big)+$$
$$+\frac{1}{3}Q_2\Big(4(-3iG_{13}+3G_{23}+9iG_{31}-9G_{32}-D_1-iD_2)L_1+2(3i(G_{11}+iG_{12}-iG_{21}+G_{22}+3G_{33})+D_3)L_2$$
$$+(-9iG_{11}+9G_{12}-21G_{21}-21iG_{22}+15iG_{33}-D_3)L_5+(9iG_{13}+15G_{23}-21iG_{31}+21G_{32}+D_1+iD_2)L_6\Big)$$
$$+J_{13}\Big((9(-iG_{13}+G_{23}+5iG_{31}-5G_{32})-5D_1-5iD_2)L_1+(-3iG_{11}-9G_{12}-3G_{21}+9i(G_{22}+G_{33})+D_3)L_2$$
$$+(-27iG_{11}+9G_{12}-21G_{21}-27iG_{22}+9iG_{33}-D_3)L_5+(3(3iG_{13}+3G_{23}-7iG_{31}+9G_{32}+D_1)+iD_2)L_6\Big)$$
$$-2iJ_{31}\Big((3G_{13}+3iG_{23}-9G_{31}-9iG_{32}-iD_1+D_2)L_1+3(3(G_{11}+iG_{12}-iG_{21}+G_{22}+3G_{33})-iD_3)L_2$$
$$+2(9G_{11}+9iG_{12}+6G_{33}-iD_3)L_5-2(9G_{13}+6iG_{23}-iD_1+D_2)L_6\Big)+J_{32}\Big(-4(3G_{13}+3iG_{23}-9G_{31}$$
$$-9iG_{32}-iD_1+D_2)L_1-6(3(G_{11}+iG_{12}-iG_{21}+G_{22}+3G_{33})-iD_3)L_2-(9(5G_{11}+5iG_{12}-iG_{21}+G_{22})+21G_{33}-5iD_3)L_5$$
$$+(45G_{13}+21iG_{23}-9G_{31}-9iG_{32}-5iD_1+5D_2)L_6\Big)+J_{23}\Big((9(3G_{13}+3iG_{23}+G_{31}+iG_{32})+iD_1-D_2)L_1$$
$$+(-27G_{11}+33iG_{12}+27iG_{21}+33G_{22}+9G_{33}-iD_3)L_2-(45G_{11}-45iG_{12}+9iG_{21}+21G_{22}+9G_{33}+5iD_3)L_5$$
$$-(45G_{13}-9iG_{23}-9G_{31}+21iG_{32}+5iD_1+5D_2)L_6\Big)+2J_{21}\Big((9G_{11}+15iG_{12}+9iG_{21}-15G_{22}-9G_{33}-iD_3)L_1$$
$$+(9G_{13}-9i(G_{23}+iG_{31}+G_{32})-iD_1-D_2)L_2+2(9G_{13}-6iG_{32}-iD_1+D_2)L_5-2(9G_{11}+9iG_{12}+6G_{22}-iD_3)L_6\Big)$$
$$+\frac{1}{3}Q_3\Big(-3(5G_{11}-iG_{12}+5iG_{21}+G_{22}+9G_{33}+iD_3)L_1-3(5G_{13}-5iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2$$
$$-(9G_{13}+21iG_{23}-21G_{31}+15iG_{32}-iD_1+D_2)L_5+(9G_{11}+9iG_{12}-21iG_{21}-15G_{22}+21G_{33}-iD_3)L_6\Big)$$
$$+J_{12}\Big(4(6G_{11}+6iG_{21}+9G_{33}+iD_3)L_1+2(9G_{13}-9iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2$$
$$+(9G_{13}+27iG_{23}-21G_{31}+9iG_{32}-3iD_1+D_2)L_5+(-27G_{11}-9iG_{12}+21iG_{21}+9G_{22}-27G_{33}+iD_3)L_6\Big)$$
$$+J_{33}\Big(3(7iG_{11}+11G_{12}-7G_{21}+11iG_{22}+27iG_{33}-3D_3)L_1+3(-5iG_{13}-5G_{23}+9iG_{31}+9G_{32}+D_1-iD_2)L_2$$
$$+(-63iG_{13}-33G_{23}+3iG_{31}-3G_{32}+5D_1-7iD_2)L_5+(-45iG_{11}-63G_{12}+3G_{21}+3i(G_{22}-11G_{33})+7D_3)L_6\Big)$$
$$+J_{22}\Big(4(6(-iG_{11}+2G_{12}+G_{21}+2iG_{22})+9iG_{33}-D_3)L_1+2(-15iG_{13}-15G_{23}-9iG_{31}-9G_{32}-D_1+iD_2)L_2$$
$$+(-63iG_{13}-3G_{23}+3iG_{31}-33G_{32}-5D_1-7iD_2)L_5+(45iG_{11}-63G_{12}+3G_{21}+33iG_{22}-3iG_{33}+7D_3)L_6\Big)\Big]=0,$$

$7 \Longrightarrow 2)$

$$-6iJ_{10}L_1+2Q_0L_2+6iJ_{30}\Big(3L_2+2L_5\Big)+6J_{20}\Big(L_1+2L_6\Big)$$
$$+\frac{1}{M}\Big[2J_{11}\Big((-3iG_{13}+3G_{23}+9iG_{31}-9G_{32}-D_1-iD_2)L_1+(3i(G_{11}+iG_{12}-iG_{21}+G_{22}+3G_{33})$$
$$+D_3)L_2-6(G_{21}+i(G_{22}-G_{33}))L_5+6(G_{23}-iG_{31}+G_{32})L_6\Big)-\frac{2}{3}Q_1\Big((3(G_{11}-iG_{12}+iG_{21}+G_{22}+3G_{33})$$
$$+iD_3)L_1+(3G_{13}-3iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2+6i(G_{23}+iG_{31}+G_{32})L_5+6(iG_{21}+G_{22}-G_{33})L_6\Big)$$
$$+J_{33}\Big(3(5iG_{13}-5G_{23}-9iG_{31}+9G_{32}+D_1+iD_2)L_1+3(7iG_{11}-11G_{12}+7G_{21}+11iG_{22}+27iG_{33}+3D_3)L_2$$
$$+(45iG_{11}-63G_{12}+3G_{21}-3iG_{22}+33iG_{33}+7D_3)L_5+(-63iG_{13}+33G_{23}+3iG_{31}+3G_{32}-5D_1-7iD_2)L_6\Big)$$
$$+J_{22}\Big(2i(15G_{13}+15iG_{23}+9G_{31}+9iG_{32}+iD_1-D_2)L_1+4(-6iG_{11}-6(2G_{12}+G_{21}-2iG_{22})+9iG_{33}+D_3)L_2$$
$$+(-45iG_{11}-63G_{12}+3G_{21}-33iG_{22}+3iG_{33}+7D_3)L_5+(-63iG_{13}+3G_{23}+3iG_{31}+33G_{32}+5D_1-7iD_2)L_6\Big)$$
$$-2J_{21}\Big(-\Big((9(G_{13}+iG_{23}+G_{31}+iG_{32})+iD_1-D_2)L_1\Big)+(9G_{11}-15iG_{12}-9iG_{21}-15G_{22}-9G_{33}+iD_3)L_2$$
$$+2(9G_{11}-9iG_{12}+6G_{22}+iD_3)L_5+2(9G_{13}+6iG_{32}+iD_1+D_2)L_6\Big)+\frac{1}{3}Q_3\Big(-3(5G_{13}$$
$$+5iG_{23}-9G_{31}-9iG_{32}-iD_1+D_2)L_1+3(5G_{11}+iG_{12}-5iG_{21}+G_{22}+9G_{33}-iD_3)L_2+(9G_{11}-9iG_{12}$$
$$+21iG_{21}-15G_{22}+21G_{33}+iD_3)L_5+(9G_{13}-21iG_{23}-21G_{31}-15iG_{32}+iD_1+D_2)L_6\Big)+J_{12}\Big(2(9G_{13}$$
$$+9iG_{23}-9G_{31}-9iG_{32}-iD_1+D_2)L_1+4(-6G_{11}+6iG_{21}-9G_{33}+iD_3)L_2-(27G_{11}-9iG_{12}+21iG_{21}-9G_{22}$$
$$+27G_{33}+iD_3)L_5-(9G_{13}-27iG_{23}-21G_{31}-9iG_{32}+3iD_1+D_2)L_6\Big)+J_{23}\Big((-27G_{11}-33iG_{12}-27iG_{21}$$

$$+33G_{22}+9G_{33}+iD_3)L_1+(-27G_{13}+27iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2+(-45G_{13}-9iG_{23}+9G_{31}+21iG_{32}$$
$$+5iD_1-5D_2)L_5+(45G_{11}+45iG_{12}-9iG_{21}+21G_{22}+9G_{33}-5iD_3)L_6\Big)+J_{32}\Big(-6(3(G_{11}-iG_{12}+iG_{21}$$
$$+G_{22}+3G_{33})+iD_3)L_1+4(3G_{13}-3iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2+(45G_{13}-21iG_{23}-9G_{31}+9iG_{32}+5iD_1$$
$$+5D_2)L_5+(9(5G_{11}-5iG_{12}+iG_{21}+G_{22})+21G_{33}+5iD_3)L_6\Big)+J_{31}\Big(6(3i(G_{11}-iG_{12}+iG_{21}+G_{22}$$
$$+3G_{33})-D_3)L_1+2(-3iG_{13}-3G_{23}+9iG_{31}+9G_{32}+D_1-iD_2)L_2+4(-9iG_{13}-6G_{23}+D_1-iD_2)L_5$$
$$+4(-9iG_{11}-9G_{12}-6iG_{33}+D_3)L_6\Big)+J_{13}\Big((3iG_{11}-3(3G_{12}+G_{21}+3i(G_{22}+G_{33}))+D_3)L_1+(-9iG_{13}-9G_{23}$$
$$+45iG_{31}+45G_{32}+5D_1-5iD_2)L_2+(3(-3iG_{13}+3G_{23}+7iG_{31}+9G_{32}+D_1)-iD_2)L_5+(-27iG_{11}-9G_{12}$$
$$+21G_{21}-27iG_{22}+9iG_{33}+D_3)L_6\Big)+\frac{1}{3}Q_2\Big(2(D_3-3i(G_{11}-iG_{12}+iG_{21}+G_{22}+3G_{33}))L_1$$
$$+4(-3iG_{13}-3G_{23}+9iG_{31}+9G_{32}+D_1-iD_2)L_2+(-9iG_{13}+15G_{23}+21iG_{31}+21G_{32}+D_1-iD_2)L_5+(-9iG_{11}-9G_{12}$$
$$+21G_{21}-21iG_{22}+15iG_{33}+D_3)L_6\Big)\Big]=0,$$

$9 \Longrightarrow 3)$

$$6iJ_{20}\Big(L_2+L_5\Big)-6J_{10}\Big(L_2+3L_5\Big)+J_{30}\Big(18L_1-6L_6\Big)+2iQ_0\Big(L_1-L_6\Big)$$
$$+\frac{1}{M}\Big[Q_1\Big(\frac{4}{3}(3iG_{13}-3G_{23}-9iG_{31}+9G_{32}+D_1+iD_2)L_1+\frac{2}{3}(3i(G_{11}+iG_{12}-iG_{21}+G_{22}$$
$$+3G_{33})+D_3)L_2+(9iG_{11}-9G_{12}+5G_{21}+i(5G_{22}+G_{33})+D_3)L_5+(-9iG_{13}+G_{23}+5iG_{31}-5G_{32}-D_1-iD_2)L_6\Big)$$
$$+J_{23}\Big((9(-iG_{13}+G_{23}+5iG_{31}-5G_{32})-5D_1-5iD_2)L_1+(-9iG_{11}-3G_{12}-9G_{21}+3iG_{22}-9iG_{33}-D_3)L_2$$
$$+2(-18iG_{11}+9G_{12}-9G_{21}-12iG_{22}-D_3)L_5+2(9iG_{13}-9iG_{31}+12G_{32}+2D_1+iD_2)L_6\Big)$$
$$+\frac{2}{3}Q_2\Big((3G_{13}+3iG_{23}-9G_{31}-9iG_{32}-iD_1+D_2)L_1+(3(G_{11}+iG_{12}-iG_{21}+G_{22}+3G_{33})-iD_3)L_2$$
$$+(3(3G_{11}+3iG_{12}-iG_{21}+G_{22}+G_{33})-iD_3)L_5-(9G_{13}+3iG_{23}-3G_{31}-3iG_{32}-iD_1+D_2)L_6\Big)$$
$$+J_{31}\Big(4(3G_{13}+3iG_{23}-9G_{31}-9iG_{32}-iD_1+D_2)L_1-6(3(G_{11}+iG_{12}-iG_{21}+G_{22}+3G_{33})-iD_3)L_2-(9G_{11}$$
$$+9i(G_{12}+3G_{21}+3iG_{22})+33G_{33}-iD_3)L_5+(9G_{13}+33iG_{23}+27G_{31}+27iG_{32}-iD_1+D_2)L_6\Big)$$
$$+J_{13}\Big(-\Big((9(3G_{13}+3iG_{23}+G_{31}+iG_{32})+iD_1-D_2)L_1\Big)+(33G_{11}-27iG_{12}-33iG_{21}-27G_{22}+9G_{33}-iD_3)L_2$$
$$+2(27G_{11}-18iG_{12}+6iG_{21}+9(G_{22}+G_{33})+2iD_3)L_5+2(18G_{13}-9iG_{23}-6G_{31}+9iG_{32}+3iD_1+2D_2)L_6\Big)$$
$$+2J_{32}\Big((-3iG_{13}+3G_{23}+9iG_{31}-9G_{32}-D_1-iD_2)L_1+3(3i(G_{11}+iG_{12}-iG_{21}+G_{22}+3G_{33})+D_3)L_2$$
$$+(9iG_{11}-9(G_{12}+G_{21}+iG_{22})+15iG_{33}+D_3)L_5-(D_1+i(9G_{13}+15iG_{23}+9G_{31}+9iG_{32}+D_2))L_6\Big)$$
$$+J_{21}\Big(-4(6G_{12}+6iG_{22}+9iG_{33}-D_3)L_1+2i(9G_{13}-9iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2+(45iG_{13}$$
$$+9G_{23}-9iG_{31}+3G_{32}-D_1+5iD_2)L_5+(9iG_{11}+45G_{12}-9G_{21}-3iG_{22}+9iG_{33}-5D_3)L_6\Big)$$
$$+2J_{12}\Big(i(15G_{11}+9iG_{12}+15iG_{21}-9G_{22}+9G_{33}+iD_3)L_1+(9(iG_{13}+G_{23}+iG_{31}+G_{32})+D_1-iD_2)L_2$$
$$+(9iG_{13}-9G_{23}-3iG_{31}+9G_{32}+3D_1+iD_2)L_5+(-27iG_{11}+9G_{12}-3G_{21}-9i(G_{22}+G_{33})-D_3)L_6\Big)$$
$$+J_{11}\Big(-4(12G_{11}+6iG_{12}+12iG_{21}-6G_{22}+9G_{33}+iD_3)L_1+2(-15G_{13}+15iG_{23}-9G_{31}+9iG_{32}+iD_1$$
$$+D_2)L_2-3(9G_{13}+11iG_{23}-5G_{31}-7iG_{32}-3iD_1+D_2)L_5+3(27G_{11}+9iG_{12}-5iG_{21}+7G_{22}+11G_{33}-iD_3)L_6\Big)$$
$$+2J_{22}\Big(-\Big((3(G_{11}-iG_{12}+iG_{21}+G_{22}+3G_{33})+iD_3)L_1\Big)+(3G_{13}-3iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2$$
$$+(9G_{13}-3iG_{23}-3G_{31}+3iG_{32}+iD_1+D_2)L_5+(3(3G_{11}-3iG_{12}+iG_{21}+G_{22}+G_{33})+iD_3)L_6\Big)$$
$$+J_{33}\Big(-3(11G_{11}-7iG_{12}+11iG_{21}+7G_{22}+27G_{33}+3iD_3)L_1-3(5G_{13}-5iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2$$
$$+2(9G_{13}-24iG_{23}+15G_{31}-12iG_{32}+2iD_1+D_2)L_5+2(3(6G_{11}-3iG_{12}-5iG_{21}-4G_{22}+8G_{33})+iD_3)L_6\Big)$$
$$+Q_3\Big((-iG_{11}-5G_{12}+G_{21}-5iG_{22}-9iG_{33}+D_3)L_1+(5iG_{13}+5G_{23}-9iG_{31}-9G_{32}-D_1+iD_2)L_2$$
$$+\frac{2}{3}i\Big((18G_{13}-3iG_{23}-6G_{31}+3iG_{32}+iD_1+2D_2)L_5+(3(3G_{11}-6iG_{12}+2iG_{21}+G_{22}+G_{33})+2iD_3)L_6\Big)\Big)\Big]=0,$$

$11 \Longrightarrow 4)$

$$-2iQ_0\Big(L_2+L_5\Big)+6J_{30}\Big(3L_2+L_5\Big)+6J_{10}\Big(L_1-3L_6\Big)+6iJ_{20}\Big(L_1-L_6\Big)$$
$$+\frac{1}{M}\Big[2J_{12}\Big((9(-iG_{13}+G_{23}-iG_{31}+G_{32})+D_1+iD_2)L_1+(15iG_{11}+9G_{12}+15G_{21}-9iG_{22}+9iG_{33}+D_3)L_2$$
$$+(27iG_{11}+9G_{12}-3G_{21}+9i(G_{22}+G_{33})-D_3)L_5+(9iG_{13}+9G_{23}-3iG_{31}-9G_{32}-3D_1+iD_2)L_6\Big)$$
$$+J_{21}\Big(2(-9iG_{13}+9G_{23}+9iG_{31}-9G_{32}-D_1-iD_2)L_1+4(6G_{12}-3i(2G_{22}+3G_{33})-D_3)L_2+(-9iG_{11}$$
$$+45G_{12}-9G_{21}+3i(G_{22}-3G_{33})-5D_3)L_5+(45iG_{13}-3(3G_{23}+3iG_{31}+G_{32})+D_1+5iD_2)L_6\Big)$$
$$+2J_{22}\Big((3G_{13}+3iG_{23}-9G_{31}-9iG_{32}-iD_1+D_2)L_1+(3(G_{11}+iG_{12}-iG_{21}+G_{22}+3G_{33})-iD_3)L_2$$
$$+(3(3G_{11}+3iG_{12}-iG_{21}+G_{22}+G_{33})-iD_3)L_5-(9G_{13}+3iG_{23}-3G_{31}-3iG_{32}-iD_1+D_2)L_6\Big)$$
$$+J_{33}\Big(-3(5G_{13}+5iG_{23}-9G_{31}-9iG_{32}-iD_1+D_2)L_1+3(11G_{11}+7iG_{12}-11iG_{21}+7G_{22}+27G_{33}-3iD_3)L_2$$
$$+2(3(6G_{11}+3iG_{12}+5iG_{21}-4G_{22}+8G_{33})-iD_3)L_5-2(9G_{13}+24iG_{23}+15G_{31}+12iG_{32}-2iD_1+D_2)L_6\Big)$$
$$+J_{11}\Big(-2(15G_{13}+15iG_{23}+9G_{31}+9iG_{32}+iD_1-D_2)L_1+4(12G_{11}-6iG_{12}-12iG_{21}-6G_{22}+9G_{33}-iD_3)L_2$$
$$+3(27G_{11}-9iG_{12}+5iG_{21}+7G_{22}+11G_{33}+iD_3)L_5+3(9G_{13}-11iG_{23}-5G_{31}+7iG_{32}+3iD_1+D_2)L_6\Big)$$
$$+2J_{32}\Big(3(D_3-3i(G_{11}-iG_{12}+iG_{21}+G_{22}+3G_{33}))L_1+(-3iG_{13}-3G_{23}+9iG_{31}+9G_{32}+D_1-iD_2)L_2$$
$$+(9iG_{13}+15G_{23}+9iG_{31}+9G_{32}-D_1+iD_2)L_5+(9(iG_{11}+G_{12}+G_{21}-iG_{22})+15iG_{33}-D_3)L_6\Big)$$
$$+Q_1\Big(\frac{2}{3}(D_3-3i(G_{11}-iG_{12}+iG_{21}+G_{22}+3G_{33}))L_1+\frac{4}{3}i(3G_{13}-3iG_{23}-9G_{31}+9iG_{32}+iD_1$$
$$+D_2)L_2+(9iG_{13}+G_{23}-5iG_{31}-5G_{32}-D_1+iD_2)L_5+(9iG_{11}+9G_{12}-5G_{21}+i(5G_{22}+G_{33})-D_3)L_6\Big)$$
$$+J_{31}\Big(-6(3(G_{11}-iG_{12}+iG_{21}+G_{22}+3G_{33})+iD_3)L_1-4(3G_{13}-3iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2$$
$$+(9G_{13}-33iG_{23}+27G_{31}-27iG_{32}+iD_1+D_2)L_5+(9(G_{11}-i(G_{12}+3G_{21})-3G_{22})+33G_{33}+iD_3)L_6\Big)$$
$$+\frac{1}{3}Q_2\Big(2(3(G_{11}-iG_{12}+iG_{21}+G_{22}+3G_{33})+iD_3)L_1-2(3G_{13}-3iG_{23}-9G_{31}+9iG_{32}+iD_1$$
$$+D_2)L_2-2(9G_{13}-3iG_{23}-3G_{31}+3iG_{32}+iD_1+D_2)L_5-2(3(3G_{11}-3iG_{12}+iG_{21}+G_{22}+G_{33})+iD_3)L_6\Big)$$
$$+J_{13}\Big((33G_{11}+27iG_{12}+33iG_{21}-27G_{22}+9G_{33}+iD_3)L_1+(9(3G_{13}-3iG_{23}+G_{31}-iG_{32})-iD_1-D_2)L_2+$$
$$+2(18G_{13}+9iG_{23}-6G_{31}-9iG_{32}-3iD_1+2D_2)L_5-2(27G_{11}+18iG_{12}-6iG_{21}+9(G_{22}+G_{33})-2iD_3)L_6\Big)$$
$$+J_{23}\Big(i(9G_{11}+3iG_{12}+9iG_{21}-3G_{22}+9G_{33}+iD_3)L_1+(-9iG_{13}-9G_{23}+45iG_{31}+45G_{32}$$
$$+5D_1-5iD_2)L_2-2i(9G_{13}-9G_{31}+12iG_{32}+2iD_1+D_2)L_5+2(-18iG_{11}-9G_{12}+9G_{21}-12iG_{22}+D_3)L_6\Big)$$
$$+Q_3\Big((-5iG_{13}+5G_{23}+9iG_{31}-9G_{32}-D_1-iD_2)L_1+(-i(G_{11}+5iG_{12}-iG_{21}+5G_{22}+9G_{33})-D_3)L_2$$
$$+\frac{2}{3}\Big((-3i(3G_{11}+6iG_{12}-2iG_{21}+G_{22}+G_{33})-2D_3)L_5+(18iG_{13}-3G_{23}-6iG_{31}+3G_{32}+D_1+2iD_2)L_6\Big)\Big)\Big]=0,$$

$13 \Longrightarrow 5)$

$$-6iJ_{30}L_5+2Q_0L_5+6iJ_{10}\Big(2L_1-3L_6\Big)-6J_{20}\Big(2L_1+L_6\Big)$$
$$+\frac{1}{M}\Big[J_{12}\Big((9G_{13}+9i(G_{23}+5iG_{31}-5G_{32})-5iD_1+5D_2)L_1+(21G_{11}+9iG_{12}-21iG_{21}+9G_{22}$$
$$+45G_{33}-5iD_3)L_2+2(27G_{11}+18iG_{12}-6iG_{21}+9(G_{22}+G_{33})-2iD_3)L_5+2(-18G_{13}-9iG_{23}+6G_{31}+9iG_{32}$$
$$+3iD_1-2D_2)L_6\Big)+J_{11}\Big((3iG_{13}-3G_{23}-63iG_{31}+63G_{32}+7D_1+7iD_2)L_1+(33iG_{11}+3G_{12}+33G_{21}-3iG_{22}$$
$$+45iG_{33}+5D_3)L_2+3(27iG_{11}-9G_{12}+5G_{21}+11iG_{22}+7iG_{33}+D_3)L_5+3(-9iG_{13}+7G_{23}+5iG_{31}-11G_{32}$$
$$-3D_1-iD_2)L_6\Big)+2J_{33}\Big(6(G_{23}-iG_{13})L_1+6(iG_{11}+G_{12}+G_{21}-iG_{22})L_2+(3i(3G_{11}-3iG_{12}+iG_{21}$$
$$+G_{22}+G_{33})-D_3)L_5+(9iG_{13}+3G_{23}-3iG_{31}-3G_{32}-D_1+iD_2)L_6\Big)+\frac{2}{3}Q_3\Big(6(G_{13}+iG_{23})L_1$$
$$+6(-G_{11}+i(G_{12}+G_{21})+G_{22})L_2-(3(3G_{11}-3iG_{12}+iG_{21}+G_{22}+G_{33})+iD_3)L_5-(9G_{13}-3iG_{23}-3G_{31}$$

$$+3iG_{32}+iD_1+D_2)L_6\Big)+J_{21}\Big((-9G_{13}-9iG_{23}+45G_{31}+45iG_{32}+5iD_1-5D_2)L_1+(9G_{11}$$
$$+21iG_{12}-9iG_{21}+21G_{22}+45G_{33}-5iD_3)L_2+(-9G_{11}+9iG_{12}+27iG_{21}-33G_{22}+27G_{33}-iD_3)L_5-(9(G_{13}$$
$$+3iG_{23}+3G_{31})+33iG_{32}+iD_1+D_2)L_6\Big)+J_{22}\Big((3iG_{13}-3G_{23}-63iG_{31}+63G_{32}+7D_1+7iD_2)L_1$$
$$+(3iG_{11}+33G_{12}+3G_{21}-33iG_{22}-45iG_{33}-5D_3)L_2+2(3(6iG_{11}+3G_{12}+5G_{21}+8iG_{22}-4iG_{33})-D_3)L_5$$
$$+2i(9G_{13}+12iG_{23}+15G_{31}+24iG_{32}+2iD_1+D_2)L_6\Big)+J_{31}\Big(3(3iG_{11}-3(3G_{12}+G_{21}+3i(G_{22}+G_{33}))$$
$$+D_3)L_1+(21iG_{13}+21G_{23}-9iG_{31}-9G_{32}-D_1+iD_2)L_2+(45iG_{13}+3G_{23}-9iG_{31}+9G_{32}+D_1+5iD_2)L_5$$
$$+(-9iG_{11}+45G_{12}-9G_{21}-9iG_{22}+3iG_{33}-5D_3)L_6\Big)+J_{23}\Big(4(-6iG_{12}+6G_{22}+9G_{33}+iD_3)L_1$$
$$+4(-9G_{31}+9iG_{32}+iD_1+D_2)L_2-2(9(G_{13}-iG_{23}+G_{31})-15iG_{32}-iD_1+D_2)L_5+2(9G_{11}+9iG_{12}+9iG_{21}$$
$$+15G_{22}-9G_{33}-iD_3)L_6\Big)+Q_1\Big(\Big(7G_{11}+5iG_{12}+7iG_{21}-5G_{22}+3G_{33}+\frac{1}{3}iD_3\Big)L_1$$
$$+\frac{1}{3}(21G_{13}-21iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2+(9G_{13}+5iG_{23}-5G_{31}-iG_{32}-iD_1+D_2)L_5-(9G_{11}$$
$$+9iG_{12}-5iG_{21}+G_{22}+5G_{33}-iD_3)L_6\Big)+J_{32}\Big(3(9G_{11}+3iG_{12}+9iG_{21}-3G_{22}+9G_{33}+iD_3)L_1$$
$$+(21G_{13}-21iG_{23}-9G_{31}+9iG_{32}+iD_1+D_2)L_2+2(9G_{13}+12iG_{23}-9G_{31}-2iD_1+D_2)L_5-2(18G_{11}$$
$$+9i(G_{12}-G_{21})+12G_{33}-iD_3)L_6\Big)+\frac{1}{3}Q_2\Big((3(-5iG_{11}+7G_{12}+5G_{21}+7iG_{22}+3iG_{33})-D_3)L_1$$
$$+(-21iG_{13}-21G_{23}+9iG_{31}+9G_{32}+D_1-iD_2)L_2+2(-18iG_{13}+3G_{23}+6iG_{31}-3G_{32}-D_1-2iD_2)L_5$$
$$+2(3i(3G_{11}+6iG_{12}-2iG_{21}+G_{22}+G_{33})+2D_3)L_6\Big)+J_{13}\Big(4(-6iG_{11}+6G_{21}-9iG_{33}+D_3)L_1$$
$$+2i\Big(2(-9G_{31}+9iG_{32}+iD_1+D_2)L_2+(9G_{13}-9iG_{23}-3G_{31}+9iG_{32}+3iD_1+D_2)L_5$$
$$+(27G_{11}-9iG_{12}+3iG_{21}+9(G_{22}+G_{33})+iD_3)L_6\Big)\Big)\Big]=0,$$

$15 \Longrightarrow 6)$

$$6J_{20}\Big(L_5-2L_2\Big)-6iJ_{10}\Big(2L_2+3L_5\Big)+6iJ_{30}L_6+2Q_0L_6$$
$$+\frac{1}{M}\Big[\frac{1}{3}Q_2\Big((21iG_{13}-21G_{23}-9iG_{31}+9G_{32}+D_1+iD_2)L_1$$
$$+(3(-5iG_{11}-7G_{12}-5G_{21}+7iG_{22}+3iG_{33})+D_3)L_2+2(2D_3-3i(3G_{11}-6iG_{12}+2iG_{21}+G_{22}+G_{33}))L_5$$
$$+2(-18iG_{13}-3G_{23}+6iG_{31}+3G_{32}+D_1-2iD_2)L_6\Big)+Q_1\Big(\frac{1}{3}(21G_{13}+21iG_{23}-9G_{31}-9iG_{32}-iD_1$$
$$+D_2)L_1+\Big(-7G_{11}+5iG_{12}+7iG_{21}+5G_{22}-3G_{33}+\frac{1}{3}iD_3\Big)L_2-(9G_{11}-9iG_{12}+5iG_{21}+G_{22}+5G_{33}$$
$$+iD_3)L_5-(9G_{13}-5iG_{23}-5G_{31}+iG_{32}+iD_1+D_2)L_6\Big)+2J_{23}\Big(-2(9G_{31}+9iG_{32}+iD_1-D_2)L_1$$
$$+2(-6iG_{12}-6G_{22}-9G_{33}+iD_3)L_2+(9G_{11}-9iG_{12}-9iG_{21}+15G_{22}-9G_{33}+iD_3)L_5+(9(G_{13}+iG_{23}+G_{31})$$
$$+15iG_{32}+iD_1+D_2)L_6\Big)+J_{32}\Big((21G_{13}+21iG_{23}-9G_{31}-9iG_{32}-iD_1+D_2)L_1+3(-9G_{11}+3iG_{12}$$
$$+9iG_{21}+3G_{22}-9G_{33}+iD_3)L_2-2(18G_{11}-9iG_{12}+9iG_{21}+12G_{33}+iD_3)L_5-2(9G_{13}-12iG_{23}-9G_{31}$$
$$+2iD_1+D_2)L_6\Big)-2iJ_{13}\Big(-2(9G_{31}+9iG_{32}+iD_1-D_2)L_1+2(6G_{11}-6iG_{21}+9G_{33}-iD_3)L_2+(27G_{11}$$
$$+9iG_{12}-3iG_{21}+9(G_{22}+G_{33})-iD_3)L_5-(9G_{13}+9iG_{23}-3G_{31}-9iG_{32}-3iD_1+D_2)L_6\Big)$$
$$+J_{31}\Big((-21iG_{13}+21G_{23}+9iG_{31}-9G_{32}-D_1-iD_2)L_1+3(3iG_{11}+9G_{12}+3G_{21}-9i(G_{22}+G_{33})-D_3)L_2$$
$$+(9iG_{11}+45G_{12}-9G_{21}+9iG_{22}-3iG_{33}-5D_3)L_5+i(45G_{13}+3iG_{23}-9G_{31}+9iG_{32}+iD_1+5D_2)L_6\Big)$$
$$+J_{11}\Big((-33iG_{11}+3G_{12}+33G_{21}+3i(G_{22}-15G_{33})+5D_3)L_1+(3iG_{13}+3G_{23}-63iG_{31}-63G_{32}-7D_1+7iD_2)L_2$$
$$+3i(9G_{13}-7iG_{23}-5G_{31}+11iG_{32}+3iD_1+D_2)L_5+3(27iG_{11}+9G_{12}-5G_{21}+11iG_{22}+7iG_{33}-D_3)L_6\Big)$$
$$+J_{21}\Big((9G_{11}-21iG_{12}+9iG_{21}+21G_{22}+45G_{33}+5iD_3)L_1+(9(G_{13}-iG_{23}-5G_{31}+5iG_{32})+5iD_1$$
$$+5D_2)L_2-(9(G_{13}-3iG_{23}+3G_{31})-33iG_{32}-iD_1+D_2)L_5+(9G_{11}+9iG_{12}+27iG_{21}+33G_{22}-27G_{33}-iD_3)L_6\Big)$$
$$+\frac{2}{3}Q_3\Big(-6(G_{11}+i(G_{12}+G_{21}+iG_{22}))L_1-6(G_{13}-iG_{23})L_2-(9G_{13}+3iG_{23}-3G_{31}-3iG_{32}-iD_1+D_2)L_5$$
$$+(3(3G_{11}+3iG_{12}-iG_{21}+G_{22}+G_{33})-iD_3)L_6\Big)+J_{12}\Big((21G_{11}-9iG_{12}+21iG_{21}+9G_{22}+45G_{33}$$

$$+5iD_3)L_1 - (9(G_{13} - iG_{23} - 5G_{31} + 5iG_{32}) + 5iD_1 + 5D_2)L_2 - 2(18G_{13} - 9iG_{23} - 6G_{31} + 9iG_{32} + 3iD_1$$
$$+2D_2)L_5 - 2(27G_{11} - 18iG_{12} + 6iG_{21} + 9(G_{22} + G_{33}) + 2iD_3)L_6\Big) + J_{22}\Big((-3iG_{11} + 33G_{12} + 3G_{21}$$
$$+33iG_{22} + 45iG_{33} - 5D_3)L_1 + (3iG_{13} + 3G_{23} - 63iG_{31} - 63G_{32} - 7D_1 + 7iD_2)L_2 - 2i(9G_{13} - 12iG_{23}$$
$$+15G_{31} - 24iG_{32} - 2iD_1 + D_2)L_5 + 2(3i(6G_{11} + 3iG_{12} + 5iG_{21} + 8G_{22} - 4G_{33}) + D_3)L_6\Big)+$$
$$+2J_{33}\Big(6(-iG_{11} + G_{12} + G_{21} + iG_{22})L_1 + (-6iG_{13} - 6G_{23})L_2 + (-9iG_{13} + 3G_{23} + 3iG_{31} - 3G_{32} - D_1 - iD_2)L_5$$
$$+(3i(3G_{11} + 3iG_{12} - iG_{21} + G_{22} + G_{33}) + D_3)L_6\Big)\Big] = 0,$$

Let us combine equations as follows

$$I = 1) + 4), \quad II = 1) - i \cdot 5), \quad III = 4) + i \cdot 5),$$

$$IV = 2) - 3), \quad V = 2) + i \cdot 6), \quad VI = 3) + i \cdot 6)$$

We readily verify that $III = I - II$ and $VI = V - IV$, so exist only four independent equations; we apply new numeration and introduce new variables

$$L_1 \to \frac{1}{3}\left(\Psi_1 + \Psi_3\right), L_2 \to \frac{1}{3}\left(\Psi_2 + \Psi_4\right), L_5 \to \frac{1}{3}\left(\Psi_4 - 2\Psi_2\right), L_6 \to \frac{1}{3}\left(2\Psi_1 - \Psi_3\right);$$

this results in

1)

$$\Big(+ 2D_0M + 6iG_{30}M - iD_1D_1 - iD_2D_2 - iD_3D_3 + \frac{D_2D_1}{3} - \frac{D_1D_2}{3}$$
$$+3G_{13}D_1 - 3iG_{23}D_1 + G_{31}D_1 - 5iG_{32}D_1 - iG_{13}D_2 - G_{23}D_2 + 5iG_{31}D_2 + G_{32}D_2 - G_{11}D_3 + iG_{12}D_3 + 3iG_{21}D_3 + 3G_{22}D_3 + 3G_{33}D_3$$
$$+3D_3G_{11} + 5iD_3G_{12} - 3D_1G_{13} - 5iD_2G_{13} - 9iD_3G_{21} - D_3G_{22} + 9iD_1G_{23} + D_2G_{23} + 3D_1G_{31} + iD_2G_{31} - iD_1G_{32} + 3D_2G_{32} + 3D_3G_{33}$$
$$+45iG_{11}G_{11} - 39G_{12}G_{11} - 9G_{21}G_{11} + 45iG_{22}G_{11} - 15iG_{33}G_{11} + 21G_{11}G_{12} + 15iG_{12}G_{12} - 15iG_{21}G_{12}$$
$$-27G_{22}G_{12} - 39G_{33}G_{12} + 15iG_{13}G_{13} - 9G_{23}G_{13} + 45iG_{31}G_{13} + 21G_{32}G_{13} - 9G_{11}G_{21} + 21iG_{12}G_{21} + 27iG_{21}G_{21}$$
$$-9G_{22}G_{21} + 3G_{33}G_{21} + 33iG_{11}G_{22} - 3G_{12}G_{22} + 3G_{21}G_{22} + 81iG_{22}G_{22} + 21iG_{33}G_{22} - 3G_{13}G_{23} + 27iG_{23}G_{23}$$
$$-9G_{31}G_{23} + 33iG_{32}G_{23} + 9iG_{13}G_{31} + 9G_{23}G_{31} + 3iG_{31}G_{31} + 15G_{32}G_{31} + 3G_{13}G_{32} - 3iG_{23}G_{32} - 15G_{31}G_{32} + 3iG_{32}G_{32}$$
$$-3iG_{11}G_{33} - 3G_{12}G_{33} - 9G_{21}G_{33} + 9iG_{22}G_{33} + 9iG_{33}G_{33}\Big)\Psi_1$$
$$+\Big(+ 6iG_{10}M - 18G_{20}M - iD_3D_1 + iD_1D_3 - \frac{D_2D_3}{3} + \frac{D_3D_2}{3}$$
$$+3G_{11}D_1 + 5iG_{12}D_1 + 3iG_{21}D_1 + 3G_{22}D_1 - G_{33}D_1 + iG_{11}D_2 + G_{12}D_2$$
$$-G_{21}D_2 + 9iG_{22}D_2 + 5iG_{33}D_2 + G_{13}D_3 + 3iG_{23}D_3 + 3G_{31}D_3 + iG_{32}D_3$$
$$+3D_1G_{11} + 5iD_2G_{11} - 5iD_1G_{12} + 3D_2G_{12} + 3D_3G_{13} + 9iD_1G_{21} + D_2G_{21} - D_1G_{22}$$
$$+9iD_2G_{22} + 9iD_3G_{23} - 3D_3G_{31} - iD_3G_{32} + 3D_1G_{33} + iD_2G_{33}$$
$$+45iG_{13}G_{11} + 9G_{23}G_{11} + 15iG_{31}G_{11} + 39G_{32}G_{11} - 21G_{13}G_{12} - 15iG_{23}G_{12} - 39G_{31}G_{12} + 15iG_{32}G_{12}$$
$$-15iG_{11}G_{13} + 21G_{12}G_{13} - 9G_{21}G_{13} + 45iG_{22}G_{13} + 45iG_{33}G_{13} + 9G_{13}G_{21} + 27iG_{23}G_{21} + 3G_{31}G_{21}$$
$$+21iG_{32}G_{21} + 33iG_{13}G_{22} - 3G_{23}G_{22} - 21iG_{31}G_{22} + 3G_{32}G_{22} - 3G_{11}G_{23} - 33iG_{12}G_{23} - 27iG_{21}G_{23}$$
$$+9G_{22}G_{23} + 9G_{33}G_{23} - 9iG_{11}G_{31} + 15G_{12}G_{31} + 9G_{21}G_{31} - 9iG_{22}G_{31} + 3iG_{33}G_{31} + 3G_{11}G_{32}$$
$$-3iG_{12}G_{32} + 3iG_{21}G_{32} + 27G_{22}G_{32} + 15G_{33}G_{32} - 3iG_{13}G_{33} + 9G_{23}G_{33} - 9iG_{31}G_{33} + 3G_{32}G_{33}\Big)\Psi_2$$
$$+\Big(- 12iG_{30}M + \frac{2D_1D_2}{3} - \frac{2D_2D_1}{3}$$
$$-6G_{13}D_1 - 2G_{31}D_1 + 4iG_{32}D_1 - 4iG_{13}D_2 + 2G_{23}D_2 - 4iG_{31}D_2 - 2G_{32}D_2 + 2G_{11}D_3 + 4iG_{12}D_3 - 6G_{22}D_3 - 6G_{33}D_3$$
$$-6D_3G_{11} - 4iD_3G_{12} + 6D_1G_{13} + 4iD_2G_{13} + 2D_3G_{22} - 2D_2G_{23} - 6D_1G_{31} + 4iD_2G_{31} - 4iD_1G_{32} - 6D_2G_{32} - 6D_3G_{33}$$
$$-36iG_{11}G_{11} + 42G_{12}G_{11} + 18G_{21}G_{11} - 36iG_{22}G_{11} + 12iG_{33}G_{11} - 6G_{11}G_{12} - 12iG_{12}G_{12} + 12iG_{21}G_{12}$$
$$+54G_{22}G_{12} + 42G_{33}G_{12} - 12iG_{13}G_{13} + 18G_{23}G_{13} - 36iG_{31}G_{13} - 6G_{32}G_{13} + 18G_{11}G_{21} + 12iG_{12}G_{21}$$
$$+18G_{22}G_{21} - 6G_{33}G_{21} - 12iG_{11}G_{22} + 6G_{12}G_{22} - 6G_{21}G_{22} + 12iG_{33}G_{22} + 6G_{13}G_{23} + 18G_{31}G_{23}$$
$$-12iG_{32}G_{23} + 36iG_{13}G_{31} - 18G_{23}G_{31} + 12iG_{31}G_{31} + 6G_{32}G_{31} - 42G_{13}G_{32} - 12iG_{23}G_{32} - 6G_{31}G_{32}$$
$$+12iG_{32}G_{32} - 12iG_{11}G_{33} + 42G_{12}G_{33} + 18G_{21}G_{33} + 36iG_{22}G_{33} + 36iG_{33}G_{33}\Big)\Psi_3$$
$$+\Big(- 12iG_{10}M + \frac{2D_2D_3}{3} - \frac{2D_3D_2}{3}$$

$$-6G_{11}D_1-4iG_{12}D_1-6G_{22}D_1+2G_{33}D_1+4iG_{11}D_2-2G_{12}D_2+2G_{21}D_2-4iG_{33}D_2-2G_{13}D_3-6G_{31}D_3+4iG_{32}D_3$$
$$-6D_1G_{11}-4iD_2G_{11}+4iD_1G_{12}-6D_2G_{12}-6D_3G_{13}-2D_2G_{21}+2D_1G_{22}+6D_3G_{31}-4iD_3G_{32}-6D_1G_{33}+4iD_2G_{33}$$
$$-36iG_{13}G_{11}-18G_{23}G_{11}-12iG_{31}G_{11}-42G_{32}G_{11}+6G_{13}G_{12}+12iG_{23}G_{12}+42G_{31}G_{12}-12iG_{32}G_{12}$$
$$+12iG_{11}G_{13}-6G_{12}G_{13}+18G_{21}G_{13}-36iG_{22}G_{13}-36iG_{33}G_{13}-18G_{13}G_{21}-6G_{31}G_{21}+12iG_{32}G_{21}$$
$$-12iG_{13}G_{22}+6G_{23}G_{22}-12iG_{31}G_{22}-6G_{32}G_{22}+6G_{11}G_{23}+12iG_{12}G_{23}-18G_{22}G_{23}-18G_{33}G_{23}$$
$$-36iG_{11}G_{31}+6G_{12}G_{31}-18G_{21}G_{31}-36iG_{22}G_{31}+12iG_{33}G_{31}-42G_{11}G_{32}-12iG_{12}G_{32}+12iG_{21}G_{32}$$
$$-54G_{22}G_{32}+6G_{33}G_{32}-12iG_{13}G_{33}-18G_{23}G_{33}-36iG_{31}G_{33}-42G_{32}G_{33}\Big)\Psi_4=0,$$

2)

$$\Big(4G_{13}D_1-4iG_{23}D_1-4iG_{13}D_2-4G_{23}D_2-4G_{11}D_3+4iG_{12}D_3+4iG_{21}D_3+4G_{22}D_3$$
$$+4D_3G_{11}-4iD_3G_{12}-4D_1G_{13}+4iD_2G_{13}-4iD_3G_{21}-4D_3G_{22}+4iD_1G_{23}+4D_2G_{23}$$
$$-36iG_{11}G_{11}-36G_{12}G_{11}-12G_{21}G_{11}+12iG_{22}G_{11}-36iG_{33}G_{11}+12G_{11}G_{12}-12iG_{12}G_{12}-36iG_{21}G_{12}$$
$$-36G_{22}G_{12}-36G_{33}G_{12}-12iG_{13}G_{13}-12G_{23}G_{13}+12iG_{31}G_{13}+12G_{32}G_{13}-36G_{11}G_{21}+36iG_{12}G_{21}$$
$$+12iG_{21}G_{21}+12G_{22}G_{21}-36G_{33}G_{21}-12iG_{11}G_{22}-12G_{12}G_{22}-36G_{21}G_{22}+36iG_{22}G_{22}+36iG_{33}G_{22}$$
$$-12G_{13}G_{23}+12iG_{23}G_{23}+12G_{31}G_{23}-12iG_{32}G_{23}+12iG_{13}G_{31}+12G_{23}G_{31}+12G_{13}G_{32}-12iG_{23}G_{32}-$$
$$-12iG_{11}G_{33}-12G_{12}G_{33}-12G_{21}G_{33}+12iG_{22}G_{33}\Big)\Psi_1+$$
$$+\Big(-12iG_{10}M-12G_{20}M-\frac{2}{3}iD_3D_1+\frac{2iD_1D_3}{3}+\frac{2D_2D_3}{3}-\frac{2D_3D_2}{3}$$
$$-6G_{11}D_1+6iG_{12}D_1+2iG_{21}D_1+2G_{22}D_1-6G_{33}D_1-2iG_{11}D_2-2G_{12}D_2-6G_{21}D_2$$
$$+6iG_{22}D_2+6iG_{33}D_2-2G_{13}D_3+2iG_{23}D_3+2G_{31}D_3-2iG_{32}D_3$$
$$-6D_1G_{11}+6iD_2G_{11}-6iD_1G_{12}-6D_2G_{12}-6D_3G_{13}+6iD_1G_{21}+6D_2G_{21}-6D_1G_{22}$$
$$+6iD_2G_{22}+6iD_3G_{23}-2D_3G_{31}+2iD_3G_{32}+2D_1G_{33}-2iD_2G_{33}$$
$$+42iG_{13}G_{11}+42G_{23}G_{11}+18iG_{31}G_{11}+18G_{32}G_{11}+6G_{13}G_{12}-6iG_{23}G_{12}-18G_{31}G_{12}+18iG_{32}G_{12}$$
$$-6iG_{11}G_{13}-6G_{12}G_{13}-42G_{21}G_{13}+42iG_{22}G_{13}+54iG_{33}G_{13}-6G_{13}G_{21}+6iG_{23}G_{21}+18G_{31}G_{21}$$
$$-18iG_{32}G_{21}+42iG_{13}G_{22}+42G_{23}G_{22}+18iG_{31}G_{22}+18G_{32}G_{22}+42G_{11}G_{23}-42iG_{12}G_{23}-6iG_{21}G_{23}$$
$$-6G_{22}G_{23}+54G_{33}G_{23}+18iG_{11}G_{31}+18G_{12}G_{31}+6G_{21}G_{31}-6iG_{22}G_{31}+18iG_{33}G_{31}-6G_{11}G_{32}$$
$$+6iG_{12}G_{32}+18iG_{21}G_{32}+18G_{22}G_{32}+18G_{33}G_{32}+6iG_{13}G_{33}+6G_{23}G_{33}-6iG_{31}G_{33}-6G_{32}G_{33}\Big)\Psi_2$$
$$+\Big(+2D_0M-18iG_{30}M-iD_1D_1-iD_2D_2-iD_3D_3+D_1D_2-D_2D_1-$$
$$-5G_{13}D_1+5iG_{23}D_1-3G_{31}D_1+3iG_{32}D_1-iG_{13}D_2-G_{23}D_2-3iG_{31}D_2-3G_{32}D_2-G_{11}D_3+iG_{12}D_3-5iG_{21}D_3-5G_{22}D_3-9G_{33}D_3$$
$$-5D_3G_{11}+5iD_3G_{12}+5D_1G_{13}-5iD_2G_{13}-iD_3G_{21}-D_3G_{22}+iD_1G_{23}+D_2G_{23}-9D_1G_{31}+9iD_2G_{31}-9iD_1G_{32}-9D_2G_{32}-9D_3G_{33}$$
$$+45iG_{11}G_{11}+45G_{12}G_{11}+15G_{21}G_{11}-15iG_{22}G_{11}+45iG_{33}G_{11}-15G_{11}G_{12}+15iG_{12}G_{12}+45iG_{21}G_{12}$$
$$+45G_{22}G_{12}+45G_{33}G_{12}+15iG_{13}G_{13}+15G_{23}G_{13}-15iG_{31}G_{13}-15G_{32}G_{13}-9G_{11}G_{21}+9iG_{12}G_{21}$$
$$+3iG_{21}G_{21}+3G_{22}G_{21}-9G_{33}G_{21}-3iG_{11}G_{22}-3G_{12}G_{22}-9G_{21}G_{22}+9iG_{22}G_{22}+9iG_{33}G_{22}-3G_{13}G_{23}$$
$$+3iG_{23}G_{23}+3G_{31}G_{23}-3iG_{32}G_{23}+21iG_{13}G_{31}+21G_{23}G_{31}+27iG_{31}G_{31}+27G_{32}G_{31}-33G_{13}G_{32}$$
$$+33iG_{23}G_{32}-27G_{31}G_{32}+27iG_{32}G_{32}+33iG_{11}G_{33}+33G_{12}G_{33}-21G_{21}G_{33}+21iG_{22}G_{33}+81iG_{33}G_{33}\Big)\Psi_3$$
$$+\Big(+6iG_{10}M+6G_{20}M+\frac{iD_3D_1}{3}-\frac{iD_1D_3}{3}-\frac{D_2D_3}{3}+\frac{D_3D_2}{3}$$
$$+3G_{11}D_1-3iG_{12}D_1-iG_{21}D_1-G_{22}D_1+3G_{33}D_1+iG_{11}D_2+G_{12}D_2$$
$$+3G_{21}D_2-3iG_{22}D_2-3iG_{33}D_2+G_{13}D_3-iG_{23}D_3-G_{31}D_3+iG_{32}D_3$$
$$+3D_1G_{11}-3iD_2G_{11}+3iD_1G_{12}+3D_2G_{12}+3D_3G_{13}-3iD_1G_{21}-$$
$$-3D_2G_{21}+3D_1G_{22}-3iD_2G_{22}-3iD_3G_{23}+D_3G_{31}-iD_3G_{32}-D_1G_{33}+iD_2G_{33}-$$
$$-39iG_{13}G_{11}-39G_{23}G_{11}-9iG_{31}G_{11}-9G_{32}G_{11}-21G_{13}G_{12}+21iG_{23}G_{12}+9G_{31}G_{12}-9iG_{32}G_{12}$$
$$+21iG_{11}G_{13}+21G_{12}G_{13}+39G_{21}G_{13}-39iG_{22}G_{13}-27iG_{33}G_{13}-15G_{13}G_{21}+15iG_{23}G_{21}-9G_{31}G_{21}$$
$$+9iG_{32}G_{21}-3iG_{13}G_{22}-3G_{23}G_{22}-9iG_{31}G_{22}-9G_{32}G_{22}-3G_{11}G_{23}+3iG_{12}G_{23}-15iG_{21}G_{23}$$
$$-15G_{22}G_{23}-27G_{33}G_{23}-9iG_{11}G_{31}-9G_{12}G_{31}-3G_{21}G_{31}+3iG_{22}G_{31}-9iG_{33}G_{31}+3G_{11}G_{32}$$
$$-3iG_{12}G_{32}-9iG_{21}G_{32}-9G_{22}G_{32}-9G_{33}G_{32}-3iG_{13}G_{33}-3G_{23}G_{33}+3iG_{31}G_{33}+3G_{32}G_{33}\Big)\Psi_4=0,$$

3)

$$\Big(6iG_{10}M+18G_{20}M+iD_3D_1-iD_1D_3-\frac{D_2D_3}{3}+\frac{D_3D_2}{3}$$
$$+3G_{11}D_1-5iG_{12}D_1-3iG_{21}D_1+3G_{22}D_1-G_{33}D_1-iG_{11}D_2$$

$$+G_{12}D_2 - G_{21}D_2 - 9iG_{22}D_2 - 5iG_{33}D_2 + G_{13}D_3 - 3iG_{23}D_3 + 3G_{31}D_3 - iG_{32}D_3$$
$$+3D_1G_{11} - 5iD_2G_{11} + 5iD_1G_{12} + 3D_2G_{12} + 3D_3G_{13} - 9iD_1G_{21}$$
$$+D_2G_{21} - D_1G_{22} - 9iD_2G_{22} - 9iD_3G_{23} - 3D_3G_{31} + iD_3G_{32} + 3D_1G_{33} - iD_2G_{33}$$
$$-45iG_{13}G_{11} + 9G_{23}G_{11} - 15iG_{31}G_{11} + 39G_{32}G_{11} - 21G_{13}G_{12} + 15iG_{23}G_{12} - 39G_{31}G_{12} - 15iG_{32}G_{12}$$
$$+15iG_{11}G_{13} + 21G_{12}G_{13} - 9G_{21}G_{13} - 45iG_{22}G_{13} - 45iG_{33}G_{13} + 9G_{13}G_{21} - 27iG_{23}G_{21} + 3G_{31}G_{21}$$
$$-21iG_{32}G_{21} - 33iG_{13}G_{22} - 3G_{23}G_{22} + 21iG_{31}G_{22} + 3G_{32}G_{22} - 3G_{11}G_{23} + 33iG_{12}G_{23} + 27iG_{21}G_{23}$$
$$+9G_{22}G_{23} + 9G_{33}G_{23} + 9iG_{11}G_{31} + 15G_{12}G_{31} + 9G_{21}G_{31} + 9iG_{22}G_{31} - 3iG_{33}G_{31} + 3G_{11}G_{32}$$
$$+3iG_{12}G_{32} - 3iG_{21}G_{32} + 27G_{22}G_{32} + 15G_{33}G_{32} + 3iG_{13}G_{33} + 9G_{23}G_{33} + 9iG_{31}G_{33} + 3G_{32}G_{33} + \Big)\Psi_1$$
$$+\Big(+ 2D_0M - 6iG_{30}M - iD_1D_1 - iD_2D_2 - iD_3D_3 + \frac{D_1D_2}{3} - \frac{D_2D_1}{3} -$$
$$-3G_{13}D_1 - 3iG_{23}D_1 - G_{31}D_1 - 5iG_{32}D_1 - iG_{13}D_2 + G_{23}D_2 + 5iG_{31}D_2 - G_{32}D_2 + G_{11}D_3 + iG_{12}D_3 + 3iG_{21}D_3 - 3G_{22}D_3 - 3G_{33}D_3$$
$$-3D_3G_{11} + 5iD_3G_{12} + 3D_1G_{13} - 5iD_2G_{13} - 9iD_3G_{21} + D_3G_{22} + 9iD_1G_{23} - D_2G_{23} - 3D_1G_{31} + iD_2G_{31} - iD_1G_{32} - 3D_2G_{32} - 3D_3G_{33}$$
$$+45iG_{11}G_{11} + 39G_{12}G_{11} + 9G_{21}G_{11} + 45iG_{22}G_{11} - 15iG_{33}G_{11} - 21G_{11}G_{12} + 15iG_{12}G_{12} - 15iG_{21}G_{12}$$
$$+27G_{22}G_{12} + 39G_{33}G_{12} + 15iG_{13}G_{13} + 9G_{23}G_{13} + 45iG_{31}G_{13} - 21G_{32}G_{13} + 9G_{11}G_{21} + 21iG_{12}G_{21} + 27iG_{21}G_{21}$$
$$+9G_{22}G_{21} - 3G_{33}G_{21} + 33iG_{11}G_{22} + 3G_{12}G_{22} - 3G_{21}G_{22} + 81iG_{22}G_{22} + 21iG_{33}G_{22} + 3G_{13}G_{23}$$
$$+27iG_{23}G_{23} + 9G_{31}G_{23} + 33iG_{32}G_{23} + 9iG_{13}G_{31} - 9G_{23}G_{31} + 3iG_{31}G_{31} - 15G_{32}G_{31} - 3G_{13}G_{32} - 3iG_{23}G_{32}$$
$$+15G_{31}G_{32} + 3iG_{32}G_{32} - 3iG_{11}G_{33} + 3G_{12}G_{33} + 9G_{21}G_{33} + 9iG_{22}G_{33} + 9iG_{33}G_{33}\Big)\Psi_2$$
$$+\Big(- 12iG_{10}M + \frac{2D_2D_3}{3} - \frac{2D_3D_2}{3}$$
$$-6G_{11}D_1 + 4iG_{12}D_1 - 6G_{22}D_1 + 2G_{33}D_1 - 4iG_{11}D_2 - 2G_{12}D_2 + 2G_{21}D_2 + 4iG_{33}D_2 - 2G_{13}D_3 - 6G_{31}D_3 - 4iG_{32}D_3$$
$$-6D_1G_{11} + 4iD_2G_{11} - 4iD_1G_{12} - 6D_2G_{12} - 6D_3G_{13} - 2D_2G_{21} + 2D_1G_{22} + 6D_3G_{31} + 4iD_3G_{32} - 6D_1G_{33} - 4iD_2G_{33}$$
$$+36iG_{13}G_{11} - 18G_{23}G_{11} + 12iG_{31}G_{11} - 42G_{32}G_{11} + 6G_{13}G_{12} - 12iG_{23}G_{12} + 42G_{31}G_{12} + 12iG_{32}G_{12}$$
$$-12iG_{11}G_{13} - 6G_{12}G_{13} + 18G_{21}G_{13} + 36iG_{22}G_{13} + 36iG_{33}G_{13} - 18G_{13}G_{21} - 6G_{31}G_{21} - 12iG_{32}G_{21}$$
$$+12iG_{13}G_{22} + 6G_{23}G_{22} + 12iG_{31}G_{22} - 6G_{32}G_{22} + 6G_{11}G_{23} - 12iG_{12}G_{23} - 18G_{22}G_{23} - 18G_{33}G_{23}$$
$$+36iG_{11}G_{31} + 6G_{12}G_{31} - 18G_{21}G_{31} + 36iG_{22}G_{31} - 12iG_{33}G_{31} - 42G_{11}G_{32} + 12iG_{12}G_{32} - 12iG_{21}G_{32}$$
$$-54G_{22}G_{32} + 6G_{33}G_{32} + 12iG_{13}G_{33} - 18G_{23}G_{33} + 36iG_{31}G_{33} - 42G_{32}G_{33}\Big)\Psi_3$$
$$+\Big(+ 12iG_{30}M - \frac{2D_1D_2}{3} + \frac{2D_2D_1}{3} +$$
$$+6G_{13}D_1 + 2G_{31}D_1 + 4iG_{32}D_1 - 4iG_{13}D_2 - 2G_{23}D_2 - 4iG_{31}D_2 + 2G_{32}D_2 - 2G_{11}D_3 + 4iG_{12}D_3 + 6G_{22}D_3 + 6G_{33}D_3$$
$$+6D_3G_{11} - 4iD_3G_{12} - 6D_1G_{13} + 4iD_2G_{13} - 2D_3G_{22} + 2D_2G_{23} + 6D_1G_{31} + 4iD_2G_{31} - 4iD_1G_{32} + 6D_2G_{32} + 6D_3G_{33}$$
$$-36iG_{11}G_{11} - 42G_{12}G_{11} - 18G_{21}G_{11} - 36iG_{22}G_{11} + 12iG_{33}G_{11} + 6G_{11}G_{12} - 12iG_{12}G_{12} + 12iG_{21}G_{12}$$
$$-54G_{22}G_{12} - 42G_{33}G_{12} - 12iG_{13}G_{13} - 18G_{23}G_{13} - 36iG_{31}G_{13} + 6G_{32}G_{13} - 18G_{11}G_{21} + 12iG_{12}G_{21} - 18G_{22}G_{21}$$
$$+6G_{33}G_{21} - 12iG_{11}G_{22} - 6G_{12}G_{22} + 6G_{21}G_{22} + 12iG_{33}G_{22} - 6G_{13}G_{23} - 18G_{31}G_{23} - 12iG_{32}G_{23}$$
$$+36iG_{13}G_{31} + 18G_{23}G_{31} + 12iG_{31}G_{31} - 6G_{32}G_{31} + 42G_{13}G_{32} - 12iG_{23}G_{32} + 6G_{31}G_{32} + 12iG_{32}G_{32} - 12iG_{11}G_{33}$$
$$-42G_{12}G_{33} - 18G_{21}G_{33} + 36iG_{22}G_{33} + 36iG_{33}G_{33}\Big)\Psi_4 = 0,$$

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$$\Big(- 12iG_{10}M + 12G_{20}M + \frac{2iD_3D_1}{3} - \frac{2iD_1D_3}{3} + \frac{2D_2D_3}{3} - \frac{2D_3D_2}{3}$$
$$-6G_{11}D_1 - 6iG_{12}D_1 - 2iG_{21}D_1 + 2G_{22}D_1 - 6G_{33}D_1 + 2iG_{11}D_2 - 2G_{12}D_2$$
$$-6G_{21}D_2 - 6iG_{22}D_2 - 6iG_{33}D_2 - 2G_{13}D_3 - 2iG_{23}D_3 + 2G_{31}D_3 + 2iG_{32}D_3$$
$$-6D_1G_{11} - 6iD_2G_{11} + 6iD_1G_{12} - 6D_2G_{12} - 6D_3G_{13} - 6iD_1G_{21} + 6D_2G_{21}$$
$$-6D_1G_{22} - 6iD_2G_{22} - 6iD_3G_{23} - 2D_3G_{31} - 2iD_3G_{32} + 2D_1G_{33} + 2iD_2G_{33}$$
$$-42iG_{13}G_{11} + 42G_{23}G_{11} - 18iG_{31}G_{11} + 18G_{32}G_{11} + 6G_{13}G_{12} + 6iG_{23}G_{12} - 18G_{31}G_{12} - 18iG_{32}G_{12}$$
$$+6iG_{11}G_{13} - 6G_{12}G_{13} - 42G_{21}G_{13} - 42iG_{22}G_{13} - 54iG_{33}G_{13} - 6G_{13}G_{21} - 6iG_{23}G_{21} + 18G_{31}G_{21}$$
$$+18iG_{32}G_{21} - 42iG_{13}G_{22} + 42G_{23}G_{22} - 18iG_{31}G_{22} + 18G_{32}G_{22} + 42G_{11}G_{23} + 42iG_{12}G_{23} + 6iG_{21}G_{23}$$
$$-6G_{22}G_{23} + 54G_{33}G_{23} - 18iG_{11}G_{31} + 18G_{12}G_{31} + 6G_{21}G_{31} + 6iG_{22}G_{31} - 18iG_{33}G_{31} - 6G_{11}G_{32} - 6iG_{12}G_{32}$$
$$-18iG_{21}G_{32} + 18G_{22}G_{32} + 18G_{33}G_{32} - 6iG_{13}G_{33} + 6G_{23}G_{33} + 6iG_{31}G_{33} - 6G_{32}G_{33}\Big)\Psi_1$$
$$+\Big(- 4G_{13}D_1 - 4iG_{23}D_1 - 4iG_{13}D_2 + 4G_{23}D_2 + 4G_{11}D_3 + 4iG_{12}D_3 + 4iG_{21}D_3 - 4G_{22}D_3$$
$$-4D_3G_{11} - 4iD_3G_{12} + 4D_1G_{13} + 4iD_2G_{13} - 4iD_3G_{21} + 4D_3G_{22} + 4iD_1G_{23} - 4D_2G_{23}$$
$$-36iG_{11}G_{11} + 36G_{12}G_{11} + 12G_{21}G_{11} + 12iG_{22}G_{11} - 36iG_{33}G_{11} - 12G_{11}G_{12} - 12iG_{12}G_{12} - 36iG_{21}G_{12}$$

$$+36G_{22}G_{12}+36G_{33}G_{12}-12iG_{13}G_{13}+12G_{23}G_{13}+12iG_{31}G_{13}-12G_{32}G_{13}+36G_{11}G_{21}+36iG_{12}G_{21}$$
$$+12iG_{21}G_{21}-12G_{22}G_{21}+36G_{33}G_{21}-12iG_{11}G_{22}+12G_{12}G_{22}+36G_{21}G_{22}+36iG_{22}G_{22}+36iG_{33}G_{22}$$
$$+12G_{13}G_{23}+12iG_{23}G_{23}-12G_{31}G_{23}-12iG_{32}G_{23}+12iG_{13}G_{31}-12G_{23}G_{31}-12G_{13}G_{32}-12iG_{23}G_{32}$$
$$-12iG_{11}G_{33}+12G_{12}G_{33}+12G_{21}G_{33}+12iG_{22}G_{33}\Big)\Psi_2$$
$$+\Big(+6iG_{10}M-6G_{20}M-\frac{1}{3}iD_3D_1+\frac{iD_1D_3}{3}-\frac{D_2D_3}{3}+\frac{D_3D_2}{3}$$
$$+3G_{11}D_1+3iG_{12}D_1+iG_{21}D_1-G_{22}D_1+3G_{33}D_1-iG_{11}D_2+G_{12}D_2+3G_{21}D_2$$
$$+3iG_{22}D_2+3iG_{33}D_2+G_{13}D_3+iG_{23}D_3-G_{31}D_3-iG_{32}D_3+$$
$$+3D_1G_{11}+3iD_2G_{11}-3iD_1G_{12}+3D_2G_{12}+3D_3G_{13}+3iD_1G_{21}-3D_2G_{21}$$
$$+3D_1G_{22}+3iD_2G_{22}+3iD_3G_{23}+D_3G_{31}+iD_3G_{32}-D_1G_{33}-iD_2G_{33}$$
$$+39iG_{13}G_{11}-39G_{23}G_{11}+9iG_{31}G_{11}-9G_{32}G_{11}-21G_{13}G_{12}-21iG_{23}G_{12}+9G_{31}G_{12}+9iG_{32}G_{12}$$
$$-21iG_{11}G_{13}+21G_{12}G_{13}+39G_{21}G_{13}+39iG_{22}G_{13}+27iG_{33}G_{13}-15G_{13}G_{21}-15iG_{23}G_{21}-9G_{31}G_{21}$$
$$-9iG_{32}G_{21}+3iG_{13}G_{22}-3G_{23}G_{22}+9iG_{31}G_{22}-9G_{32}G_{22}-3G_{11}G_{23}-3iG_{12}G_{23}+15iG_{21}G_{23}$$
$$-15G_{22}G_{23}-27G_{33}G_{23}+9iG_{11}G_{31}-9G_{12}G_{31}-3G_{21}G_{31}-3iG_{22}G_{31}+9iG_{33}G_{31}+3G_{11}G_{32}+3iG_{12}G_{32}$$
$$+9iG_{21}G_{32}-9G_{22}G_{32}-9G_{33}G_{32}+3iG_{13}G_{33}-3G_{23}G_{33}-3iG_{31}G_{33}+3G_{32}G_{33}\Big)\Psi_3$$
$$+\Big(+2D_0M+18iG_{30}M-iD_1D_1-iD_2D_2-iD_3D_3-D_1D_2+D_2D_1$$
$$+5G_{13}D_1+5iG_{23}D_1+3G_{31}D_1+3iG_{32}D_1-iG_{13}D_2+G_{23}D_2-3iG_{31}D_2+3G_{32}D_2+G_{11}D_3+iG_{12}D_3-5iG_{21}D_3+5G_{22}D_3+9G_{33}D_3$$
$$+5D_3G_{11}+5iD_3G_{12}-5D_1G_{13}-5iD_2G_{13}-iD_3G_{21}+D_3G_{22}+iD_1G_{23}-D_2G_{23}+9D_1G_{31}+9iD_2G_{31}-9iD_1G_{32}+9D_2G_{32}+9D_3G_{33}$$
$$+45iG_{11}G_{11}-45G_{12}G_{11}-15G_{21}G_{11}-15iG_{22}G_{11}+45iG_{33}G_{11}+15G_{11}G_{12}+15iG_{12}G_{12}+45iG_{21}G_{12}$$
$$-45G_{22}G_{12}-45G_{33}G_{12}+15iG_{13}G_{13}-15G_{23}G_{13}-15iG_{31}G_{13}+15G_{32}G_{13}+9G_{11}G_{21}+9iG_{12}G_{21}+3iG_{21}G_{21}$$
$$-3G_{22}G_{21}+9G_{33}G_{21}-3iG_{11}G_{22}+3G_{12}G_{22}+9G_{21}G_{22}+9iG_{22}G_{22}+9iG_{33}G_{22}+3G_{13}G_{23}+3iG_{23}G_{23}$$
$$-3G_{31}G_{23}-3iG_{32}G_{23}+21iG_{13}G_{31}-21G_{23}G_{31}+27iG_{31}G_{31}-27G_{32}G_{31}+33G_{13}G_{32}+33iG_{23}G_{32}$$
$$+27G_{31}G_{32}+27iG_{32}G_{32}+33iG_{11}G_{33}-33G_{12}G_{33}+21G_{21}G_{33}+21iG_{22}G_{33}+81iG_{33}G_{33}\Big)\Psi_4=0,$$

Re-grouping the terms, and introducing the new notations

$$D_1^2+D_2^2+D_3^2=\Delta,\quad D_1D_2-D_2D_1=D_{12},\quad D_2D_3-D_3D_2=D_{23},\quad D_3D_1-D_1D_3=D_{31},$$

we obtain

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$$\Big\{2M(D_0+3iG_{30})-i\Delta-\frac{D_{12}}{3}$$
$$+D_1(-3G_{13}+9iG_{23}+3G_{31}-iG_{32})+D_2(-5iG_{13}+G_{23}+iG_{31}+3G_{32})+D_3(3G_{11}+5iG_{12}-9iG_{21}-G_{22}+3G_{33})$$
$$+(3G_{13}-3iG_{23}+G_{31}-5iG_{32})D_1+(-iG_{13}-G_{23}+5iG_{31}+G_{32})D_2+(-G_{11}+iG_{12}+3iG_{21}+3G_{22}+3G_{33})D_3$$
$$+\Big(24G_{13}G_{32}+30iG_{23}G_{32}-42G_{12}G_{33}-6G_{21}G_{33}+30iG_{22}G_{33}+45iG_{11}^2-18G_{12}G_{11}-18G_{21}G_{11}$$
$$+78iG_{22}G_{11}-18iG_{33}G_{11}+15iG_{12}^2+15iG_{13}^2+27iG_{21}^2+81iG_{22}^2+27iG_{23}^2+3iG_{31}^2+3iG_{32}^2$$
$$+9iG_{33}^2+6iG_{12}G_{21}-30G_{12}G_{22}-6G_{21}G_{22}-12G_{13}G_{23}+54iG_{13}G_{31}\Big)\Big\}\Psi_1$$

$$+\Big\{+(6iG_{10}-18G_{20})M-\frac{D_{23}}{3}-iD_{31}$$
$$+D_1(3G_{11}-5iG_{12}+9iG_{21}-G_{22}+3G_{33})+D_2(5iG_{11}+3G_{12}+G_{21}+9iG_{22}+iG_{33})+D_3(3G_{13}+9iG_{23}-3G_{31}-iG_{32})$$
$$+(3G_{11}+5iG_{12}+3iG_{21}+3G_{22}-G_{33})D_1+(iG_{11}+G_{12}-G_{21}+9iG_{22}+5iG_{33})D_2+(G_{13}+3iG_{23}+3G_{31}+iG_{32})D_3$$
$$+\Big(30iG_{11}G_{13}+78iG_{13}G_{22}+6G_{11}G_{23}-48iG_{12}G_{23}+6G_{22}G_{23}+6iG_{11}G_{31}-24G_{12}G_{31}$$
$$+12G_{21}G_{31}-30iG_{22}G_{31}+42G_{11}G_{32}+12iG_{12}G_{32}+24iG_{21}G_{32}+30G_{22}G_{32}+42iG_{13}G_{33}+18G_{23}G_{33}-6iG_{31}G_{33}+18G_{32}G_{33}\Big)\Big\}\Psi_2$$

$$+\Big\{-12iG_{30}M+\frac{2D_{12}}{3}$$
$$+D_1(6G_{13}-6G_{31}-4iG_{32})+D_2(4iG_{13}-2G_{23}+4iG_{31}-6G_{32})+D_3(-6G_{11}-4iG_{12}+2G_{22}-6G_{33})$$
$$+(-6G_{13}-2G_{31}+4iG_{32})D_1+(-4iG_{13}+2G_{23}-4iG_{31}-2G_{32})D_2+(2G_{11}+4iG_{12}-6G_{22}-6G_{33})D_3$$
$$+\Big(-36iG_{11}^2+36G_{12}G_{11}+36G_{21}G_{11}-48iG_{22}G_{11}-12iG_{12}^2-12iG_{13}^2+12iG_{31}^2+12iG_{32}^2+36iG_{33}^2$$
$$+24iG_{12}G_{21}+60G_{12}G_{22}+12G_{21}G_{22}+24G_{13}G_{23}-48G_{13}G_{32}-24iG_{23}G_{32}+84G_{12}G_{33}+12G_{21}G_{33}+48iG_{22}G_{33}\Big)\Big\}\Psi_3$$

$$+\Big\{ -12iG_{10}M + \frac{2D_{23}}{3}$$
$$+D_1(-6G_{11}+4iG_{12}+2G_{22}-6G_{33}) + D_2(-4iG_{11}-6G_{12}-2G_{21}+4iG_{33}) + D_3(-6G_{13}+6G_{31}-4iG_{32})$$
$$+(-6G_{11}-4iG_{12}-6G_{22}+2G_{33})D_1 + (4iG_{11}-2G_{12}+2G_{21}-4iG_{33})D_2 + (-2G_{13}-6G_{31}+4iG_{32})D_3$$
$$+\Big(-24iG_{11}G_{13}-48iG_{13}G_{22}-12G_{11}G_{23}+24iG_{12}G_{23}-12G_{22}G_{23}-48iG_{11}G_{31}+48G_{12}G_{31}-24G_{21}G_{31}-48iG_{22}G_{31}-84G_{11}G_{32}$$
$$-24iG_{12}G_{32}+24iG_{21}G_{32}-60G_{22}G_{32}-48iG_{13}G_{33}-36G_{23}G_{33}-24iG_{31}G_{33}-36G_{32}G_{33}\Big)\Big\}\Psi_4 = 0,$$

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$$\Big\{ +(6iG_{10}+18G_{20})M - \frac{D_{23}}{3} + iD_{31}$$
$$+D_1(3G_{11}+5iG_{12}-9iG_{21}-G_{22}+3G_{33})+D_2(-5iG_{11}+3G_{12}+G_{21}-9iG_{22}-iG_{33})+D_3(3G_{13}-9iG_{23}-3G_{31}+iG_{32})$$
$$+(3G_{11}-5iG_{12}-3iG_{21}+3G_{22}-G_{33})D_1+(-iG_{11}+G_{12}-G_{21}-9iG_{22}-5iG_{33})D_2+(G_{13}-3iG_{23}+3G_{31}-iG_{32})D_3$$
$$+\Big(-30iG_{11}G_{13}-78iG_{13}G_{22}+6G_{11}G_{23}+48iG_{12}G_{23}+6G_{22}G_{23}-6iG_{11}G_{31}-24G_{12}G_{31}+12G_{21}G_{31}+30iG_{22}G_{31}+42G_{11}G_{32}$$
$$-12iG_{12}G_{32}-24iG_{21}G_{32}+30G_{22}G_{32}-42iG_{13}G_{33}+18G_{23}G_{33}+6iG_{31}G_{33}+18G_{32}G_{33}\Big)\Big\}\Psi_1$$

$$+\Big\{2M(D_0-3iG_{30}) - i\Delta + \frac{D_{12}}{3}$$
$$+D_1(3G_{13}+9iG_{23}-3G_{31}-iG_{32}) + D_2(-5iG_{13}-G_{23}+iG_{31}-3G_{32}) + D_3(-3G_{11}+5iG_{12}-9iG_{21}+G_{22}-3G_{33})$$
$$+(-3G_{13}-3iG_{23}-G_{31}-5iG_{32})D_1 + (-iG_{13}+G_{23}+5iG_{31}-G_{32})D_2 + (G_{11}+iG_{12}+3iG_{21}-3G_{22}-3G_{33})D_3$$
$$+\Big(45iG_{11}^2+18G_{12}G_{11}+18G_{21}G_{11}+78iG_{22}G_{11}-18iG_{33}G_{11}+15iG_{12}^2+15iG_{13}^2+27iG_{21}^2+81iG_{22}^2+27iG_{23}^2+3iG_{31}^2+3iG_{32}^2$$
$$+9iG_{33}^2+6iG_{12}G_{21}+30G_{12}G_{22}+6G_{21}G_{22}+12G_{13}G_{23}$$
$$+54iG_{13}G_{31}-24G_{13}G_{32}+30iG_{23}G_{32}+42G_{12}G_{33}+6G_{21}G_{33}+30iG_{22}G_{33}\Big)\Big\}\Psi_2$$

$$+\Big\{ -12iG_{10}M + \frac{2D_{23}}{3}$$
$$+D_1(-6G_{11}-4iG_{12}+2G_{22}-6G_{33}) + D_2(4iG_{11}-6G_{12}-2G_{21}-4iG_{33}) + D_3(-6G_{13}+6G_{31}+4iG_{32})$$
$$+(-6G_{11}+4iG_{12}-6G_{22}+2G_{33})D_1 + (-4iG_{11}-2G_{12}+2G_{21}+4iG_{33})D_2 + (-2G_{13}-6G_{31}-4iG_{32})D_3$$
$$+\Big(24iG_{11}G_{13}+48iG_{13}G_{22}-12G_{11}G_{23}-24iG_{12}G_{23}-12G_{22}G_{23}+48iG_{11}G_{31}+48G_{12}G_{31}-24G_{21}G_{31}+48iG_{22}G_{31}-84G_{11}G_{32}$$
$$+24iG_{12}G_{32}-24iG_{21}G_{32}-60G_{22}G_{32}+48iG_{13}G_{33}-36G_{23}G_{33}+24iG_{31}G_{33}-36G_{32}G_{33}\Big)\Big\}\Psi_3$$

$$+\Big\{ +12iG_{30}M - \frac{2D_{12}}{3}$$
$$+D_1(-6G_{13}+6G_{31}-4iG_{32}) + D_2(4iG_{13}+2G_{23}+4iG_{31}+6G_{32}) + D_3(6G_{11}-4iG_{12}-2G_{22}+6G_{33})$$
$$+(6G_{13}+2G_{31}+4iG_{32})D_1 + (-4iG_{13}-2G_{23}-4iG_{31}+2G_{32})D_2 + (-2G_{11}+4iG_{12}+6G_{22}+6G_{33})D_3$$
$$+\Big(-36iG_{11}^2-36G_{12}G_{11}-36G_{21}G_{11}-48iG_{22}G_{11}-12iG_{12}^2-12iG_{13}^2+12iG_{31}^2+12iG_{32}^2+36iG_{33}^2+24iG_{12}G_{21}-60G_{12}G_{22}-12G_{21}G_{22}$$
$$-24G_{13}G_{23}+48G_{13}G_{32}-24iG_{23}G_{32}-84G_{12}G_{33}-12G_{21}G_{33}+48iG_{22}G_{33}\Big)\Big\}\Psi_4 = 0,$$

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$$\Big\{ +D_1(4iG_{23}-4G_{13}) + D_2(4iG_{13}+4G_{23}) + D_3(4G_{11}-4iG_{12}-4iG_{21}-4G_{22})$$
$$+(4G_{13}-4iG_{23})D_1 + (-4iG_{13}-4G_{23})D_2 + (-4G_{11}+4iG_{12}+4iG_{21}+4G_{22})D_3$$
$$+\Big(-36iG_{11}^2-24G_{12}G_{11}-48G_{21}G_{11}-48iG_{33}G_{11}-12iG_{12}^2-12iG_{13}^2+12iG_{21}^2+36iG_{22}^2+12iG_{23}^2-48G_{12}G_{22}-24G_{21}G_{22}-24G_{13}G_{23}$$
$$+24iG_{13}G_{31}+24G_{23}G_{31}+24G_{13}G_{32}-24iG_{23}G_{32}-48G_{12}G_{33}-48G_{21}G_{33}+48iG_{22}G_{33}\Big)\Big\}\Psi_1$$

$$+\Big\{(-12iG_{10}-12G_{20})M + \frac{2D_{23}}{3} - \frac{2iD_{31}}{3}$$
$$+D_1(-6G_{11}-6iG_{12}+6iG_{21}-6G_{22}+2G_{33})+D_2(6iG_{11}-6G_{12}+6G_{21}+6iG_{22}-2iG_{33})+D_3(-6G_{13}+6iG_{23}-2G_{31}+2iG_{32})$$
$$+(-6G_{11}+6iG_{12}+2iG_{21}+2G_{22}-6G_{33})D_1+(-2iG_{11}-2G_{12}-6G_{21}+6iG_{22}+6iG_{33})D_2+(-2G_{13}+2iG_{23}+2G_{31}-2iG_{32})D_3$$
$$+\Big(36iG_{11}G_{13}-48G_{13}G_{21}+84iG_{13}G_{22}+84G_{11}G_{23}-48iG_{12}G_{23}+36G_{22}G_{23}+36iG_{11}G_{31}+24G_{21}G_{31}+12iG_{22}G_{31}+12G_{11}G_{32}$$
$$+24iG_{12}G_{32}+36G_{22}G_{32}+60iG_{13}G_{33}+60G_{23}G_{33}+12iG_{31}G_{33}+12G_{32}G_{33}\Big)\Big\}\Psi_2$$

$$+\Big\{2M(D_0-9iG_{30})M - i\Delta + D_{12}$$

$$+D_1(5G_{13}+iG_{23}-9G_{31}-9iG_{32})+D_2(-5iG_{13}+G_{23}+9iG_{31}-9G_{32})+D_3(-5G_{11}+5iG_{12}-iG_{21}-G_{22}-9G_{33})$$
$$+(-5G_{13}+5iG_{23}-3G_{31}+3iG_{32})D_1+(-iG_{13}-G_{23}-3iG_{31}-3G_{32})D_2+(-G_{11}+iG_{12}-5iG_{21}-5G_{22}-9G_{33})D_3$$
$$+\Big(45iG_{11}^2+30G_{12}G_{11}+6G_{21}G_{11}-18iG_{22}G_{11}+78iG_{33}G_{11}+15iG_{12}^2+15iG_{13}^2+3iG_{21}^2+9iG_{22}^2+3iG_{23}^2+27iG_{31}^2+27iG_{32}^2$$
$$+81iG_{33}^2+54iG_{12}G_{21}+42G_{12}G_{22}-6G_{21}G_{22}+12G_{13}G_{23}+6iG_{13}G_{31}$$
$$+24G_{23}G_{31}-48G_{13}G_{32}+30iG_{23}G_{32}+78G_{12}G_{33}-30G_{21}G_{33}+30iG_{22}G_{33}\Big)\Big\}\Psi_3$$

$$+\Big\{+(6iG_{10}+6G_{20})M-\frac{D_{23}}{3}+\frac{iD_{31}}{3}$$
$$+D_1(3G_{11}+3iG_{12}-3iG_{21}+3G_{22}-G_{33})+D_2(-3iG_{11}+3G_{12}-3G_{21}-3iG_{22}+iG_{33})+D_3(3G_{13}-3iG_{23}+G_{31}-iG_{32})$$
$$+(3G_{11}-3iG_{12}-iG_{21}-G_{22}+3G_{33})D_1+(iG_{11}+G_{12}+3G_{21}-3iG_{22}-3iG_{33})D_2+(G_{13}-iG_{23}-G_{31}+iG_{32})D_3$$
$$+\Big(-18iG_{11}G_{13}+24G_{13}G_{21}-42iG_{13}G_{22}-42G_{11}G_{23}+24iG_{12}G_{23}-18G_{22}G_{23}-18iG_{11}G_{31}-12G_{21}G_{31}-6iG_{22}G_{31}-6G_{11}G_{32}$$
$$-12iG_{12}G_{32}-18G_{22}G_{32}-30iG_{13}G_{33}-30G_{23}G_{33}-6iG_{31}G_{33}-6G_{32}G_{33}\Big)\Big\}\Psi_4=0,$$

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$$\Big\{+(12G_{20}-12iG_{10})M+\frac{2D_{23}}{3}+\frac{2iD_{31}}{3}$$
$$+D_1(-6G_{11}+6iG_{12}-6iG_{21}-6G_{22}+2G_{33})+D_2(-6iG_{11}-6G_{12}+6G_{21}-6iG_{22}+2iG_{33})+D_3(-6G_{13}-6iG_{23}-2G_{31}-2iG_{32})$$
$$+(-6G_{11}-6iG_{12}-2iG_{21}+2G_{22}-6G_{33})D_1+(2iG_{11}-2G_{12}-6G_{21}-6iG_{22}-6iG_{33})D_2+(-2G_{13}-2iG_{23}+2G_{31}+2iG_{32})D_3$$
$$+\Big(-36iG_{11}G_{13}-48G_{13}G_{21}-84iG_{13}G_{22}+84G_{11}G_{23}+48iG_{12}G_{23}+36G_{22}G_{23}-36iG_{11}G_{31}+24G_{21}G_{31}-12iG_{22}G_{31}+12G_{11}G_{32}$$
$$-24iG_{12}G_{32}+36G_{22}G_{32}-60iG_{13}G_{33}+60G_{23}G_{33}-12iG_{31}G_{33}+12G_{32}G_{33}\Big)\Big\}\Psi_1$$

$$+\Big\{+D_1(4G_{13}+4iG_{23})+D_2(4iG_{13}-4G_{23})+D_3(-4G_{11}-4iG_{12}-4iG_{21}+4G_{22})$$
$$+(-4G_{13}-4iG_{23})D_1+(4G_{23}-4iG_{13})D_2+(4G_{11}+4iG_{12}+4iG_{21}-4G_{22})D_3$$
$$+\Big(-36iG_{11}^2+24G_{12}G_{11}+48G_{21}G_{11}-48iG_{33}G_{11}-12iG_{12}^2-12iG_{13}^2+12iG_{21}^2+36iG_{22}^2+12iG_{23}^2+48G_{12}G_{22}+24G_{21}G_{22}+24G_{13}G_{23}$$
$$+24iG_{13}G_{31}-24G_{23}G_{31}-24G_{13}G_{32}-24iG_{23}G_{32}+48G_{12}G_{33}+48G_{21}G_{33}+48iG_{22}G_{33}\Big)\Big\}\Psi_2$$

$$+\Big\{+(6iG_{10}-6G_{20})M-\frac{D_{23}}{3}-\frac{iD_{31}}{3}$$
$$+D_1(3G_{11}-3iG_{12}+3iG_{21}+3G_{22}-G_{33})+D_2(3iG_{11}+3G_{12}-3G_{21}+3iG_{22}-iG_{33})+D_3(3G_{13}+3iG_{23}+G_{31}+iG_{32})$$
$$+(3G_{11}+3iG_{12}+iG_{21}-G_{22}+3G_{33})D_1+(-iG_{11}+G_{12}+3G_{21}+3iG_{22}+3iG_{33})D_2+(G_{13}+iG_{23}-G_{31}-iG_{32})D_3$$
$$+\Big(18iG_{11}G_{13}+24G_{13}G_{21}+42iG_{13}G_{22}-42G_{11}G_{23}-24iG_{12}G_{23}-18G_{22}G_{23}+18iG_{11}G_{31}-12G_{21}G_{31}+6iG_{22}G_{31}-6G_{11}G_{32}$$
$$+12iG_{12}G_{32}-18G_{22}G_{32}+30iG_{13}G_{33}-30G_{23}G_{33}+6iG_{31}G_{33}-6G_{32}G_{33}\Big)\Big\}\Psi_3$$

$$+\Big\{2M(D_0+9iG_{30})-i\Delta-D_{12}$$
$$+D_1(-5G_{13}+iG_{23}+9G_{31}-9iG_{32})+D_2(-5iG_{13}-G_{23}+9iG_{31}+9G_{32})+D_3(5G_{11}+5iG_{12}-iG_{21}+G_{22}+9G_{33})$$
$$+(5G_{13}+5iG_{23}+3G_{31}+3iG_{32})D_1+(-iG_{13}+G_{23}-3iG_{31}+3G_{32})D_2+(G_{11}+iG_{12}-5iG_{21}+5G_{22}+9G_{33})D_3$$
$$+\Big(45iG_{11}^2-30G_{12}G_{11}-6G_{21}G_{11}-18iG_{22}G_{11}+78iG_{33}G_{11}+15iG_{12}^2+15iG_{13}^2+3iG_{21}^2+9iG_{22}^2+3iG_{23}^2+27iG_{31}^2+27iG_{32}^2$$
$$+81iG_{33}^2+54iG_{12}G_{21}-42G_{12}G_{22}+6G_{21}G_{22}-12G_{13}G_{23}+6iG_{13}G_{31}-24G_{23}G_{31}$$
$$+48G_{13}G_{32}+30iG_{23}G_{32}-78G_{12}G_{33}+30G_{21}G_{33}+30iG_{22}G_{33}\Big)\Big\}\Psi_4=0,$$

We can present this system in the matrix form

$$2M(D_0+A_0)\Psi-i\Delta\Psi+(a_1D_{23}+a_2D_{31}+a_3D_{12})\Psi$$
$$+D_1A_1\Psi+B_1D_1\Psi+D_2A_2\Psi+B_2D_2\Psi+D_3A_3\Psi+B_3D_3\Psi+\Sigma\Psi=0, \qquad (34)$$

where

$$\Psi=\left|\begin{array}{c}\Psi_1\\ \Psi_2\\ \Psi_3\\ \Psi_4\end{array}\right|,\quad A_0=\left|\begin{array}{cccc}3iG_{30} & 3i(G_{10}+3iG_{20}) & -6iG_{30} & -6iG_{10}\\ 3i(G_{10}-3iG_{20}) & -3iG_{30} & -6iG_{10} & 6iG_{30}\\ 0 & -6i(G_{10}-iG_{20}) & -9iG_{30} & 3i(G_{10}-iG_{20})\\ 6(G_{20}-iG_{10}) & 0 & -3(G_{20}-iG_{10}) & 9iG_{30}\end{array}\right|,$$

and so on.

For 4×4 matrices a_1, a_2, a_3, the commutators are valid

$$a_1 a_2 - a_2 a_1 = i\frac{2}{3} a_3, \ ...;$$

by changing normalization

$$S_1 = \frac{3}{2} a_1, \quad S_2 = \frac{3}{2} a_2, \quad S_3 = \frac{3}{2} a_3,$$

we obtain three spin matrices with the needed commutation relations

$$S_1 S_2 - S_2 S_1 = i S_3, ... \ .$$

Explicitly, they read

$$S_1 = \begin{vmatrix} 0 & -\frac{1}{2} & 0 & 1 \\ -\frac{1}{2} & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 1 & 0 & -\frac{1}{2} & 0 \end{vmatrix}, \quad S_2 = \begin{vmatrix} 0 & -\frac{3i}{2} & 0 & 0 \\ \frac{3i}{2} & 0 & 0 & 0 \\ 0 & -i & 0 & \frac{i}{2} \\ i & 0 & -\frac{i}{2} & 0 \end{vmatrix}, \quad S_3 = \begin{vmatrix} -\frac{1}{2} & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & -1 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{vmatrix}. \tag{35}$$

We can readily find transformation

$$\bar{\Psi} = S\Psi, \quad S = \begin{vmatrix} 0 & 0 & 0 & 1 \\ -2 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}; \tag{36}$$

to a basis, in which the matrix $\bar{S}_3$ becomes diagonal and new spin components are given by the formulas

$$\bar{S}_1 = \begin{vmatrix} 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{3}{2} & 0 & -1 & 0 \\ 0 & -1 & 0 & -\frac{3}{2} \\ 0 & 0 & -\frac{1}{2} & 0 \end{vmatrix}, \quad \bar{S}_2 = \begin{vmatrix} 0 & -\frac{i}{2} & 0 & 0 \\ \frac{3i}{2} & 0 & -i & 0 \\ 0 & i & 0 & -\frac{3i}{2} \\ 0 & 0 & \frac{i}{2} & 0 \end{vmatrix}, \quad \bar{S}_3 = \begin{vmatrix} -3/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 3/2 \end{vmatrix}. \tag{37}$$

This basis is called as cyclic one; sometimes it is more convenient basis than the Cartesian one.

The Pauli like equation for spin 3/2 particle in curved space has the structure

$$i(D_0 + A_0)\Psi = -\frac{1}{2M}\Delta\Psi - \frac{i}{3M}\Big(S_1 D_{23} + S_2 D_{31} + S_3 D_{12}\Big)\Psi -$$
$$-\frac{i}{2M}\Big[D_1 A_1 + D_2 A_2 + D_3 A_3 + B_1 D_1 + B_2 D_2 + B_3 D_3 + +\Sigma\Big]\Psi, \tag{38}$$

this equation evidently include the tetrad components of the Ricci tensor R_{ab} and scalar $R^a_a = R$.

Transformation to cyclic basis is reached as follows

$$i(D_0 + \bar{A}_0)\bar{\Psi} = -\frac{1}{2M}\Delta\bar{\Psi} - \frac{i}{3M}\Big(\bar{S}_1 D_{23} + \bar{S}_2 D_{31} + \bar{S}_3 D_{12}\Big)\bar{\Psi} -$$
$$-\frac{i}{2M}\Big[D_1 \bar{A}_1 + D_2 \bar{A}_2 + D_3 \bar{A}_3 + \bar{B}_1 D_1 + \bar{B}_2 D_2 + \bar{B}_3 D_3 + \bar{\Sigma}\Big]\bar{\Psi}, \tag{39}$$

where

$$\bar{\Psi} = S\Psi, \quad \Psi = S^{-1}\bar{\Psi}, \quad \bar{A}_0 = SA_0S^{-1}, , \quad \bar{S}_i = SS_iS^{-1},$$

$$\bar{A}_i = SA_iS^{-1}, \quad \bar{B}_i = SB_iS^{-1}, \quad \bar{\Sigma} = S\Sigma S^{-1}.$$

5. Detailed structure of the resulting equation

The matrix A_0 contains the Ricci rotation coefficients G_{j0}, which differ from zero only in non-static metrics. The matrices A_1, A_2, A_3 and B_1, B_2, B_3 are determined by 9 Ricci coefficients

$$G_{11}, \quad G_{12}, \quad G_{13}, \quad G_{21}, \quad G_{22}, \quad G_{23}, \quad G_{21}, \quad G_{22}, \quad G_{23}. \tag{40}$$

So, we can search for linear expansions of all these matrices in terms of 9 basic elements:

$$
\begin{array}{c}
S_1 = t_1, \quad S_2 = t_2, \quad S_3 = t_3, \\
S_1^2 = t_4, \quad S_2^2 = t_5, \quad S_3^2 = t_6, \\
S_2 S_3 = t_7, \quad S_3 S_1 = t_8, \quad S_1 S_2 = t_9,
\end{array}
\tag{41}
$$

We readily find the needed expansions:

$$
A_0 = x_1 t_1 + x_2 t_2 + x_3 t_3 + ... + x_9 t_9,
$$

$$
x_1 = -6iG_{10}, \quad x_2 = -6iG_{20}, \quad x_3 = -6iG_{30},
$$

$$
x_4 = 0, \; x_5 = 0, \; x_6 = 0, \; x_7 = 0, \; x_8 = 0, \; x_9 = 0,
$$

$$
2iM(D_0 + A_0)\Psi = 2iM\Big[D_0 + 6(G_{10}S_1 + G_{20}S_2 + G_{30}S_3)\Big]\Psi; \tag{42}
$$

$$
A_1 = y_1 t_1 + y_2 t_2 + y_3 t_3 + ... + y_9 t_9,
$$

$$
y_1 = -6G_{11} - 4G_{22}, \quad y_2 = -6G_{21}, \quad y_3 = 4G_{13} - 6G_{31},
$$

$$
y_4 = 0, \quad y_5 = 4iG_{23}, \quad y_6 = -4iG_{32},
$$

$$
y_7 = -4i(G_{22} - G_{33}), \quad y_8 = -4iG_{12}, \quad y_9 = 4iG_{13};
$$

$$
A_2 = z_1 t_1 + z_2 t_2 + z_3 t_3 + ... + z_9 t_9,
$$

$$
z_1 = 4G_{21} - 6G_{12}, \quad z_2 = -6G_{22} - 4G_{33}, \quad z_3 = -6G_{32},
$$

$$
z_4 = -4iG_{13}, \quad z_5 = 0, \quad z_6 = 4iG_{31},
$$

$$
z_7 = 4iG_{21}, \quad z_8 = 4i(G_{11} - G_{33}), \quad z_9 = -4iG_{23};
$$

$$
A_3 = h_1 t_1 + h_2 t_2 + h_3 t_3 + ... + h_9 t_9,
$$

$$
h_1 = -6G_{13}, \quad h_2 = 4G_{32} - 6G_{23}, \quad h_3 = -4G_{11} - 6G_{33},
$$

$$
h_4 = 4iG_{12}, \quad h_5 = -4iG_{21}, \quad h_6 = 0,
$$

$$
h_7 = -4iG_{31}, \quad h_8 = 4iG_{32}, \quad h_9 = -4i(G_{11} - G_{22});
$$

$$
B_1 = y_1 t_1 + y_2 t_2 + y_3 t_3 + ... + y_9 t_9,
$$

$$
y_1 = -6G_{11} - 4G_{33}, \quad y_2 = -2G_{21}, \quad y_3 = -2(2G_{13} + G_{31}),
$$

$$
y_4 = \frac{8}{5}i(G_{23} - G_{32}), \quad y_5 = -\frac{4}{5}i(3G_{23} + 2G_{32}), \quad y_6 = \frac{4}{5}i(2G_{23} + 3G_{32}),
$$

$$
y_7 = 4i(G_{22} - G_{33}), \quad y_8 = 4iG_{12}, \quad y_9 = -4iG_{13};
$$

$$B_2 = z_1 t_1 + z_2 t_2 + z_3 t_3 + ... + z_9 t_9,$$

$$z_1 = -2(G_{12} + 2G_{21}), \quad z_2 = -4G_{11} - 6G_{22}, \quad z_3 = -2G_{32},$$

$$z_4 = \frac{4}{5}i(3G_{13} + 2G_{31}), \quad z_5 = -\frac{8}{5}i(G_{13} - G_{31}), \quad z_6 = -\frac{4}{5}i(2G_{13} + 3G_{31}),$$

$$z_7 = -4iG_{21}, \quad z_8 = -4i(G_{11} - G_{33}), \quad z_9 = 4iG_{23};$$

$$B_3 = h_1 t_1 + h_2 t_2 + h_3 t_3 + ... + h_9 t_9,$$

$$h_1 = -2G_{13}, \quad h_2 = -2(G_{23} + 2G_{32}), \quad h_3 = -4G_{22} - 6G_{33},$$

$$h_4 = -\frac{4}{5}i(3G_{12} + 2G_{21}), \quad h_5 = \frac{4}{5}i(2G_{12} + 3G_{21}), \quad h_6 = \frac{8}{5}i(G_{12} - G_{21}),$$

$$h_7 = 4iG_{31}, \quad h_8 = -4iG_{32}, \quad h_9 = 4i(G_{11} - G_{22});$$

$$A_1 + B_1 = y_1 t_1 + y_2 t_2 + y_3 t_3 + ... + y_9 t_9,$$

$$y_1 = -4(3G_{11} + G_{22} + G_{33}), \quad y_2 = -8G_{21}, \quad y_3 = -8G_{31},$$

$$y_4 = \frac{8}{5}i(G_{23} - G_{32}), \quad y_5 = \frac{8}{5}i(G_{23} - G_{32}), \quad y_6 = \frac{8}{5}i(G_{23} - G_{32}),$$

$$y_7 = 0, \quad y_8 = 0, \quad y_9 = 0;$$

$$A_2 + B_2 = z_1 t_1 + z_2 t_2 + z_3 t_3 + ... + z_9 t_9,$$

$$z_1 = -8G_{12}, \quad z_2 = -4(3G_{22} + G_{11} + G_{33}), \quad z_3 = -8G_{32},$$

$$z_4 = -\frac{8}{5}i(G_{13} - G_{31}), \quad z_5 = -\frac{8}{5}i(G_{13} - G_{31}), \quad z_6 = -\frac{8}{5}i(G_{13} - G_{31}),$$

$$z_7 = 0, \quad z_8 = 0, \quad z_9 = 0;$$

$$A_3 + B_3 = h_1 t_1 + h_2 t_2 + h_3 t_3 + ... + h_9 t_9,$$

$$h_1 = -8G_{13}, \quad h_2 = -8G_{23}, \quad h_3 = -4(3G_{33} + G_{11} + G_{22}),$$

$$h_4 = \frac{8}{5}i(G_{12} - G_{21}), \quad h_5 = \frac{8}{5}i(G_{12} - G_{21}), \quad h_6 = \frac{8}{5}i(G_{12} - G_{21}),$$

$$h_7 = 0, \quad h_8 = 0, \quad h_9 = 0;$$

$$\Sigma = x_1 t_1 + x_2 t_2 + x_3 t_3 + ... + x_9 t_9,$$

$$x_1 = 12\Big(5G_{11}G_{23} - 3G_{13}G_{21} + 3G_{33}G_{23} + 2G_{22}G_{23} - G_{11}G_{32} + G_{31}G_{12} + G_{31}G_{21} + G_{22}G_{32}\Big),$$

$$x_2 = 12\Big(5G_{22}G_{31} - 3G_{21}G_{32} + 3G_{11}G_{31} + 2G_{33}G_{31} - G_{22}G_{13} + G_{12}G_{23} + G_{12}G_{32} + G_{33}G_{13}\Big),$$

$$x_3 = 12\Big(5G_{33}G_{12} - 3G_{13}G_{32} + 3G_{22}G_{12} + 2G_{11}G_{12} - G_{33}G_{21} + G_{23}G_{13} + G_{23}G_{31} + G_{11}G_{21}\Big),$$

$$x_4 = 12i\Big(3G_{11}^2 + 2G_{11}G_{22} + 2G_{33}G_{11} - 2G_{22}G_{33} + 2G_{23}G_{32} + G_{12}^2 + G_{13}^2\Big),$$

$$x_5 = 12i\Big(3G_{22}^2 + 2G_{11}G_{22} - 2G_{33}G_{11} + 2G_{22}G_{33} + 2G_{13}G_{31} + G_{21}^2 + G_{23}^2\Big),$$

$$x_6 = 12i\Big(3G_{33}^2 + 2G_{11}G_{33} + 2G_{33}G_{22} - 2G_{22}G_{11} + 2G_{12}G_{21} + G_{31}^2 + G_{32}^2\Big),$$

$$x_7 = 24i\Big(2G_{11}G_{23} + 2G_{11}G_{32} + 2G_{22}G_{32} + 2G_{33}G_{23} - G_{12}G_{31} - G_{13}G_{21} + G_{21}G_{31} + G_{32}G_{33} + G_{22}G_{23}\Big),$$

$$x_8 = 24i\Big(2G_{11}G_{31} + 2G_{22}G_{13} + 2G_{22}G_{31} + 2G_{33}G_{13} + G_{11}G_{13} - G_{12}G_{23} + G_{12}G_{32} - G_{21}G_{32} + G_{33}G_{31}\Big),$$

$$x_9 = 24i\Big(2G_{11}G_{21} + 2G_{33}G_{12} + 2G_{22}G_{12} + 2G_{33}G_{21} + G_{13}G_{23} - G_{13}G_{32} + G_{21}G_{22} - G_{23}G_{31} + G_{11}G_{12}\Big).$$

6. Examples: magnetic and electric uniform fields

Cylindrical coordinates $x^\alpha = (t, r, \phi, z)$, the relevant tetrad, Ricci rotation coefficients, and the uniform electric field are determined by relations

$$dS^2 = dt^2 - dr^2 - r^2 d\phi^2 - dz^2, g_{\alpha\beta} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}, \; e^\alpha_{(a)} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix},$$

$$0 \Longrightarrow t, \quad 1 \Longrightarrow r, \quad 2 \Longrightarrow \phi, \quad 3 \Longrightarrow z, \quad \gamma_{122} = G_{32} = +\frac{1}{r}, \tag{43}$$

$$D_0 = \partial_0 + ieA_0, \quad D_1 = \partial_r + ieA_r, \quad D_2 = \frac{1}{r}(\partial_\phi + ieA_\phi), \quad D_3 = \partial_z + ieA_z,$$

$$\begin{gathered} \gamma_{230} = G_{10} = 0, \quad \gamma_{310} = G_{20} = 0, \quad \gamma_{120} = G_{30} = 0, \\ \gamma_{231} = G_{11} = 0, \quad \gamma_{311} = G_{21} = 0, \quad \gamma_{121} = G_{31} = 0, \\ \gamma_{232} = G_{12} = 0, \quad \gamma_{312} = G_{22} = 0, \quad \gamma_{122} = G_{32} = \frac{1}{r}, \\ \gamma_{233} = G_{13} = 0, \quad \gamma_{313} = G_{23} = 0, \quad \gamma_{123} = G_{33} = 0. \end{gathered} \tag{44}$$

To simplify the problem, let us consider the situation of presence of uniform magnetic and electric fields along the exes z; then we have

$$D_0 = \partial_t + ieEz, \quad D_1 = \partial_r, \quad D_2 = \frac{1}{r}\Big(\partial_\phi + ie\frac{Br^2}{2}\Big), \quad D_3 = \partial_z,$$

$$A_0 = 0, \quad \Delta = \frac{\partial^2}{\partial r^2} + \Big(\frac{1}{r}\partial_\phi + \frac{ieB}{2}r\Big)^2 + \frac{\partial^2}{\partial z^2},$$

$$D_{23} = 0, \quad D_{31} = 0, \quad D_{12} = \partial_r\Big(\frac{1}{r}\partial_\phi + \frac{ieB}{2}r\Big) - \Big(\frac{1}{r}\partial_\phi + \frac{ieB}{2}r\Big)\partial_r = -\frac{1}{r^2}\Big(\partial_\phi - ie\frac{Br^2}{2}\Big),$$

$$(\partial_r A_1) = \frac{i}{r^2}\begin{vmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 9 \end{vmatrix}, \quad A_1 + B_1 = -\frac{6i}{r}\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = -\frac{6i}{r}I,$$

$$A_2 + B_2 = \frac{4}{r}\begin{vmatrix} 1 & 0 & -2 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 3 \end{vmatrix}, \quad A_3 + B_3 = 0, \quad \Sigma = \frac{3i}{r^2}\begin{vmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 9 \end{vmatrix}$$

So the main equation takes the form

$$i(\partial_t + ieEz)\Psi = -\frac{1}{2M}\Big[\frac{\partial^2}{\partial r^2} + \Big(\frac{1}{r}\partial_\phi + \frac{ieB}{2}r\Big)^2 + \frac{\partial^2}{\partial z^2}\Big]\Psi + \frac{i}{3M}\frac{1}{r^2}\Big(\partial_\phi - ie\frac{Br^2}{2}\Big)S_3\Psi -$$
$$-\frac{i}{2M}\Big[(\partial_r A_1) + (A_1 + B_1)\partial_r + (A_2 + B_2)\frac{1}{r}\Big(\partial_\phi + ie\frac{Br^2}{2}\Big) + (A_3 + B_3)\partial_z + \Sigma\Big]\Psi = 0. \tag{45}$$

В ситуации с циклической симметрией удобнее использовать циклический базис; основное уравнение преобразуется так:

$$i(\partial_t + ieEz)\bar\Psi = -\frac{1}{2M}\Big[\frac{\partial^2}{\partial r^2} + \Big(\frac{1}{r}\partial_\phi + \frac{ieB}{2}r\Big)^2 + \frac{\partial^2}{\partial z^2}\Big]\bar\Psi + \frac{i}{3M}\frac{1}{r^2}\Big(\partial_\phi - ie\frac{Br^2}{2}\Big)\bar S_3\bar\Psi -$$

$$-\frac{i}{2M}\Big[(\partial_r \bar A_1) + (\bar A_1 + \bar B_1)\partial_r + (\bar A_2 + \bar B_2)\frac{1}{r}\Big(\partial_\phi + ie\frac{Br^2}{2}\Big) + (\bar A_3 + \bar B_3)\partial_z + \bar\Sigma\Big]\bar\Psi = 0, \tag{46}$$

правило преобразования такое:

$$\bar\Psi = S\Psi, \quad \Psi = S^{-1}\bar\Psi, \quad \bar A_0 = SA_0S^{-1},, \quad \bar S_i = SS_iS^{-1},$$

$$\bar A_i = SA_iS^{-1}, \quad \bar B_i = SB_iS^{-1}, \quad \bar\Sigma = S\Sigma S^{-1}, \quad S = \begin{vmatrix} 0 & 0 & 0 & 1 \\ -2 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix};$$

Явный вид преобразованных матриц следующий:

$$(\partial_r \bar A_1) = \frac{i}{r^2}\begin{vmatrix} 9 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 9 \end{vmatrix}, \ \bar A_1 + \bar B_1 = -\frac{6i}{r}I, \ \bar A_2 + \bar B_2 = -\frac{8}{r}\begin{vmatrix} -3/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 3/2 \end{vmatrix},$$
$$\bar A_3 + \bar B_3 = 0, \quad \bar\Sigma = \frac{3i}{r^2}\begin{vmatrix} 9 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 9 \end{vmatrix}, \quad \bar S_3 = \begin{vmatrix} -3/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 3/2 \end{vmatrix}. \tag{47}$$

Полученная структура нерелятивистского уравнения (46) означает, что уравнение для 4-х компонентной волновой функции разбивается на 4 несвязанных уравнения с похожей структурой.

7. Example: spherical coordinates

Let us consider the case of spherical coordinates $x^\alpha = (t, r, \theta, \phi)$:

$$dS^2 = dt^2 - r^2 d\theta^2 - r^2\sin^2\theta d\phi^2 - dr^2\,, \quad g_{\alpha\beta} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2\sin^2\theta & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}, \tag{48}$$

and the diagonal tetrad is taken as

$$e^\alpha_{(0)} = (1,0,0,0)\,, \quad e^\alpha_{(3)} = (0,1,0,0)\,, \quad e^\alpha_{(1)} = (0,0,\frac{1}{r},0)\,, \quad e^\alpha_{(2)} = (1,0,0,\frac{1}{r\sin\theta})\,. \tag{49}$$

To this tetrad there correspond the following Ricci rotation coefficients

$$\gamma_{ab0}=0,\gamma_{ab3}=0\,,\;\gamma_{ab1}=\begin{vmatrix}0&0&0&0\\0&0&0&-\frac{1}{r}\\0&0&0&0\\0&+\frac{1}{r}&0&0\end{vmatrix},\gamma_{ab2}=\begin{vmatrix}0&0&0&0\\0&0&+\frac{\cot\theta}{r}&0\\0&-\frac{\cot\theta}{r}&0&-\frac{1}{r}\\0&0&+\frac{1}{r}&0\end{vmatrix}. \qquad (50)$$

Therefore we have

$$\begin{gathered}0\Longrightarrow t,\quad 1\Longrightarrow\theta,\quad 2\Longrightarrow\phi,\quad 3\Longrightarrow r,\quad i(D_0+ieA_0)\Longrightarrow(E+\frac{\alpha}{r}),\\ D_1=\frac{1}{r}\partial_\theta,\quad D_2=\frac{1}{r\sin\theta}\partial_\phi,\quad D_3=\partial_r\quad \Delta=\frac{1}{r^2}\partial_\theta^2+\frac{1}{r^2\sin^2\theta}\partial_\phi^2+\partial_r^2\end{gathered} \qquad (51)$$

the Ricci rotation coefficients

$$\begin{array}{lll}\gamma_{230}=G_{10}=0, & \gamma_{310}=G_{20}=0, & \gamma_{120}=G_{30}=0,\\ \gamma_{231}=G_{11}=0, & \gamma_{311}=G_{21}=+\frac{1}{r}, & \gamma_{121}=G_{31}=0,\\ \gamma_{232}=G_{12}=-\frac{1}{r}, & \gamma_{312}=G_{22}=0, & \gamma_{122}=G_{32}=\frac{\cot\theta}{r},\\ \gamma_{233}=G_{13}=0, & \gamma_{313}=G_{23}=0, & \gamma_{123}=G_{33}=0\end{array} \qquad (52)$$

The blocks matrices take the form

$$A_1=\frac{i}{r}\begin{vmatrix}-\cot\theta&14&-4\cot\theta&-4\\-14&-\cot\theta&4&-4\cot\theta\\0&12&-9\cot\theta&-6\\-12&0&6&-9\cot\theta\end{vmatrix},\quad A_2=\frac{1}{r}\begin{vmatrix}3\cot\theta&-2&-6\cot\theta&4\\-2&-3\cot\theta&4&6\cot\theta\\0&12&-9\cot\theta&-6\\12&0&-6&9\cot\theta\end{vmatrix},$$

$$A_3=\frac{i}{r}\begin{vmatrix}-14&-\cot\theta&4&-4\cot\theta\\\cot\theta&-14&4\cot\theta&4\\0&2\cot\theta&-6&-\cot\theta\\-2\cot\theta&0&\cot\theta&-6\end{vmatrix}$$

$$D_3=\partial_3,\quad(\partial_r A_1)=\frac{i}{r^2}\begin{vmatrix}\cot\theta&-14&4\cot\theta&4\\14&\cot\theta&-4&4\cot\theta\\0&-12&9\cot\theta&6\\12&0&-6&9\cot\theta\end{vmatrix},$$

$$B_1=\frac{i}{r}\begin{vmatrix}-5\cot\theta&-2&4\cot\theta&4\\2&-5\cot\theta&-4&4\cot\theta\\0&-4&3\cot\theta&2\\4&0&-2&3\cot\theta\end{vmatrix},\quad B_2=\frac{1}{r}\begin{vmatrix}\cot\theta&-2&-2\cot\theta&4\\-2&-\cot\theta&4&2\cot\theta\\0&-4&-3\cot\theta&2\\-4&0&2&3\cot\theta\end{vmatrix},$$

$$B_3=\frac{i}{r}\begin{vmatrix}2&\cot\theta&-4&4\cot\theta\\-\cot\theta&2&-4\cot\theta&-4\\0&-2\cot\theta&-6&\cot\theta\\2\cot\theta&0&-\cot\theta&-6\end{vmatrix}$$

$$A_1+B_1=\frac{2i}{r}\begin{vmatrix}-3\cot\theta&6&0&0\\-6&-3\cot\theta&0&0\\0&4&-3\cot\theta&-2\\-4&0&2&-3\cot\theta\end{vmatrix},\quad A_2+B_2=\frac{4}{r}\begin{vmatrix}\cot\theta&-1&-2\cot\theta&2\\-1&-\cot\theta&2&2\cot\theta\\0&2&-3\cot\theta&-1\\2&0&-1&3\cot\theta\end{vmatrix},$$

$$A_3+B_3=-\frac{12i}{r}I$$

$$\Sigma=\frac{1}{r^2}\begin{vmatrix}3i\left(\cot^2\theta+12\right)&12i\cot\theta&12i\left(\cot^2\theta-3\right)&48i\cot\theta\\-12i\cot\theta&3i\left(\cot^2\theta+12\right)&-48i\cot\theta&12i\left(\cot^2\theta-3\right)\\0&-24i\cot\theta&9i\left(3\cot^2\theta-4\right)&12i\cot\theta\\24i\cot\theta&0&-12i\cot\theta&9i\left(3\cot^2\theta-4\right)\end{vmatrix}$$

Преобразуем эти матрицы к циклическому базису

$$A_1=\frac{i}{r}\begin{vmatrix}-9\cot\theta&6&0&0\\-6&-\cot\theta&8&0\\0&-8&-\cot\theta&6\\0&0&-6&-9\cot\theta\end{vmatrix},\quad A_2=\frac{1}{r}\begin{vmatrix}9\cot\theta&-6&0&0\\-6&3\cot\theta&-8&0\\0&-8&-3\cot\theta&-6\\0&0&-6&-9\cot\theta\end{vmatrix},$$

$$A_3=\frac{i}{r}\begin{vmatrix}-6&\cot\theta&0&0\\9\cot\theta&-14&-2\cot\theta&0\\0&2\cot\theta&-14&-9\cot\theta\\0&0&-\cot\theta&-6\end{vmatrix}$$

$$(\partial_r A_1) = \begin{vmatrix} \frac{9i\cot\theta}{r^2} & -\frac{6i}{r^2} & 0 & 0 \\ \frac{6i}{r^2} & \frac{i\cot\theta}{r^2} & -\frac{8i}{r^2} & 0 \\ 0 & \frac{8i}{r^2} & \frac{i\cot\theta}{r^2} & -\frac{6i}{r^2} \\ 0 & 0 & \frac{6i}{r^2} & \frac{9i\cot\theta}{r^2} \end{vmatrix} = \frac{i}{r^2} \begin{vmatrix} 9\cot\theta & -6 & 0 & 0 \\ 6 & \cot\theta & -8 & 0 \\ 0 & 8 & \cot\theta & -6 \\ 0 & 0 & 6 & 9\cot\theta \end{vmatrix},$$

$$B_1 = \begin{vmatrix} \frac{3i\cot\theta}{r} & -\frac{2i}{r} & 0 & 0 \\ -\frac{6i}{r} & -\frac{5i\cot\theta}{r} & 0 & 0 \\ 0 & 0 & -\frac{5i\cot\theta}{r} & \frac{6i}{r} \\ 0 & 0 & \frac{2i}{r} & \frac{3i\cot\theta}{r} \end{vmatrix} = \frac{i}{r} \begin{vmatrix} 3\cot\theta & -2 & 0 & 0 \\ -6 & -5\cot\theta & 0 & 0 \\ 0 & 0 & -5\cot\theta & 6 \\ 0 & 0 & 2 & 3\cot\theta \end{vmatrix},$$

$$B_2 = \begin{vmatrix} \frac{3\cot\theta}{r} & \frac{2}{r} & 0 & 0 \\ -\frac{6}{r} & \frac{\cot\theta}{r} & 0 & 0 \\ 0 & 0 & -\frac{\cot\theta}{r} & -\frac{6}{r} \\ 0 & 0 & \frac{2}{r} & -\frac{3\cot\theta}{r} \end{vmatrix} = \frac{1}{r} \begin{vmatrix} 3\cot\theta & 2 & 0 & 0 \\ -6 & \cot\theta & 0 & 0 \\ 0 & 0 & -\cot\theta & -6 \\ 0 & 0 & 2 & -3\cot\theta \end{vmatrix},$$

$$B_3 = \begin{vmatrix} -\frac{6i}{r} & -\frac{i\cot\theta}{r} & 0 & 0 \\ -\frac{9i\cot\theta}{r} & \frac{2i}{r} & \frac{2i\cot\theta}{r} & 0 \\ 0 & -\frac{2i\cot\theta}{r} & \frac{2i}{r} & \frac{9i\cot\theta}{r} \\ 0 & 0 & \frac{i\cot\theta}{r} & -\frac{6i}{r} \end{vmatrix} = \frac{i}{r} \begin{vmatrix} -6 & -\cot\theta & 0 & 0 \\ -9\cot\theta & 2 & 2\cot\theta & 0 \\ 0 & -2\cot\theta & 2 & 9\cot\theta \\ 0 & 0 & \cot\theta & -6 \end{vmatrix}$$

$$A_1 + B_1 = \frac{2i}{r} \begin{vmatrix} -3\cot\theta & 2 & 0 & 0 \\ -6 & -3\cot\theta & 4 & 0 \\ 0 & -4 & -3\cot\theta & 6 \\ 0 & 0 & -2 & -3\cot\theta \end{vmatrix}, \quad A_2 + B_2 = \frac{4}{r} \begin{vmatrix} 3\cot\theta & -1 & 0 & 0 \\ -3 & \cot\theta & -2 & 0 \\ 0 & -2 & -\cot\theta & -3 \\ 0 & 0 & -1 & -3\cot\theta \end{vmatrix},$$

$$A_3 + B_3 = \begin{vmatrix} -\frac{12i}{r} & 0 & 0 & 0 \\ 0 & -\frac{12i}{r} & 0 & 0 \\ 0 & 0 & -\frac{12i}{r} & 0 \\ 0 & 0 & 0 & -\frac{12i}{r} \end{vmatrix} = -\frac{12i}{r} I,$$

$$\Sigma = \begin{vmatrix} \frac{27i\cot^2\theta}{r^2} - \frac{36i}{r^2} & -\frac{12i\cot\theta}{r^2} & 0 & 0 \\ -\frac{108i\cot\theta}{r^2} & \frac{3i\cot^2\theta}{r^2} + \frac{36i}{r^2} & \frac{24i\cot\theta}{r^2} & \frac{24i\cot^2\theta}{r^2} - 2\left(\frac{12i\cot^2\theta}{r^2} - \frac{36i}{r^2}\right) - \frac{72i}{r^2} \\ \frac{24i\cot^2\theta}{r^2} - 2\left(\frac{12i\cot^2\theta}{r^2} - \frac{36i}{r^2}\right) - \frac{72i}{r^2} & -\frac{24i\cot\theta}{r^2} & \frac{3i\cot^2\theta}{r^2} + \frac{36i}{r^2} & \frac{108i\cot\theta}{r^2} \\ 0 & 0 & \frac{12i\cot\theta}{r^2} & \frac{27i\cot^2\theta}{r^2} - \frac{36i}{r^2} \end{vmatrix} =$$

$$= \frac{3}{r^2} \begin{vmatrix} 3i\left(3\cot^2\theta - 4\right) & -4i\cot\theta & 0 & 0 \\ -36i\cot\theta & i\left(\cot^2\theta + 12\right) & 8i\cot\theta & 0 \\ 0 & -8i\cot\theta & i\left(\cot^2\theta + 12\right) & 36i\cot\theta \\ 0 & 0 & 4i\cot\theta & 3i\left(3\cot^2\theta - 4\right) \end{vmatrix}$$

The Pauli-like equation reads

$$(\epsilon + \frac{\alpha}{r})\Psi = -\frac{1}{2M}\Big[\partial_r^2 + \frac{1}{r^2}\partial_\phi^2 + \frac{\partial^2}{\partial z^2}\Big]\Psi -$$

$$-\frac{i}{2M}\Big[\frac{1}{3}(\partial_\theta A_1) + 0 + (\partial_3 A_3) + \frac{1}{r}(A_1 + B_1)\partial_\theta \; + \frac{1}{r\sin\theta}(A_2 + B_2)\partial_\phi + (A_3 + B_3)\partial_r + \Sigma\Big]\Psi,$$

8. Conclusions

The goal of the present paper is investigation of the non-relativistic approximation in the first order 39-component theory for a spin 2 particle, in curved space-time, and in presence of external electromagnetic fields.

For distinguishing the large and small constituents in the complete wave function, we use three projective operators constructed on the base of the minimal polynomial of the 4-th order for the matrix $\Gamma^0_{16\times16}$. The relevant large and small components are found in explicit form. Among them we have found independent variables; in particular, among the large components there exist only four independent ones.

Acting in accordance with the known general procedure, we have derived the non-relativistic system of equations for a 4-component wave function; the relevant Hamiltonian depends on electromagnetic field and additional geometrical terms are determined by the Ricci rotation coefficients (these terms should be determined by Ricci scalar R and Ricci tensor R_{ab} in tetrad form. The terms describing interaction of the magnetic moment of the spin 3/2 particle with the external magnetic field is separated; this additional term is constructed with the use of the spin matrices S_i and the components of the magnetic field $\vec{B}$.

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