Nonlinear Dynamics and Applications: Proceedings of JIPNR-Sosny. Vol. 31 (2025) 262 - 286

A Spin 1/2 Particle with Three Additional Characteristics in Presence of External Fields, General Theory within the Gel'fand–Yaglom Formalism

V.V. Kisel,* E.M. Ovsiyuk,† P.O. Sachenok,‡ and A.S. Martynenko‡
Mozyr State Pedagogical University named after I.P. Shamyakin

A.V. Bury§ and V.M. Red'kov¶
B. I. Stepanov Institute of Physics of NAS of Belarus,
68 Nezavisimosti Ave., 220072 Minsk, BELARUS

Within the general method by Gel'fand – Yaglom, starting with the extended 28-component representation of the Lorentz group (it includes four bispiniors and one spinor of the 3rd rank), for a spin 1/2 particle, we construct a relativistic P-invariant generalized system of the first order equations for a spin 1/2 particle with three additional to electric charge characteristics.

First, the model is developed for a free particle, and the system of spinor equations is derived; then it is transformed to spin-tensor form. In this spin-tensor form, we take into account the presence of external electromagnetic fields. After eliminating the accessory variables of the complete wave function, we derive the minimal 4-component Dirac-like equation, the last includes several new interaction terms which are interpreted as related to some additional electromagnetic characteristics of a spin 1/2 particle.

PACS numbers: 02.30.Gp, 02.40.Ky, 03.65Ge, 04.62.+v
Keywords: Spin 1/2, Gel'fand – Yaglom, extended wave equations, additional characteristics, external electromagnetic field, Riemannian geometry.

1. Gel'fand – Yaglom formalism, and a new Dirac-like equation

We will construct a generalized relativistic equation for a particle with spin 1/2, using the extended set of irreducible representations of the proper Lorentz group

$$T = 4[(0,1/2) \oplus (1/2,0)] \oplus [(1/2,1) \oplus (1,1/2)]; \tag{1}$$

with the linking scheme

$$\begin{array}{ccc} 4(0,1/2) & - & 4(1/2,0) \\ | & & | \\ (1/2,1) & - & (1,1/2) \end{array} \tag{2}$$

First we will construct a matrix equation for a free particle [1]

$$(\Gamma_\mu \partial_\mu + M)\Psi = 0, \quad \mu = 1,2,3,4. \tag{3}$$

In modified Gel'fand – Yaglom basis [2], the components of the complete wave function are determined as follows

$$\Psi^{\text{G-Y(mod)}} = \Big\{ (\Psi^{(1)})^{(0,1/2)}_{1/2,1/2}, (\Psi^{(1)})^{(0,1/2)}_{1/2,-1/2}, (\Psi^{(1)})^{(1/2,0)}_{1/2,1/2}, (\Psi^{(1)})^{(1/2,0)}_{1/2,-1/2};$$
$$(\Psi^{(2)})^{(0,1/2)}_{1/2,1/2}, (\Psi^{(2)})^{(0,1/2)}_{1/2,-1/2}, (\Psi^{(2)})^{(1/2,0)}_{1/2,1/2}, (\Psi^{(2)})^{(1/2,0)}_{1/2,-1/2};$$
$$(\Psi^{(3)})^{(0,1/2)}_{1/2,1/2}, (\Psi^{(3)})^{(0,1/2)}_{1/2,-1/2}, (\Psi^{(3)})^{(1/2,0)}_{1/2,1/2}, (\Psi^{(3)})^{(1/2,0)}_{1/2,-1/2};$$
$$(\Psi^{(4)})^{(0,1/2)}_{1/2,1/2}, (\Psi^{(4)})^{(0,1/2)}_{1/2,-1/2}, (\Psi^{(4)})^{(1/2,0)}_{1/2,1/2}, (\Psi^{(4)})^{(1/2,0)}_{1/2,-1/2};$$

*E-mail: vasiliy-bspu@mail.ru
†E-mail: e.ovsiyuk@mail.ru
‡E-mail: polinasacenok@gmail.com
§E-mail: anton.buryy.97@mail.ru
¶E-mail: v.redkov@ifanbel.bas-net.by
[1] Applying the *ict*-metric.

$$\Psi^{(1,1/2)}_{1/2,1/2}, \Psi^{(1,1/2)}_{1/2,-1/2}, \Psi^{(1/2,1)}_{1/2,1/2}, \Psi^{(1/2,1)}_{1/2,-1/2};$$
$$\Psi^{(1,1/2)}_{3/2,3/2}, \Psi^{(1,1/2)}_{3/2,-3/2}, \Psi^{(1/2,1)}_{3/2,3/2}, \Psi^{(1/2,1)}_{3/2,-3/2};$$
$$\Psi^{(1,1/2)}_{3/2,1/2}, \Psi^{(1,1/2)}_{3/2,-1/2}, \Psi^{(1/2,1)}_{3/2,1/2}, \Psi^{(1/2,1)}_{3/2,-1/2}\Big\}_T; \tag{4}$$

the symbol T designates the matrix transposition; the matrix Γ_4 may be presented in the form

$$\Gamma_4 = \left| \begin{array}{cc} c^{(1/2)} \otimes \gamma_4 & 0 \\ 0 & c^{(3/2)} \otimes I_2 \otimes \gamma_4 \end{array} \right|, \tag{5}$$

where the spin blocks $c^{(1/2)}, c^{(3/2)}$ have the structure

$$c^{(1/2)} = \left| \begin{array}{ccccc} c^{(1/2)}_{11'} & c^{(1/2)}_{12'} & c^{(1/2)}_{13'} & c^{(1/2)}_{14'} & c^{(1/2)}_{15'} \\ c^{(1/2)}_{21'} & c^{(1/2)}_{22'} & c^{(1/2)}_{23'} & c^{(1/2)}_{24'} & c^{(1/2)}_{25'} \\ c^{(1/2)}_{31'} & c^{(1/2)}_{32'} & c^{(1/2)}_{33'} & c^{(1/2)}_{34'} & c^{(1/2)}_{35'} \\ c^{(1/2)}_{41'} & c^{(1/2)}_{42'} & c^{(1/2)}_{43'} & c^{(1/2)}_{44'} & c^{(1/2)}_{45'} \\ c^{(1/2)}_{51'} & c^{(1/2)}_{52'} & c^{(1/2)}_{53'} & c^{(1/2)}_{54'} & c^{(1/2)}_{55'} \end{array} \right|, \quad C^{(3/2)} = C^{(3/2)}_{55'}, \quad \gamma_4 = \left| \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right|; \tag{6}$$

I_2 is a 2×2 unit matrix; the involved irreducible representations of the proper Lorentz group are numerated as follows

$$1, 2, 3, 4 \Longrightarrow (0, 1/2), \quad 1', 2', 3', 4' \Longrightarrow (1/2, 0), \quad 5 \Longrightarrow (1, 1/2), \quad 5' \Longrightarrow (1/2, 1).$$

According to [1, 2], we have

$$c^{(1/2)} = \left| \begin{array}{ccccc} c_{11'} & c_{12'} & c_{13'} & c_{14'} & i\sqrt{3}c_{15'} \\ c_{21'} & c_{22'} & c_{23'} & c_{24'} & i\sqrt{3}c_{25'} \\ c_{31'} & c_{32'} & c_{33'} & c_{34'} & i\sqrt{3}c_{35'} \\ c_{41'} & c_{42'} & c_{43'} & c_{44'} & i\sqrt{3}c_{45'} \\ i\sqrt{3}c_{51'} & i\sqrt{3}c_{52'} & i\sqrt{3}c_{53'} & i\sqrt{3}c_{54'} & c_{55'} \end{array} \right|, \quad C^{(3/2)} = 2C_{55'}, \tag{7}$$

where $c_{ij'}$ are some numerical parameters, on which special constraints will be imposed later on.

In order to simplify the calculations bellow, let us take into account the possibility to break the linking between repeated representations [1]; this is achieved by vanishing the corresponding parameters in the block $c^{(1/2)}$:

$$c^{(1/2)} = \left| \begin{array}{ccccc} c_{11'} & 0 & 0 & 0 & i\sqrt{3}c_{15'} \\ 0 & c_{22'} & 0 & 0 & i\sqrt{3}c_{25'} \\ 0 & 0 & c_{3'} & 0 & i\sqrt{3}c_{35'} \\ 0 & 0 & 0 & c_{44'} & i\sqrt{3}c_{45'} \\ i\sqrt{3}c_{51'} & i\sqrt{3}c_{52'} & i\sqrt{3}c_{53'} & i\sqrt{3}c_{54'} & c_{55'} \end{array} \right|, \quad c^{(3/2)} = 2c_{55'}. \tag{8}$$

Because we assume the absence states with spin 3/2, we should set $c_{55'} = 0$; then we arrive at

$$c^{(1/2)} = \left| \begin{array}{ccccc} c_{11'} & 0 & 0 & 0 & i\sqrt{3}c_{15'} \\ 0 & c_{22'} & 0 & 0 & i\sqrt{3}c_{25'} \\ 0 & 0 & c_{3'} & 0 & i\sqrt{3}c_{35'} \\ 0 & 0 & 0 & c_{44'} & i\sqrt{3}c_{45'} \\ i\sqrt{3}c_{51'} & i\sqrt{3}c_{52'} & i\sqrt{3}c_{53'} & i\sqrt{3}c_{54'} & 0 \end{array} \right|, \quad c^{(3/2)} = 0. \tag{9}$$

It is convenient to introduce the new notations:

$$\lambda_1 = c_{11'}, \quad \lambda_2 = c_{22'}, \quad \lambda_3 = c_{33'}, \quad \lambda_4 = c_{44'}, \tag{10}$$

$$\begin{array}{llll} \dfrac{\beta_1}{\sqrt{2}} = -ic_{15'}, & \dfrac{\beta_2}{\sqrt{2}} = -ic_{25'}, & \dfrac{\beta_3}{\sqrt{2}} = -ic_{35'}, & \dfrac{\beta_4}{\sqrt{2}} = -ic_{45'}, \\ \dfrac{\beta_5}{\sqrt{2}} = ic_{51'}, & \dfrac{\beta_6}{\sqrt{2}} = ic_{52'}, & \dfrac{\beta_7}{\sqrt{2}} = ic_{53'}, & \dfrac{\beta_8}{\sqrt{2}} = ic_{54'}; \end{array} \tag{11}$$

so we obtain

$$c^{(1/2)} = \left| \begin{array}{ccccc} \lambda_1 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}}\beta_1 \\ 0 & \lambda_2 & 0 & 0 & -\sqrt{\frac{3}{2}}\beta_2 \\ 0 & 0 & \lambda_3 & 0 & -\sqrt{\frac{3}{2}}\beta_3 \\ 0 & 0 & 0 & \lambda_4 & -\sqrt{\frac{3}{2}}\beta_4 \\ \sqrt{\frac{3}{2}}\beta_5 & \sqrt{\frac{3}{2}}\beta_6 & \sqrt{\frac{3}{2}}\beta_7 & \sqrt{\frac{3}{2}}\beta_8 & 0 \end{array} \right| . \qquad (12)$$

The matrices of the bilinear form should have the structure

$$\eta = \left| \begin{array}{cc} \eta^{(1/2)} \otimes \gamma_4 & 0 \\ 0 & \eta^{(3/2)} \otimes I_2 \otimes \gamma_4 \end{array} \right| , \quad \eta^{(1/2)} = \left| \begin{array}{ccccc} k_1 & 0 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 0 & 0 \\ 0 & 0 & 0 & k_4 & 0 \\ 0 & 0 & 0 & 0 & k_5 \end{array} \right| , \quad \eta^{(3/2)} = -\text{diag}(k_i), \qquad (13)$$

where $k_i = \pm 1$. Due to the known constraint $(\eta\Gamma_4)^+ = \eta\Gamma_4$, we find restrictions on parameters λ_i, β_i:

$$\begin{gathered} \lambda_1^* = \lambda_1, \quad \lambda_2^* = \lambda_2, \quad \lambda_3^* = \lambda_3, \quad \lambda_4^* = \lambda_4, \\ \beta_5 = -k_1 k_5 \beta_1^*, \quad \beta_6 = -k_2 k_5 \beta_2^*, \\ \beta_7 = -k_3 k_5 \beta_3^*, \quad \beta_8 = -k_4 k_5 \beta_4^*. \end{gathered} \qquad (14)$$

Because the particle under consideration has only one mass parameter M, and the spin equals $1/2$, the matrix $c^{(1/2)}$ (10) should have only one non-vanishing eigenvalue $(+1)$. We use this property to derive additional restrictions on parameters λ_i, β_i:

$$S_p c^{(1/2)} = 1, \quad S_p[(c^{(1/2)})^2] = 1, \quad S_p(c^{(1/2)})^3 = 1, \quad S_p(c^{(1/2)})^4 = 1, \quad \det c^{(1/2)} = 0. \qquad (15)$$

From expression for $c^{(1/2)}$ (11) and condition (14), it follows

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1. \qquad (16)$$

From equality

$$(c^{(1/2)})^2$$

$$= \left| \begin{array}{ccccc} \lambda_1^2 - \frac{3}{2}\beta_1\beta_5 & -\frac{3}{2}\beta_1\beta_6 & -\frac{3}{2}\beta_1\beta_7 & -\frac{3}{2}\beta_1\beta_8 & -\sqrt{\frac{3}{2}}\lambda_1\beta_1 \\ -\frac{3}{2}\beta_2\beta_5 & \lambda_2^2 - \frac{3}{2}\beta_2\beta_6 & -\frac{3}{2}\beta_2\beta_7 & -\frac{3}{2}\beta_2\beta_8 & -\sqrt{\frac{3}{2}}\lambda_2\beta_2 \\ -\frac{3}{2}\beta_3\beta_5 & -\frac{3}{2}\beta_3\beta_6 & \lambda_3^2 - \frac{3}{2}\beta_3\beta_7 & -\frac{3}{2}\beta_3\beta_8 & -\sqrt{\frac{3}{2}}\lambda_3\beta_3 \\ -\frac{3}{2}\beta_4\beta_5 & -\frac{3}{2}\beta_4\beta_6 & -\frac{3}{2}\beta_4\beta_7 & \lambda_4^2 - \frac{3}{2}\beta_4\beta_8 & -\sqrt{\frac{3}{2}}\lambda_4\beta_4 \\ \sqrt{\frac{3}{2}}\lambda_1\beta_5 & \sqrt{\frac{3}{2}}\lambda_2\beta_6 & \sqrt{\frac{3}{2}}\lambda_3\beta_7 & \sqrt{\frac{3}{2}}\lambda_4\beta_8 & -\frac{3}{2}(\beta_1\beta_5 + \beta_2\beta_6 + \beta_3\beta_7 + \beta_4\beta_8) \end{array} \right| \qquad (17)$$

it follows (see (14))

$$(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2) - 3(\beta_1\beta_5 + \beta_2\beta_6 + \beta_3\beta_7 + \beta_4\beta_8) = 1. \qquad (18)$$

Due to (15), we have $(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)^2 = 1$, or

$$(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2) + 2(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4) = 1. \qquad (19)$$

From (17),(19) we obtain

$$(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4) + \frac{3}{2}(\beta_1\beta_5 + \beta_2\beta_6 + \beta_3\beta_7 + \beta_4\beta_8) = 0. \qquad (20)$$

From definition for $(C^{(1/2)})^3$:

$$(c^{(1/2)})^3 = \left| \begin{array}{c} \lambda_1(\lambda_1^2 - 3\beta_1\beta_5) \\ -\frac{3}{2}\beta_2\beta_5(\lambda_1 + \lambda_2) \\ -\frac{3}{2}\beta_3\beta_5(\lambda_1 + \lambda_3) \\ -\frac{3}{2}\beta_4\beta_5(\lambda_1 + \lambda_4) \\ \sqrt{\frac{3}{2}}\beta_5[\lambda_1^2 - \frac{3}{2}(\beta_1\beta_5 + \beta_2\beta_6 + \beta_3\beta_7 + \beta_4\beta_8)] \end{array} \right.$$

$$
\begin{gathered}
-\tfrac{3}{2}\beta_1\beta_6(\lambda_1+\lambda_2)\\
\lambda_2(\lambda_2^2-3\beta_2\beta_6)\\
-\tfrac{3}{2}\beta_3\beta_6(\lambda_2+\lambda_3)\\
-\tfrac{3}{2}\beta_4\beta_6(\lambda_2+\lambda_4)\\
\sqrt{\tfrac{3}{2}}\beta_6[\lambda_2^2-\tfrac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]
\end{gathered}
$$

$$
\begin{gathered}
-\tfrac{3}{2}\beta_1\beta_7(\lambda_1+\lambda_3)\\
-\tfrac{3}{2}\beta_2\beta_7(\lambda_2+\lambda_3)\\
\lambda_3(\lambda_3^2-3\beta_3\beta_7)\\
-\tfrac{3}{2}\beta_4\beta_7(\lambda_3+\lambda_4)\\
\sqrt{\tfrac{3}{2}}\beta_7[\lambda_3^2-\tfrac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]
\end{gathered}
$$

$$
\begin{gathered}
-\tfrac{3}{2}\beta_1\beta_8(\lambda_1+\lambda_4)\\
-\tfrac{3}{2}\beta_2\beta_8(\lambda_2+\lambda_4)\\
-\tfrac{3}{2}\beta_3\beta_8(\lambda_3+\lambda_4)\\
\lambda_4(\lambda_4^2-3\beta_4\beta_8)\\
\sqrt{\tfrac{3}{2}}\beta_7[\lambda_4^2-\tfrac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]
\end{gathered}
$$

$$
\left.
\begin{gathered}
-\sqrt{\tfrac{3}{2}}\beta_1[\lambda_1^2-\tfrac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\sqrt{\tfrac{3}{2}}\beta_2[\lambda_2^2-\tfrac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\sqrt{\tfrac{3}{2}}\beta_3[\lambda_3^2-\tfrac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\sqrt{\tfrac{3}{2}}\beta_4[\lambda_4^2-\tfrac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\tfrac{3}{2}[\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8]
\end{gathered}
\right|, \tag{21}
$$

we get

$$
(\lambda_1^3+\lambda_2^3+\lambda_3^3)-\frac{9}{2}(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)=1. \tag{22}
$$

Taking in mind (15), we find $(\lambda_1+\lambda_2+\lambda_3+\lambda_4)^3=1$, or

$$
(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_2+\lambda_3+\lambda_4)^2
$$

$$
=(\lambda_1+\lambda_2+\lambda_3+\lambda_4)[(\lambda_1^2+\lambda_2^2+\lambda_3^2+\lambda_4^2)+2(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)]=1,
$$

so we arrive at

$$
\begin{gathered}
(\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3)+3\lambda_1(\lambda_2^2+\lambda_3^2+\lambda_4^2)+3\lambda_2(\lambda_1^2+\lambda_3^2+\lambda_4^2)+3\lambda_3(\lambda_1^2+\lambda_2^2+\lambda_4^2)\\
+3\lambda_4(\lambda_1^2+\lambda_2^2+\lambda_3^2)+6(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)=1.
\end{gathered} \tag{23}
$$

Relation (23) may be presented as follows

$$
\begin{gathered}
(\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3)-3(\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3)+3(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1^2+\lambda_2^2+\lambda_3^2+\lambda_4^2)\\
+6(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)=1,
\end{gathered}
$$

whence (taking in mind (16)) it follows

$$
\begin{gathered}
-2(\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3)+3(\lambda_1^2+\lambda_2^2+\lambda_3^2+\lambda_4^2)\\
+6(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)=1.
\end{gathered} \tag{24}
$$

Therefore, we get (taking in mind (19))

$$
\begin{aligned}
(\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3)=&\,3(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)\\
&-3(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)+1.
\end{aligned} \tag{25}
$$

Allowing for (20), we transform (25) to other form

$$
\begin{gathered}
(\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3)=3(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4\\
+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)+\frac{9}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)+1.
\end{gathered} \tag{26}
$$

From (26), (22) we derive

$$\begin{aligned}(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4)\\ +\frac{3}{2}[(1-\lambda_1)\beta_1\beta_5 + (1-\lambda_2)\beta_2\beta_6 + (1-\lambda_3)\beta_3\beta_7 + (1-\lambda_4)\beta_4\beta_8] = 0,\end{aligned} \tag{27}$$

or differently

$$\begin{aligned}(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4)\\ +\frac{3}{2}[(\lambda_2+\lambda_3+\lambda_4)\beta_1\beta_5 + (\lambda_1+\lambda_3+\lambda_4)\beta_2\beta_6 + (\lambda_1+\lambda_2+\lambda_4)\beta_3\beta_7\\ +(\lambda_1+\lambda_2+\lambda_3)\beta_4\beta_8] = 0.\end{aligned} \tag{28}$$

Allowing for (25) and (22), we transform the last equation to the new form

$$\begin{aligned}(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4) - (\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4)\\ -\frac{3}{2}(\lambda_1\beta_1\beta_5 + \lambda_2\beta_2\beta_6 + \lambda_3\beta_3\beta_7 + \lambda_4\beta_4\beta_8) = 0.\end{aligned} \tag{29}$$

With the use of explicit form of $(C^{(1/2)})^4$ (we present the matrix by columns):

$$(C^{(1/2)})^4$$

$$= \left| \begin{matrix}
\lambda_1^4 - \frac{9}{2}\beta_1\beta_5[\lambda_1^2 - \frac{1}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\frac{3}{2}\beta_2\beta_5[\lambda_1^2+\lambda_2^2+\lambda_1\lambda_2 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\frac{3}{2}\beta_3\beta_5[\lambda_1^2+\lambda_3^2+\lambda_1\lambda_3 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\frac{3}{2}\beta_4\beta_5[\lambda_1^2+\lambda_4^2+\lambda_1\lambda_4 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
\sqrt{\frac{3}{2}}\beta_5[\lambda_1^3 - \frac{3}{2}\lambda_1(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8) - \frac{3}{2}(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)]
\end{matrix}\right.$$

$$\begin{matrix}
-\frac{3}{2}\beta_1\beta_6[\lambda_1^2+\lambda_2^2+\lambda_1\lambda_2 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
\lambda_2^4 - \frac{9}{2}\beta_2\beta_6[\lambda_2^2 - \frac{1}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\frac{3}{2}\beta_3\beta_6[\lambda_2^2+\lambda_3^2+\lambda_2\lambda_3 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\frac{3}{2}\beta_4\beta_6[\lambda_2^2+\lambda_4^2+\lambda_2\lambda_4 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
\sqrt{\frac{3}{2}}\beta_6[\lambda_2^3 - \frac{3}{2}\lambda_2(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8) - \frac{3}{2}(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)]
\end{matrix}$$

$$\begin{matrix}
-\frac{3}{2}\beta_1\beta_7[\lambda_1^2+\lambda_3^2+\lambda_1\lambda_3 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\frac{3}{2}\beta_2\beta_7[\lambda_2^2+\lambda_3^2+\lambda_2\lambda_3 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
\lambda_3^4 - \frac{9}{2}\beta_3\beta_7[\lambda_3^2 - \frac{1}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\frac{3}{2}\beta_4\beta_7[\lambda_3^2+\lambda_4^2+\lambda_3\lambda_4 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
\sqrt{\frac{3}{2}}\beta_7[\lambda_3^3 - \frac{3}{2}\lambda_3(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8) - \frac{3}{2}(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)]
\end{matrix}$$

$$\begin{matrix}
-\frac{3}{2}\beta_1\beta_8[\lambda_1^2+\lambda_4^2+\lambda_1\lambda_4 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\frac{3}{2}\beta_2\beta_8[\lambda_2^2+\lambda_4^2+\lambda_2\lambda_4 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
-\frac{3}{2}\beta_3\beta_8[\lambda_3^2+\lambda_4^2+\lambda_3\lambda_4 - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
\lambda_4^4 - \frac{9}{2}\beta_4\beta_8[\lambda_4^2 - \frac{1}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\\
\sqrt{\frac{3}{2}}\beta_8[\lambda_4^3 - \frac{3}{2}\lambda_4(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8) - \frac{3}{2}(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)]
\end{matrix}$$

$$\left.\begin{matrix}
-\sqrt{\frac{3}{2}}\beta_1[\lambda_1^3 - \frac{3}{2}\lambda_1(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8) - \frac{3}{2}(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)]\\
-\sqrt{\frac{3}{2}}\beta_2[\lambda_2^3 - \frac{3}{2}\lambda_2(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8) - \frac{3}{2}(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)]\\
-\sqrt{\frac{3}{2}}\beta_3[\lambda_3^3 - \frac{3}{2}\lambda_3(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8) - \frac{3}{2}(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)]\\
-\sqrt{\frac{3}{2}}\beta_4[\lambda_4^3 - \frac{3}{2}\lambda_4(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8) - \frac{3}{2}(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)]\\
-\frac{3}{2}[(\lambda_1^2\beta_1\beta_5+\lambda_2^2\beta_2\beta_6+\lambda_3^2\beta_3\beta_7+\lambda_4^2\beta_4\beta_8) - \frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)^2]
\end{matrix}\right| \tag{30}$$

we derive

$$\begin{aligned}(\lambda_1^4+\lambda_2^4+\lambda_3^4+\lambda_4^4) - 6(\lambda_1^2\beta_1\beta_5+\lambda_2^2\beta_2\beta_6+\lambda_3^2\beta_3\beta_7+\lambda_4^2\beta_4\beta_8)\\ +\frac{9}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)^2 = 1.\end{aligned} \tag{31}$$

Taking in mind (20), the last equation (31) may be transformed to other form

$$(\lambda_1^4+\lambda_2^4+\lambda_3^4+\lambda_4^4)-6(\lambda_1^2\beta_1\beta_5+\lambda_2^2\beta_2\beta_6+\lambda_3^2\beta_3\beta_7+\lambda_4^2\beta_4\beta_8)$$
$$+2(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)^2=1. \tag{32}$$

Whence, taking into account (16), we derive $(\lambda_1+\lambda_2+\lambda_3+\lambda_4)^4=1$, or differently

$$(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_2+\lambda_3+\lambda_4)^3$$
$$=(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\Big\{(\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3)+3\lambda_1^2(\lambda_2+\lambda_3+\lambda_4)$$
$$+3\lambda_2^2(\lambda_1+\lambda_3+\lambda_4)+3\lambda_3^2(\lambda_1+\lambda_2+\lambda_4)+3\lambda_4^2(\lambda_2+\lambda_3+\lambda_3)$$
$$+6(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)\Big\}$$
$$=(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\Big\{-2(\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3)+3\lambda_1^2(\lambda_2+\lambda_3+\lambda_4)$$
$$+3(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1^2+\lambda_2^2+\lambda_3^2+\lambda_4^2)+6(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)\Big\}$$
$$=-2(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3)+3\lambda_1^2(\lambda_2+\lambda_3+\lambda_4)$$
$$+3(\lambda_1+\lambda_2+\lambda_3+\lambda_4)^2(\lambda_1^2+\lambda_2^2+\lambda_3^2+\lambda_4^2)+6(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)$$
$$=(\lambda_1^4+\lambda_2^4+\lambda_3^4+\lambda_4^4)-2\lambda_1(\lambda_2^3+\lambda_3^3+\lambda_4^3)-2\lambda_2(\lambda_1^3+\lambda_3^3+\lambda_4^3)$$
$$-2\lambda_3(\lambda_1^3+\lambda_2^3+\lambda_4^3)-2\lambda_4(\lambda_1^3+\lambda_2^3+\lambda_3^3)+6(\lambda_1^2\lambda_2^2+\lambda_1^2\lambda_3^2+\lambda_1^2\lambda_4^2$$
$$+\lambda_2^2\lambda_3^2+\lambda_2^2\lambda_4^2+\lambda_3^2\lambda_4^2)+6\lambda_1^3(\lambda_2+\lambda_3+\lambda_4)+6\lambda_2^3(\lambda_1+\lambda_3+\lambda_4)$$
$$+6\lambda_3^3(\lambda_1+\lambda 2+\lambda_4)+6\lambda_4^3(\lambda_1+\lambda_2+\lambda_3)+12\lambda_1^2(\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)$$
$$+6\lambda_3^3(\lambda_1+\lambda_2+\lambda_4)+6\lambda_4^3(\lambda_1+\lambda_2+\lambda_3)+12\lambda_1^2(\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)$$
$$12\lambda_2^2(\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_3\lambda_4)+12\lambda_3^2(\lambda_1\lambda_2+\lambda_1\lambda_4+\lambda_2\lambda_4)+12\lambda_4^2(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)+24\lambda_1\lambda_2\lambda_3\lambda_4$$
$$=(\lambda_1^4+\lambda_2^4+\lambda_3^4+\lambda_4^4)-2\lambda_1(\lambda_2^3+\lambda_3^3+\lambda_4^3)-2\lambda_2(\lambda_1^3+\lambda_3^3+\lambda_4^3)$$
$$-2\lambda_3(\lambda_1^3+\lambda_2^3+\lambda_4^3)-2\lambda_4(\lambda_1^3+\lambda_2^3+\lambda_3^3)+6(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4$$
$$+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)^2+6\lambda_1^3(\lambda_2+\lambda_3+\lambda_4)+6\lambda_2^3(\lambda_1+\lambda_3+\lambda_4)$$
$$+6\lambda_3^3(\lambda_1+\lambda_2+\lambda_4)+6\lambda_4^3(\lambda_1+\lambda_2+\lambda_3)$$
$$=-3(\lambda_1^4+\lambda_2^4+\lambda_3^4+\lambda_4^4)+4(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3)$$
$$+6(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)^2-12\lambda_1\lambda_2\lambda_3\lambda_4=1.$$

From the last equation, with the us of (15) and (25), we obtain

$$(\lambda_1^4+\lambda_2^4+\lambda_3^4+\lambda_4^4)=1+4(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)$$
$$-4(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)$$
$$+2(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)^2-4\lambda_1\lambda_2\lambda_3\lambda_4, \tag{33}$$

whence it follows (see (32), (33))

$$(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)+(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)^2$$
$$-(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)$$
$$-\frac{3}{2}(\lambda_1^2\beta_1\beta_5+\lambda_2^2\beta_2\beta_6+\lambda_3^2\beta_3\beta_7+\lambda_4^2\beta_4\beta_8)-4\lambda_1\lambda_2\lambda_3\lambda_4=0. \tag{34}$$

Equations (33) and (34) may be transformed to other form with the use of (20), (29):

$$(\lambda_1^4+\lambda_2^4+\lambda_3^4+\lambda_4^4)=1+6(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)$$
$$+\frac{9}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)^2-4\lambda_1\lambda_2\lambda_3\lambda_4, \tag{35}$$

$$(\lambda_1\beta_1\beta_5+\lambda_2\beta_2\beta_6+\lambda_3\beta_3\beta_7+\lambda_4\beta_4\beta_8)+\frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)^2$$
$$-(\lambda_1^2\beta_1\beta_5+\lambda_2^2\beta_2\beta_6+\lambda_3^2\beta_3\beta_7+\lambda_4^2\beta_4\beta_8)-\frac{8}{3}\lambda_1\lambda_2\lambda_3\lambda_4=0.$$

Due to the structure of eigenvalues of the matrix $C^{(1/2)}$ (10), its determinant equals zero:

$$\lambda_1\lambda_2(\lambda_4\beta_3\beta_7+\lambda_3\beta_4\beta_8)+\lambda_3\lambda_4(\lambda_1\beta_2\beta_6+\lambda_2\beta_1\beta_5)=0. \tag{36}$$

The above restrictions on parameters λ_i, β_i may be found differently: by using the characteristic equation for the matrix $C^{(1/2)}$

$$(C^{(1/2)})^5 - a_1(C^{(1/2)})^4 + a_2(C^{(1/2)})^3 - a_3(C^{(1/2)})^2 + a_4(C^{(1/2)}) - a_5 = 0, \tag{37}$$

where

$$a_1 = S_p(C^{(1/2)}), \quad a_2 = \frac{1}{2}[(S_p(C^{(1/2)}))^2 - S_p(C^{(1/2)})^2],$$

$$a_3 = \frac{1}{3}\Big\{S_p(C^{(1/2)})^3 - \frac{3}{2}(S_p(C^{(1/2)}))(S_p(C^{(1/2)})^2) + \frac{1}{2}(S_pC^{(1/2)})^3\Big\},$$

$$a_4 = \frac{1}{4}\Big\{S_p(c^{(1/2)})^4 - \frac{4}{3}(S_pC^{(1/2)})(S_p(c^{(1/2)})^3)$$

$$-\frac{1}{2}(S_p(C^{(1/2)})^2)^2 + (S_pC^{(1/2)})^2 S_p(c^{(1/2)})^2 - \frac{1}{6}(S_pc^{(1/2)})^4\Big\}, \quad a_5 = \det c^{(1/2)}. \tag{38}$$

Taking into account possible eigenvalues for matrix $C^{(1/2)}$, we get:

$$a_1 = 1, \quad a_2 = a_3 = a_4 = a_5 = 0. \tag{39}$$

At this we again arrive at relations (15),(20),(27),(37), and

$$2\lambda_1\lambda_2\lambda_3\lambda_4 + 3\beta_1\beta_5(\lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4) + 3\beta_2\beta_6(\lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_3\lambda_4)$$

$$+3\beta_3\beta_7(\lambda_1\lambda_2 + \lambda_1\lambda_4 + \lambda_2\lambda_4) + 3\beta_4\beta_8(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3) = 0. \tag{40}$$

2. Transformation to the spinor basis

Let us find explicit form of the matrix Γ_4 in spinor basis, this will permits us to determine an explicit form of the complete equation in the spinor basis. Such a transition is performed in several steps.

In canonical basis, the complete function Φ consists of the following components

$$\Psi^{(\text{канон.})} = \Big\{(\Psi^{(1)})^{(0,1/2)}_{(0,1/2)}, (\Psi^{(1)})^{(0,1/2)}_{(0,-1/2)}, (\Psi^{(1)})^{(1/2,0)}_{(1/2,0)}, (\Psi^{(1)})^{(1/2,0)}_{(-1/2,0)};$$

$$(\Psi^{(2)})^{(0,1/2)}_{(0,1/2)}, (\Psi^{(2)})^{(0,1/2)}_{(0,-1/2)}, (\Psi^{(2)})^{(1/2,0)}_{(1/2,0)}, (\Psi^{(2)})^{(1/2,0)}_{(-1/2,0)};$$

$$(\Psi^{(3)})^{(0,1/2)}_{(0,1/2)}, (\Psi^{(3)})^{(0,1/2)}_{(0,-1/2)}, (\Psi^{(3)})^{(1/2,0)}_{(1/2,0)}, (\Psi^{(3)})^{(1/2,0)}_{(-1/2,0)};$$

$$(\Psi^{(4)})^{(0,1/2)}_{(0,1/2)}, (\Psi^{(4)})^{(0,1/2)}_{(0,-1/2)}, (\Psi^{(4)})^{(1/2,0)}_{(1/2,0)}, (\Psi^{(4)})^{(1/2,0)}_{(-1/2,0)};$$

$$\Psi^{(1,1/2)}_{(1,1/2)}, \Psi^{(1,1/2)}_{(0,1/2)}, \Psi^{(1,1/2)}_{-1,1/2}, \Psi^{(1,1/2)}_{1,-1/2}, \Psi^{(1,1/2)}_{(0,-1/2)}, \Psi^{(1,1/2)}_{(-1,-1/2)};$$

$$\Psi^{(1/2,1)}_{1/2,1}, \Psi^{(1/2,1)}_{1/2,0}, \Psi^{(1/2,1)}_{1/2,-1}, \Psi^{(1/2,1)}_{-1/2,1}, \Psi^{(1/2,1)}_{-1/2,0}, \Psi^{(1/2,1)}_{-1/2,-1}\Big\}_T. \tag{41}$$

The functions $\Psi^{\text{G–Y(mod)}}$ in modified Gelfand – Yaglom basis and in canonical basis $\Psi^{(\text{can})}$ are related by the linear transformation A:

$$\Psi^{\text{G–Y(mod)}} = A\Psi^{(\text{can})}. \tag{42}$$

Explicate form of the matrix A according to the general rule

$$\Psi^{(l,l')}_{s,s_3} = \Sigma\, C^{s,s_3}_{ll_3;l'l'_3}\Psi^{(l,l')}_{(l_3,l'_3)}, \tag{43}$$

where $C^{s,s_3}_{ll_3;l'l'_3}$ are the Clebsch – Gordan coefficients; the summing is over indices l_3, l'_3 at restriction $l_3 + l'_3 = s_3$. For given case, we obtain

$$
A = \left| \begin{array}{ccccccccccccc}
I_{16\times 16} & & & & & & & & & & & & \\
 & \cdot & -\frac{1}{\sqrt{3}} & \cdot & \frac{2}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & -\sqrt{\frac{2}{3}} & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & -\sqrt{\frac{2}{3}} & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot & -\frac{1}{\sqrt{3}} & \cdot \\
 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\
 & \cdot & \sqrt{\frac{2}{3}} & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & \sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & \sqrt{\frac{2}{3}} & \cdot
\end{array} \right| ,
$$

$$
A^{+} = \left| \begin{array}{ccccccccccccc}
I_{16\times 16} & & & & & & & & & & & & \\
 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & -\frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot \\
 & \cdot & -\sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot \\
 & \sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot \\
 & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot \\
 & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} \\
 & \cdot & \cdot & -\sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot \\
 & \cdot & \cdot & \cdot & -\frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot
\end{array} \right| ,
$$

and

$$
\Psi^{(\text{can})} = A^{+}\Psi^{\text{G-Y(mod)}}, \quad \Gamma_4^{(\text{can})} = A^{+}\Gamma_4^{\text{G-Y(mod)}}A.
$$

In detailed form, the matrix $\Gamma_4^{(\text{G-Y(mod)})}$ reads

$$
\Gamma_4^{(\text{spin})} = B^{-1}\Gamma_4^{(\text{can})}B = \left| \begin{array}{cc} \Gamma_{16\times 16} & \Gamma_{12\times 16} \\ \Gamma_{16\times 12} & 0 \end{array} \right| , \tag{44}
$$

$$
\Gamma_{16\times 16} = \left| \begin{array}{cccccccccccccccc}
\cdot & \cdot & \lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 & \cdot & \cdot
\end{array} \right| ,
$$

$$\Gamma_{12\times 16} = \begin{vmatrix}
\cdot & \cdot & -\sqrt{\frac{3}{2}}\beta_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -\sqrt{\frac{3}{2}}\beta_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
-\sqrt{\frac{3}{2}}\beta_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -\sqrt{\frac{3}{2}}\beta_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -\sqrt{\frac{3}{2}}\beta_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -\sqrt{\frac{3}{2}}\beta_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
-\sqrt{\frac{3}{2}}\beta_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -\sqrt{\frac{3}{2}}\beta_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -\sqrt{\frac{3}{2}}\beta_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -\sqrt{\frac{3}{2}}\beta_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
-\sqrt{\frac{3}{2}}\beta_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -\sqrt{\frac{3}{2}}\beta_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -\sqrt{\frac{3}{2}}\beta_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -\sqrt{\frac{3}{2}}\beta_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
-\sqrt{\frac{3}{2}}\beta_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -\sqrt{\frac{3}{2}}\beta_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{vmatrix},$$

$$\Gamma_{16\times 12} = \begin{vmatrix}
\cdot & \cdot & \sqrt{\frac{3}{2}}\beta_5 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_6 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_7 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_8 & \cdot \\
\cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_5 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_6 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_7 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_8 \\
\sqrt{\frac{3}{2}}\beta_5 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_6 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_7 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_8 & \cdot & \cdot & \cdot \\
\cdot & \sqrt{\frac{3}{2}}\beta_5 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_6 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_7 & \cdot & \cdot & \cdot & \sqrt{\frac{3}{2}}\beta_8 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{vmatrix};$$

therefore

$$\Gamma_4^{(\text{can})} = \begin{vmatrix} \Gamma_{16\times 16} & \Gamma_{12\times 16} \\ \Gamma_{16\times 12} & 0 \end{vmatrix} = A^{+}\Gamma_4^{\text{G-Y(mod)}} A, \tag{45}$$

where

$$\Gamma_{16\times 16} = \begin{vmatrix}
\cdot & \cdot & \lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 & \cdot & \cdot
\end{vmatrix},$$

$$
\Gamma_{12\times 16} = \begin{vmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{\beta_1}{\sqrt{2}} & \cdot & \beta_1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_1 & \cdot & \frac{\beta_1}{\sqrt{2}} & \cdot \\
\cdot & \frac{\beta_1}{\sqrt{2}} & \cdot & -\beta_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \beta_1 & \cdot & -\frac{\beta_1}{\sqrt{2}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{\beta_2}{\sqrt{2}} & \cdot & \beta_2 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_2 & \cdot & \frac{\beta_2}{\sqrt{2}} & \cdot \\
\cdot & \frac{\beta_2}{\sqrt{2}} & \cdot & -\beta_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \beta_2 & \cdot & -\frac{\beta_2}{\sqrt{2}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{\beta_3}{\sqrt{2}} & \cdot & \beta_3 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_3 & \cdot & \frac{\beta_3}{\sqrt{2}} & \cdot \\
\cdot & \frac{\beta_3}{\sqrt{2}} & \cdot & -\beta_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \beta_3 & \cdot & -\frac{\beta_3}{\sqrt{2}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{\beta_4}{\sqrt{2}} & \cdot & \beta_4 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_4 & \cdot & \frac{\beta_4}{\sqrt{2}} & \cdot \\
\cdot & \frac{\beta_4}{\sqrt{2}} & \cdot & -\beta_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \beta_4 & \cdot & -\frac{\beta_4}{\sqrt{2}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{vmatrix},
$$

$$
\Gamma_{16\times 12} = \begin{vmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -\frac{\beta_5}{\sqrt{2}} & \cdot & \cdot & \cdot & -\frac{\beta_6}{\sqrt{2}} & \cdot & \cdot & \cdot & -\frac{\beta_7}{\sqrt{2}} & \cdot & \cdot & \cdot & -\frac{\beta_8}{\sqrt{2}} & \cdot \\
\cdot & \cdot & \cdot & -\beta_5 & \cdot & \cdot & \cdot & -\beta_6 & \cdot & \cdot & \cdot & -\beta_7 & \cdot & \cdot & \cdot & -\beta_8 \\
\cdot & \cdot & \beta_5 & \cdot & \cdot & \cdot & \beta_6 & \cdot & \cdot & \cdot & \beta_7 & \cdot & \cdot & \cdot & \beta_8 & \cdot \\
\cdot & \cdot & \cdot & \frac{\beta_5}{\sqrt{2}} & \cdot & \cdot & \cdot & \frac{\beta_6}{\sqrt{2}} & \cdot & \cdot & \cdot & \frac{\beta_7}{\sqrt{2}} & \cdot & \cdot & \cdot & \frac{\beta_8}{\sqrt{2}} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\frac{\beta_5}{\sqrt{2}} & \cdot & \cdot & \cdot & \frac{\beta_6}{\sqrt{2}} & \cdot & \cdot & \cdot & \frac{\beta_7}{\sqrt{2}} & \cdot & \cdot & \cdot & \frac{\beta_8}{\sqrt{2}} & \cdot & \cdot & \cdot \\
\cdot & \beta_5 & \cdot & \cdot & \cdot & \beta_6 & \cdot & \cdot & \cdot & \beta_7 & \cdot & \cdot & \cdot & \beta_8 & \cdot & \cdot \\
-\beta_5 & \cdot & \cdot & \cdot & -\beta_6 & \cdot & \cdot & \cdot & -\beta_7 & \cdot & \cdot & \cdot & -\beta_8 & \cdot & \cdot & \cdot \\
\cdot & -\frac{\beta_5}{\sqrt{2}} & \cdot & \cdot & \cdot & -\frac{\beta_6}{\sqrt{2}} & \cdot & \cdot & \cdot & -\frac{\beta_7}{\sqrt{2}} & \cdot & \cdot & \cdot & -\frac{\beta_8}{\sqrt{2}} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{vmatrix}.
$$

When defining the matrix $\Gamma_4^{(\text{spin})}$ in spinor basis, we apply the general formula

$$
\Psi^{(l,l')}_{(l_3,l'_3)} = \Big[\frac{(2l)!}{(l+l_3)!(l-l_3)!}\Big]^{1/2}\Big[\frac{(2l')!}{(l'+l'_3)!(l'-l'_3)!}\Big]^{1/2}\Psi^{(\dot{1}\ldots\dot{1}\dot{2}\ldots\dot{2})}_{(1\ldots 12\ldots 2)}. \tag{46}
$$

As the result we obtain the following linear transformation

$$
\Psi^{(\text{can})} = B\Psi^{(\text{spin})}, \tag{47}
$$

where

$$
\begin{gathered}
\Psi^{(\text{spin})} = \Big\{(\Psi^{(1)})^{\dot{1}}, (\Psi^{(1)})^{\dot{2}}, (\Psi^{(1)})_1, (\Psi^{(1)})_2; (\Psi^{(2)})^{\dot{1}}, (\Psi^{(2)})^{\dot{2}}, (\Psi^{(2)})_1, \\
(\Psi^{(2)})_2; (\Psi^{(3)})^{\dot{1}}, (\Psi^{(3)})^{\dot{2}}, (\Psi^{(3)})_1, (\Psi^{(3)})_2; (\Psi^{(4)})^{\dot{1}}, (\Psi^{(4)})^{\dot{2}}, \\
(\Psi^{(4)})_1, (\Psi^{(4)})_2; \Psi^{\dot{1}}_{(11)}, \Psi^{\dot{1}}_{(12)}, \Psi^{\dot{1}}_{(22)}; \Psi^{\dot{2}}_{(11)}, \Psi^{\dot{2}}_{(12)}, \Psi^{\dot{2}}_{(22)}; \\
\Psi_1^{(\dot{1}\dot{1})}, \Psi_1^{(\dot{1}\dot{2})}, \Psi_1^{(\dot{2}\dot{2})}, \Psi_2^{(\dot{1}\dot{1})}, \Psi_2^{(\dot{1}\dot{2})}, \Psi_2^{(\dot{2}\dot{2})}\Big\},
\end{gathered} \tag{48}
$$

$$
B = \begin{vmatrix}
I_{16\times 16} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \sqrt{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{2} & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{2} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1
\end{vmatrix},
$$

$$
B^{-1} = \begin{vmatrix}
I_{16\times 16} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 1/\sqrt{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1/\sqrt{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1/\sqrt{2} & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1/\sqrt{2} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1
\end{vmatrix},
$$

$$
\Psi^{(\text{spin})} = B^{-1}\Psi^{(\text{can})}, \quad \Gamma_4^{(\text{spin})} = B^{-1}\Gamma_4^{(\text{can})}B.
$$

In explicit form $\Gamma_4^{(\text{spin})}$ reads

$$
\Gamma_4^{(\text{spin})} = B^{-1}\Gamma_4^{(\text{can})}B = \begin{vmatrix} \Gamma_{16\times 16} & \Gamma_{12\times 16} \\ \Gamma_{16\times 12} & 0 \end{vmatrix}, \tag{49}
$$

$$
\Gamma_{16\times 16} = \begin{vmatrix}
\cdot & \cdot & \lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \lambda_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_4 & \cdot & \cdot
\end{vmatrix},
$$

$$
\Gamma_{12\times 16} = \begin{vmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_1 & \cdot & \beta_1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_1 & \cdot & \beta_1 & \cdot \\
\cdot & \beta_1 & \cdot & -\beta_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \beta_1 & \cdot & -\beta_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_2 & \cdot & \beta_2 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_2 & \cdot & \beta_2 & \cdot \\
\cdot & \beta_2 & \cdot & -\beta_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \beta_2 & \cdot & -\beta_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_3 & \cdot & \beta_3 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_3 & \cdot & \beta_3 & \cdot \\
\cdot & \beta_3 & \cdot & -\beta_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \beta_3 & \cdot & -\beta_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_4 & \cdot & \beta_4 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_4 & \cdot & \beta_4 & \cdot \\
\cdot & \beta_4 & \cdot & -\beta_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \beta_4 & \cdot & -\beta_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{vmatrix},
$$

$$
\Gamma_{16\times 12} = \begin{vmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -\frac{\beta_5}{2} & \cdot & \cdot & \cdot & -\frac{\beta_6}{2} & \cdot & \cdot & \cdot & -\frac{\beta_7}{2} & \cdot & \cdot & \cdot & -\frac{\beta_8}{2} & \cdot \\
\cdot & \cdot & \cdot & -\beta_5 & \cdot & \cdot & \cdot & -\beta_6 & \cdot & \cdot & \cdot & -\beta_7 & \cdot & \cdot & \cdot & -\beta_8 \\
\cdot & \cdot & \beta_5 & \cdot & \cdot & \cdot & \beta_6 & \cdot & \cdot & \cdot & \beta_7 & \cdot & \cdot & \cdot & \beta_8 & \cdot \\
\cdot & \cdot & \cdot & \frac{\beta_5}{2} & \cdot & \cdot & \cdot & \frac{\beta_6}{2} & \cdot & \cdot & \cdot & \frac{\beta_7}{2} & \cdot & \cdot & \cdot & \frac{\beta_8}{2} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\frac{\beta_5}{2} & \cdot & \cdot & \cdot & \frac{\beta_6}{2} & \cdot & \cdot & \cdot & \frac{\beta_7}{2} & \cdot & \cdot & \cdot & \frac{\beta_8}{2} & \cdot & \cdot & \cdot \\
\cdot & \beta_5 & \cdot & \cdot & \cdot & \beta_6 & \cdot & \cdot & \cdot & \beta_7 & \cdot & \cdot & \cdot & \beta_8 & \cdot & \cdot \\
-\beta_5 & \cdot & \cdot & \cdot & -\beta_6 & \cdot & \cdot & \cdot & -\beta_7 & \cdot & \cdot & \cdot & -\beta_8 & \cdot & \cdot & \cdot \\
\cdot & -\frac{\beta_5}{2} & \cdot & \cdot & \cdot & -\frac{\beta_6}{2} & \cdot & \cdot & \cdot & -\frac{\beta_7}{2} & \cdot & \cdot & \cdot & -\frac{\beta_8}{2} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{vmatrix}.
$$

The matrix $\Gamma_4^{(\text{spin})}$ may be presented with the use of the spinor elements of the complete matrix algebra:

$$\Gamma_4^{\text{spin}} = \frac{1}{i}\Big\{\lambda_1\Big[(\sigma^4)^{\dot{a}b}e^{\dot{0}_1}_{\dot{a}_1}\otimes e^{o_1}_{b_1} + (\sigma^4)_{a\dot{b}}e^{\dot{b}_1}_{\dot{0}_1}\otimes e^{a_1}_{0_1}\Big]\lambda_2\Big[(\sigma^4)^{\dot{a}b}e^{\dot{0}_2}_{\dot{a}_2}\otimes e^{o_2}_{b_2} + (\sigma^4)_{a\dot{b}}e^{\dot{b}_2}_{\dot{0}_2}\otimes e^{a_2}_{0_2}\Big]$$
$$+\lambda_3\Big[(\sigma^4)^{\dot{a}b}e^{\dot{0}_3}_{\dot{a}_3}\otimes e^{o_3}_{b_3} + (\sigma^4)_{a\dot{b}}e^{\dot{b}_3}_{\dot{0}_3}\otimes e^{a_3}_{0_3}\Big]\lambda_4\Big[(\sigma^4)^{\dot{a}b}e^{\dot{0}_4}_{\dot{a}_4}\otimes e^{o_4}_{b_4} + (\sigma^4)_{a\dot{b}}e^{\dot{b}_4}_{\dot{0}_4}\otimes e^{a_4}_{0_4}\Big]$$
$$+\beta_1(\sigma^4)^b_{\dot{c}}[e^{\dot{a}\dot{c}}_{\dot{a}_1}\otimes e^{0_1}_b + e^{\dot{c}}_{\dot{0}_1}\otimes e^{a_1}_{(ba)}]\beta_2(\sigma^4)^b_{\dot{c}}[e^{\dot{a}\dot{c}}_{\dot{a}_2}\otimes e^{0_2}_b + e^{\dot{c}}_{\dot{0}_2}\otimes e^{a_2}_{(ba)}]$$
$$+\beta_3(\sigma^4)^b_{\dot{c}}[e^{\dot{a}\dot{c}}_{\dot{a}_3}\otimes e^{0_3}_b + e^{\dot{c}}_{\dot{0}_3}\otimes e^{a_3}_{(ba)}] + \beta_4(\sigma^4)^b_{\dot{c}}[e^{\dot{a}\dot{c}}_{\dot{a}_4}\otimes e^{0_4}_b + e^{\dot{c}}_{\dot{0}_4}\otimes e^{a_4}_{(ba)}] + \beta_5(\sigma^4)^{\dot{d}}_a[e^{\dot{b}_1}_{(\dot{a}\dot{b})}\otimes e^a_{0_1} + e^{\dot{0}_1}_{\dot{d}}\otimes e^{(ac)}_{c_1}]$$
$$+\beta_6(\sigma^4)^{\dot{d}}_a[e^{\dot{b}_2}_{(\dot{a}\dot{b})}\otimes e^a_{0_2} + e^{\dot{0}_2}_{\dot{d}}\otimes e^{(ac)}_{c_2}] + \beta_7(\sigma^4)^{\dot{d}}_a[e^{\dot{b}_3}_{(\dot{a}\dot{b})}\otimes e^a_{0_3} + e^{\dot{0}_3}_{\dot{d}}\otimes e^{(ac)}_{c_3}]$$
$$+\beta_8(\sigma^4)^{\dot{d}}_a[e^{\dot{b}_4}_{(\dot{a}\dot{b})}\otimes e^a_{0_4} + e^{\dot{0}_4}_{\dot{d}}\otimes e^{(ac)}_{c_4}]\Big\}. \tag{50}$$

Similarly may be obtained all four matrices $\Gamma_\mu^{(\text{spin})}$; to this end, it suffices to make the formal change $(\sigma^4)\Longrightarrow(\sigma^\mu)$:

$$(\sigma^1)_{\dot{a}b} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}_{\dot{a}b},\ (\sigma^2)_{\dot{a}b} = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix}_{\dot{a}b},\ (\sigma^3)_{\dot{a}b} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}_{\dot{a}b},\ (\sigma^4)_{\dot{a}b}\begin{vmatrix} i & 0 \\ 0 & i \end{vmatrix}_{\dot{a}b}.$$

Recall that the spinor elements of the complete matrix algebra obey the following rules

$$(e^{\dot{A}}_{\dot{B}})^{\dot{C}}_{\dot{D}} = \delta^{\dot{A}}_{\dot{D}}\delta^{\dot{C}}_{\dot{B}},\quad e^{\dot{A}}_{\dot{B}}e^{\dot{C}}_{\dot{D}} = \delta^{\dot{A}}_{\dot{D}}e^{\dot{C}}_{\dot{B}},\quad (e^{A}_{B})^{C}_{D} = \delta^{A}_{D}\delta^{C}_{B},\quad e^{A}_{B}e^{C}_{D} = \delta^{C}_{B}e^{A}_{D}, \tag{51}$$

the digital index at spinors (like as $\dot{a}_1$) determine the number of spinor equation with respect to repeating presentations.

The complete matrix equation determined by four $\Gamma_\mu^{(\text{spin})}$ consists of 5 equations:

$$\lambda_1\Big\{\partial^{\dot{a}b}e^{\dot{0}}_{\dot{a}}\otimes e^0_b + \partial_{a\dot{b}}e^{\dot{b}}_{\dot{0}}\otimes e^a_0\Big\}\Psi^{(1)} + \beta_1\partial^b_{\dot{c}}\Big\{e^{(\dot{a}\dot{c})}_{\dot{a}}\otimes e^0_b$$
$$+e^{\dot{c}}_{\dot{0}}\otimes e^a_{(ba)}\Big\}\Psi + M\Big\{e^{\dot{0}}_{\dot{0}}\otimes e^a_a + e^{\dot{a}}_{\dot{a}}\otimes e^0_0\Big\}\Psi^{(1)} = 0, \tag{52}$$

$$\lambda_2\Big\{\partial^{\dot{a}b}e^{\dot{0}}_{\dot{a}}\otimes e^0_b + \partial_{a\dot{b}}e^{\dot{b}}_{\dot{0}}\otimes e^a_0\Big\}\Psi^{(2)} + \beta_2\partial^b_{\dot{c}}\Big\{e^{(\dot{a}\dot{c})}_{\dot{a}}\otimes e^0_b + e^{\dot{c}}_{\dot{0}}\otimes e^a_{(ba)}\Big\}\Psi$$
$$+M\Big\{e^{\dot{a}}_{\dot{a}}\otimes e^0_0 + e^{\dot{0}}_{\dot{0}}\otimes e^a_a\Big\}\Psi^{(2)} = 0, \tag{53}$$

$$\lambda_3\Big(\partial^{\dot{a}b}e^{\dot{0}}_{\dot{a}}\otimes e^0_b + \partial_{a\dot{b}}e^{\dot{b}}_{\dot{0}}\otimes e^a_0\Big)\Psi^{(3)} + \beta_3\partial^b_{\dot{c}}\Big(e^{(\dot{a}\dot{c})}_{\dot{a}}\otimes e^0_b + e^{\dot{c}}_{\dot{0}}\otimes e^a_{(ba)}\Big)\Psi$$
$$+M\Big\{e^{\dot{a}}_{\dot{a}}\otimes e^0_0 + e^{\dot{0}}_{\dot{0}}\otimes e^a_a\Big\}\Psi^{(3)} = 0, \tag{54}$$

$$\lambda_4\Big(\partial^{\dot{a}b}e^{\dot{0}}_{\dot{a}}\otimes e^0_b + \partial_{a\dot{b}}e^{\dot{b}}_{\dot{0}}\otimes e^a_0\Big)\Psi^{(4)} + \beta_4\partial^b_{\dot{c}}\Big(e^{(\dot{a}\dot{c})}_{\dot{a}}\otimes e^0_b + e^{\dot{c}}_{\dot{0}}\otimes e^a_{(ba)}\Big)\Psi$$
$$+M\Big\{e^{\dot{a}}_{\dot{a}}\otimes e^0_0 + e^{\dot{0}}_{\dot{0}}\otimes e^a_a\Big\}\Psi^{(4)} = 0, \tag{55}$$

$$\partial^{\dot{d}}_a\Big(e^{\dot{b}}_{(\dot{d}\dot{b})}\otimes e^a_0 + e^0_{\dot{d}}\otimes e^{(ac)}_c\Big)\Big(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)}$$
$$+\beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}\Big) + M\Big(e^{(\dot{a}\dot{b})}_{(\dot{a}\dot{b})}\otimes e^c_c + e^{\dot{a}}_{\dot{a}}\otimes e^{(bc)}_{(bc)}\Big)\Psi = 0, \tag{56}$$

where

$$\partial_{\dot{a}b} = \frac{1}{i}\partial_\mu(\sigma^\mu)_{\dot{a}b}. \tag{57}$$

Let us transform the above system (52)–(56) to spinor form. We start with Eq. (52), it leads to (see (51))

$$\lambda_1\Big\{\partial^{\dot{a}b}(e^{\dot{0}}_{\dot{a}})^{\dot{A}}_{\dot{B}}(e^0_b)^C_D\Psi^{(1)}{}^{\dot{B}}_C + \partial_{a\dot{b}}(e^{\dot{b}}_{\dot{0}})^{\dot{A}}_{\dot{B}}(e^a_0)^C_D\Psi^{(1)}{}^{\dot{B}}_D\Big\} + \beta_1\partial^b_{\dot{c}}\Big\{(e^{(\dot{a}\dot{c})}_{\dot{a}})^{\dot{A}}_{\dot{B}}(e^0_b)^C_D\Psi^{\dot{B}}_C + (e^{\dot{c}}_{\dot{0}})^{\dot{A}}_{\dot{B}}(e^a_{(ba)})^C_D\Psi^{\dot{B}}_C\Big\}$$
$$+M\Big\{(e^{\dot{a}}_{\dot{a}})^{\dot{A}}_{\dot{B}}(e^0_0)^C_D(\Psi^{(1)})^{\dot{B}}_C + (e^{\dot{0}}_{\dot{0}})^{\dot{A}}_{\dot{B}}(e^a_a)^C_D\Psi^{(1)}{}^{\dot{B}}_C\Big\} = 0,$$

or

$$\lambda_1 \Big\{ \partial^{\dot{a}b} \delta^{\dot{A}}_{\dot{a}} \delta^0_D \Psi^{(1)}{}^{\dot{0}}_{b} + \partial_{a\dot{b}} \delta^{\dot{A}}_{\dot{0}} \delta^a_D \Psi^{(1)}{}^{\dot{b}}_{0} \Big\} + \beta_1 \partial^b_{\dot{c}} \Big\{ \delta^{\dot{A}}_{\dot{a}} \delta^0_D \Psi^{(\dot{a}\dot{c})}_b + \delta^{\dot{A}}_{\dot{0}} \delta^a_D \Psi^{\dot{c}}_{(ba)} \Big\}$$
$$+ M \Big\{ \delta^{\dot{A}}_{\dot{a}} \delta^0_D (\Psi^{(1)})^{\dot{a}}_0 + \delta^{\dot{A}}_{\dot{0}} \delta^a_D \Psi^{(1)}{}^{\dot{0}}_{a} \Big\} = 0.$$

Let $\dot{A} = \dot{k},\ D = 0$, then we obtain

$$\lambda_1 \partial^{\dot{k}b} \Psi^{(1)}_b + \beta_1 \partial^b_{\dot{c}} \Psi^{(\dot{k}\dot{c})}_b + M \Psi^{(1)\dot{k}} = 0. \tag{58}$$

Let $\dot{A} = \dot{0},\ D = k$, then we get

$$\lambda_1 \partial_{k\dot{b}} \Psi^{(1)\dot{b}} + \beta_1 \partial^b_{\dot{c}} \Psi^{\dot{c}}_{(bk)} + M \Psi^{(1)}_k = 0. \tag{59}$$

Taking in account identities

$$\Psi^{(\dot{a}\dot{b})}_c = \delta^{(\dot{a}\dot{b})}_{(\dot{r}\dot{n})} (\sigma^\mu)^{\dot{r}}_c \Psi^{\dot{n}}_\mu = \frac{1}{2} (\delta^{\dot{a}}_{\dot{r}} \delta^{\dot{b}}_{\dot{n}} + \delta^{\dot{a}}_{\dot{n}} \delta^{\dot{b}}_{\dot{r}}) (\sigma^\mu)^{\dot{r}}_c \Psi^{\dot{n}}_\mu = \frac{1}{2} \Big\{ (\sigma^\mu)^{\dot{a}}_c \Psi^{\dot{b}}_\mu + (\sigma^\mu)^{\dot{b}}_c \Psi^{\dot{a}}_\mu \Big\}, \tag{60}$$

$$\Psi^{\dot{c}}_{(ab)} = \delta^{(rn)}_{(ab)} (\sigma^\mu)^{\dot{c}}_r \Psi_{n\mu} = \frac{1}{2} (\delta^r_a \delta^n_b + \delta^n_a \delta^r_b) (\sigma^\mu)^{\dot{c}}_r \Psi_{n\mu} = \frac{1}{2} \Big\{ (\sigma^\mu)^{\dot{c}}_a \Psi_{b\mu} + (\sigma^\mu)^{\dot{c}}_b \Psi_{a\mu} \Big\}, \tag{61}$$

we derive (see .(58) and (59)

$$\lambda_1 \partial^{\dot{k}b} \Psi^{(1)}_b + \frac{\beta_1}{2} \Big\{ (\sigma^\mu)^{\dot{k}}_b \partial^b_c \Psi^{\dot{c}}_\mu + \partial^b_{\dot{c}} (\sigma^\mu)^{\dot{c}}_b \Psi^{\dot{k}}_\mu \Big\} + M \Psi^{(1)\dot{k}} = 0,$$

$$\lambda_1 \partial_{k\dot{b}} \Psi^{(1)\dot{b}} + \frac{\beta_1}{2} \Big\{ (\sigma^\mu)^{\dot{c}}_k \partial^b_{\dot{c}} \Psi_{b\mu} + \partial^b_{\dot{c}} (\sigma^\mu)^{\dot{c}}_b \Psi_{k\mu} \Big\} + M \Psi^{(1)}_k = 0,$$

or

$$\lambda_1 \partial^{\dot{k}b} \Psi^{(1)}_b + \frac{\beta_1}{2} \Big\{ -(\sigma^\mu)^{\dot{k}b} \partial_{b\dot{c}} \Psi^{\dot{c}}_\mu - \partial^{\dot{c}b} (\sigma^\mu)_{b\dot{c}} \Psi^{\dot{k}}_\mu \Big\} + M \Psi^{(1)\dot{k}} = 0, \tag{62}$$

$$\lambda_1 \partial_{k\dot{b}} \Psi^{(1)\dot{b}} + \frac{\beta_1}{2} \Big\{ -(\sigma^\mu)_{k\dot{c}} \partial^{\dot{c}b} \Psi_{b\mu} - \partial_{b\dot{c}} (\sigma^\mu)^{\dot{c}b} \Psi_{k\mu} \Big\} + M \Psi^{(1)}_k = 0. \tag{63}$$

Joining equations (62) and (63) into a matrix one one, we get

$$\lambda_1 \left| \begin{array}{cc} 0 & \partial^{\dot{k}b} \\ \partial_{k\dot{b}} & 0 \end{array} \right| \left| \begin{array}{c} \Psi^{(1)\dot{b}} \\ \Psi^{(1)}_b \end{array} \right| + \frac{\beta_1}{2} \Big\{ - \left| \begin{array}{cc} (\sigma^\mu)^{\dot{k}b} \partial_{b\dot{c}} & 0 \\ 0 & (\sigma^\mu)_{k\dot{b}} \partial^{\dot{b}c} \end{array} \right| \left| \begin{array}{c} \Psi^{\dot{c}}_\mu \\ \Psi_{c\mu} \end{array} \right|$$
$$- \left| \begin{array}{cc} \partial^{\dot{c}b} (\sigma^\mu)_{b\dot{c}} \delta^{\dot{k}}_{\dot{c}} & 0 \\ 0 & \partial_{a\dot{b}} (\sigma^\mu)^{\dot{b}a} \delta^c_k \end{array} \right| \left| \begin{array}{c} \Psi^{\dot{c}}_\mu \\ \Psi_{c\mu} \end{array} \right| \Big\} + M \left| \begin{array}{c} \Psi^{(1)\dot{k}} \\ \Psi^{(1)}_k \end{array} \right| = 0. \tag{64}$$

Allowing for the identities

$$\sigma^{\mu\dot{a}b} \sigma^\nu_{b\dot{c}} + \sigma^{\nu\dot{a}b} \sigma^\mu_{b\dot{c}} = -2\delta_{\mu\nu} \delta^{\dot{a}}_{\dot{c}},\ \sigma^\mu_{a\dot{b}} \sigma^{\nu\dot{b}c} + \sigma^\nu_{a\dot{b}} \sigma^{\mu\dot{b}c} = -2\delta_{\mu\nu} \delta^c_a,\ \sigma^{\mu a\dot{b}} \sigma^\nu_{\dot{b}a} = \sigma^\mu_{a\dot{b}} \sigma^{\nu\dot{b}a} = -2\delta_{\mu\nu}, \tag{65}$$

we get the following presentation for Dirac matrices

$$\frac{1}{i} \left| \begin{array}{cc} 0 & \sigma^{\mu\dot{a}b} \\ \partial^\mu_{ab} & 0 \end{array} \right| = \gamma_\mu\,; \tag{66}$$

so we arrive at

$$\lambda_1 \hat{\partial} \Psi^{(1)} - 2\beta_1 \Big\{ -\partial_\mu \Psi_\mu - \frac{1}{4} \hat{\partial} (\gamma_\mu \Psi_\mu) \Big\} + M \Psi^{(1)} = 0, \tag{67}$$

where bispinors and vector-bispinor are introduced:

$$\Psi^{(1)} = \left| \begin{array}{c} \Psi^{(1)a'} \\ \Psi^{(1)}_a \end{array} \right|, \quad \Psi^{(1)} = \left| \begin{array}{c} \Psi^{\dot{a}}_\mu \\ \Psi_{a\mu} \end{array} \right|.$$

In similar way, we derive the following equations

$$\lambda_2 \partial^{\dot{k}b} \Psi_b^{(2)} + \beta_2 \partial_{\dot{c}}^b \Psi_b^{(\dot{k}\dot{c})} + M \Psi^{(2)\dot{k}} = 0, \tag{68}$$

$$\lambda_2 \partial_{k\dot{b}} \Psi^{(2)\dot{b}} + \beta_2 \partial_{\dot{b}}^c \Psi_{(kc)}^{\dot{b}} + M \Psi_k^{(2)} = 0, \tag{69}$$

$$\lambda_2 \hat{\partial} \Psi^{(2)} - i2\beta_2 \Big\{ \partial_\mu \Psi_\mu - \frac{1}{4} \hat{\partial} (\gamma_\mu \Psi_\mu) \Big\} + M \Psi^{(2)} = 0; \tag{70}$$

$$\lambda_3 \partial^{\dot{k}b} \Psi_b^{(3)} + \beta_3 \partial_{\dot{c}}^b \Psi_b^{(\dot{k}\dot{c})} + M \Psi^{(3)\dot{k}} = 0, \tag{71}$$

$$\lambda_3 \partial_{k\dot{b}} \Psi^{(3)\dot{b}} + \beta_3 \partial_{\dot{b}}^c \Psi_{(kc)}^{\dot{b}} + M \Psi_k^{(3)} = 0, \tag{72}$$

$$\lambda_3 \hat{\partial} \Psi^{(3)} - i2\beta_3 \Big\{ \partial_\mu \Psi_\mu - \frac{1}{4} \hat{\partial} (\gamma_\mu \Psi_\mu) \Big\} + M \Psi^{(3)} = 0; \tag{73}$$

$$\lambda_4 \partial^{\dot{k}b} \Psi_b^{(4)} + \beta_4 \partial_{\dot{c}}^b \Psi_b^{(\dot{k}\dot{c})} + M \Psi^{(4)\dot{k}} = 0, \tag{74}$$

$$\lambda_4 \partial_{k\dot{b}} \Psi^{(4)\dot{b}} + \beta_4 \partial_{\dot{b}}^c \Psi_{(kc)}^{\dot{b}} + M \Psi_k^{(4)} = 0, \tag{75}$$

$$\lambda_4 \hat{\partial} \Psi^{(4)} - i2\beta_4 \Big\{ \partial_\mu \Psi_\mu - \frac{1}{4} \hat{\partial} (\gamma_\mu \Psi_\mu) \Big\} + M \Psi^{(4)} = 0. \tag{76}$$

Now we turn to Eq. (56), it leads to

$$\partial_a^{\dot{d}} \Big\{ (e_{(\dot{a}\dot{b})}^{\dot{b}})_{\dot{B}}^{\dot{A}} (e_0^a)_D^C + (e_{\dot{d}}^0)_{\dot{B}}^{\dot{A}} (e_c^{(ac)})_D^C \Big\} \Big\{ \beta_5 \Psi^{(1)}{}_C^{\dot{B}}$$

$$+ \beta_6 \Psi^{(2)}{}_C^{\dot{B}} + \beta_7 \Psi^{(3)}{}_C^{\dot{B}} + \beta_8 \Psi^{(4)}{}_C^{\dot{B}} \Big\} + M \Big\{ (e_{(\dot{a}\dot{b})}^{(\dot{a}\dot{b})})(e_c^c)_D^C + (e_{\dot{a}}^{\dot{a}})_{\dot{B}}^{\dot{A}} (e_{(bc)}^{(bc)})_D^C \Big\} \Psi_C^{\dot{B}} = 0,$$

or

$$\partial_a^{\dot{d}} \delta_{(\dot{d}\dot{b})}^{\dot{A}} \delta_D^a (\beta_5 \Psi_0^{(1)\dot{b}} + \beta_6 \Psi^{(2)}{}_0^{\dot{b}} + \beta_7 \Psi^{(3)}{}_0^{\dot{b}} + \beta_8 \Psi^{(4)}{}_0^{\dot{b}})$$

$$+ \partial_a^{\dot{d}} \delta_{\dot{d}}^{\dot{A}} \delta_D^{(ac)} (\beta_5 \Psi^{(1)}{}_c^{\dot{0}} + \beta_6 \Psi^{(2)}{}_c^{\dot{0}} + \beta_7 \Psi^{(3)}{}_c^{\dot{0}} + \beta_8 \Psi^{(4)}{}_c^{0}) + M \Big\{ \delta_{(\dot{a}\dot{b})}^{\dot{A}} \delta_D^c \Psi_c^{(\dot{a}\dot{b})} + \delta_{\dot{a}}^{\dot{A}} \delta_D^{(bc)} \Psi_{(bc)}^{\dot{a}} \Big\} = 0. \tag{77}$$

Let $\dot{A} = (\dot{k}\dot{n}), \quad D = r$; then we get

$$\partial_r^{\dot{d}} \delta_{(\dot{d}\dot{b})}^{(\dot{k}\dot{n})} (\beta_5 \Psi^{(1)}{}^{\dot{b}} + \beta_6 \Psi^{(2)}{}^{\dot{b}} + \beta_7 \Psi^{(3)}{}^{(\dot{b})} + \beta_8 \Psi^{(4)}{}^{\dot{b}} + M \Psi_r^{(\dot{k}\dot{n})} = 0,$$

or

$$\frac{1}{2} \partial_r^{\dot{d}} (\delta_{\dot{d}}^{\dot{k}} \delta_{\dot{b}}^{\dot{n}} + \delta_{\dot{b}}^{\dot{k}} \delta_{\dot{d}}^{\dot{n}}) (\beta_5 \Psi^{(1)}{}^{\dot{b}} + \beta_6 \Psi^{(2)}{}^{\dot{b}} + \beta_7 \Psi^{(3)}{}^{\dot{b}} + \beta_8 \Psi^{(4)}{}^{\dot{b}}) + M \Psi_r^{(\dot{k}\dot{n})} = 0,$$

that is

$$\frac{1}{2} \Big\{ \partial_r^{\dot{k}} (\beta_5 \Psi^{(1)}{}^{\dot{n}} + \beta_6 \Psi^{(2)}{}^{\dot{n}} + \beta_7 \Psi^{(3)}{}^{\dot{n}} + \beta_8 \Psi^{(4)}{}^{\dot{n}})$$

$$+ \partial_r^{\dot{n}} (\beta_5 \Psi^{(1)}{}^{\dot{k}} + \beta_6 \Psi^{(2)}{}^{\dot{k}} + \beta_7 \Psi^{(3)}{}^{\dot{k}} + \beta_8 \Psi^{(4)}{}^{\dot{k}}) \Big\} + M \Psi_r^{(\dot{k}\dot{n})} = 0. \tag{78}$$

Allowing for Eq. (60), we obtain

$$\frac{1}{2} \Big\{ \partial_r^{\dot{k}} (\beta_5 \Psi^{(1)}{}^{\dot{n}} + \beta_6 \Psi^{(2)}{}^{(\dot{n})} + \beta_7 \Psi^{(3)}{}^{\dot{n}} + \beta_8 \Psi^{(4)}{}^{\dot{n}})$$

$$+ \partial_r^{\dot{n}} (\beta_5 \Psi^{(1)}{}^{\dot{k}} + \beta_6 \Psi^{(2)}{}^{\dot{k}} + \beta_7 \Psi^{(3)}{}^{\dot{k}} + \beta_8 \Psi^{(4)}{}^{\dot{k}}) \Big\} + \frac{1}{2} M \Big\{ (\sigma^\mu)_r^{\dot{k}} \Psi_\mu^{\dot{n}} + (\sigma^\mu)_r^{\dot{n}} \Psi_\mu^{\dot{k}} \Big\} = 0. \tag{79}$$

Let $\dot{A} = \dot{r}, D = (kn)$, then from Eq. (77) it follows

$$\frac{1}{2} \Big\{ \partial_k^{\dot{r}} (\beta_5 \Psi_n^{(1)} + \beta_6 \Psi_n^{(2)} + \beta_7 \Psi_n^{(3)} + \beta_8 \Psi_n^{(4)})$$

$$+ \partial_n^{\dot{r}} (\beta_5 \Psi_k^{(1)} + \beta_6 \Psi_k^{(2)} + \beta_7 \Psi_k^{(3)} + \beta_8 \Psi_k^{(4)}) \Big\} + M \Psi_{(kn)}^{\dot{r}} = 0,$$

$$\frac{1}{2} \Big\{ \partial_k^{\dot{r}} (\beta_5 \Psi_n^{(1)} + \beta_6 \Psi_n^{(2)} + \beta_7 \Psi_n^{(3)} + \beta_8 \Psi_n^{(4)})$$

$$+ \partial_n^{\dot{r}} (\beta_5 \Psi_k^{(1)} + \beta_6 \Psi_k^{(2)} + \beta_7 \Psi_k^{(3)} + \beta_8 \Psi_k^{(4)}) \Big\} + \frac{1}{2} M \Big\{ (\sigma^\mu)_k^{\dot{r}} \Psi_{n\mu} + (\sigma^\mu)_n^{\dot{r}} \Psi_{k\mu} \Big\} = 0. \tag{80}$$

Let us act on Eq. (79) by $(\sigma^\lambda)_{\dot{k}}^r$, and on Eq. (80) – by $(\sigma^\lambda)_{\dot{r}}^k 4$ after this we convolute the spinor indices; in this way, we arrive at

$$-\frac{1}{2} \Big\{ \sigma^{\lambda \dot{k} r} \partial_{r\dot{k}} (\beta_5 \Psi^{(1)\dot{n}} + \beta_6 \Psi^{(2)\dot{n}} + \beta_7 \Psi^{(3)\dot{n}} + \beta_8 \Psi^{(4)\dot{n}})$$

$$+\partial^{\dot{n}r}(\sigma^{\lambda})_{r\dot{k}}(\beta_5\Psi^{(1)\dot{k}}+\beta_6\Psi^{(2)\dot{k}}+\beta_7\Psi^{(3)\dot{k}}+\beta_8\Psi^{(4)\dot{k}})\Big\}$$
$$-\frac{1}{2}M\Big\{(\sigma^{\lambda})^{\dot{k}r}(\sigma^{\mu})_{r\dot{k}}\Psi^{\dot{n}}_{\mu}+(\sigma^{\mu})^{\dot{n}r}(\sigma^{\lambda})_{r\dot{k}}\Psi^{\dot{k}}_{\mu}\Big\}=0,$$
$$-\frac{1}{2}\Big\{(\sigma^{\lambda})_{k\dot{r}}\partial^{\dot{r}k}(\beta_5\Psi^{(1)}_n+\beta_6\Psi^{(2)}_n+\beta_7\Psi^{(3)}_n+\beta_8\Psi^{(4)}_n)$$
$$+\partial_{\dot{n}r}(\sigma^{\lambda})_{\dot{r}k}(\beta_5\Psi^{(1)}_k+\beta_6\Psi^{(2)}_k+\beta_7\Psi^{(3)}_k+\beta_8\Psi^{(4)}_k)\Big\}$$
$$-\frac{1}{2}M\Big\{(\sigma^{\lambda})_{k\dot{r}}(\sigma^{\mu})^{\dot{r}k}\Psi_{n\mu}+(\sigma^{\mu})_{n\dot{r}}(\sigma^{\lambda})^{\dot{r}k}\Psi_{k\mu}\Big\}=0,$$

or

$$\frac{1}{2}\Big\{\frac{2}{i}\partial_{\lambda}(\beta_5\Psi^{(1)\dot{n}}+\beta_6\Psi^{(2)\dot{n}}+\beta_7\Psi^{(3)\dot{n}}+\beta_8\Psi^{(4)\dot{n}})$$
$$-\partial^{\dot{n}r}(\sigma^{\lambda})_{r\dot{k}}(\beta_5\Psi^{(1)\dot{k}}+\beta_6\Psi^{(2)\dot{k}}+\beta_7\Psi^{(3)\dot{k}}+\beta_8\Psi^{(4)\dot{k}})\Big\}+\frac{1}{2}M\Big\{2\Psi^{\dot{n}}_{\lambda}-(\sigma^{\mu})^{\dot{n}r}(\sigma^{\lambda})_{r\dot{k}}\Psi^{\dot{k}}_{\mu}\Big\}=0, \qquad (81)$$

$$\frac{1}{2}\Big\{\frac{2}{i}\partial_{\lambda}(\beta_5\Psi^{(1)}_n+\beta_6\Psi^{(2)}_n+\beta_7\Psi^{(3)}_n+\beta_8\Psi^{(4)}_n)$$
$$-\partial_{n\dot{r}}(\sigma^{\lambda})^{\dot{r}k}(\beta_5\Psi^{(1)}_k+\beta_6\Psi^{(2)}_k+\beta_7\Psi^{(3)}_k+\beta_8\Psi^{(4)}_k)\Big\}+\frac{1}{2}M\Big\{2\Psi_{n\lambda}-(\sigma^{\mu})_{n\dot{r}}(\sigma^{\lambda})^{\dot{r}k}\Psi_{k\mu}\Big\}=0. \qquad (82)$$

Joining two last equations(81) and (82) into the matric one, we get

$$\frac{1}{2}\Big\{\frac{2}{i}\partial_{\lambda}\Big[\beta_5\left|\begin{array}{c}\Psi^{(1)\dot{n}}\\ \Psi^{(1)}_n\end{array}\right|+\beta_6\left|\begin{array}{c}\Psi^{(2)\dot{n}}\\ \Psi^{(2)}_n\end{array}\right|+\beta_7\left|\begin{array}{c}\Psi^{(3)\dot{n}}\\ \Psi^{(3)}_n\end{array}\right|+\beta_8\left|\begin{array}{c}\Psi^{(4)\dot{n}}\\ \Psi^{(4)}_n\end{array}\right|\Big]$$
$$\left|\begin{array}{cc}\partial^{\dot{n}r}(\sigma^{\lambda})_{r\dot{k}} & \cdot\\ \cdot & \partial_{n\dot{r}}(\sigma^{\lambda})^{\dot{r}k}\end{array}\right|\Big[\beta_5\left|\begin{array}{c}\Psi^{(1)\dot{k}}\\ \Psi^{(1)}_k\end{array}\right|+\beta_6\left|\begin{array}{c}\Psi^{(2)\dot{k}}\\ \Psi^{(2)}_k\end{array}\right|+\beta_7\left|\begin{array}{c}\Psi^{(3)\dot{k}}\\ \Psi^{(3)}_k\end{array}\right|+\beta_8\left|\begin{array}{c}\Psi^{(4)\dot{k}}\\ \Psi^{(4)}_k\end{array}\right|\Big]$$
$$+\frac{1}{2}M\Big\{2\left|\begin{array}{c}\Psi^{\dot{n}}_{\lambda}\\ \Psi_{n\lambda}\end{array}\right|-\left|\begin{array}{cc}(\sigma^{\mu})^{\dot{n}r}(\sigma^{\lambda})_{r\dot{k}} & \cdot\\ \cdot & (\sigma^{\mu})_{n\dot{r}}(\sigma^{\lambda})^{\dot{r}k}\end{array}\right|\left|\begin{array}{c}\Psi^{\dot{k}}_{\mu}\\ \Psi_{k\mu}\end{array}\right|\Big\}=0.$$

Allowing for (66), we rewrite the last equation in the form

$$\frac{1}{2}\Big\{-i\partial_{\lambda}+i\gamma_{\lambda}\hat{\partial}-2i\partial_{\lambda}\Big\}(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)})+\frac{1}{2}M\Big\{2\Psi_{\lambda}-\gamma_{\lambda}(\gamma_{\mu}\Psi_{\mu})+2\Psi_{\lambda}\Big\}=0,$$

or

$$-i(\partial_{\lambda}-\frac{1}{4}\gamma_{\lambda}\hat{\partial})(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)})+M\Big\{\Psi_{\lambda}-\frac{1}{4}\gamma_{\lambda}(\gamma_{\mu}\Psi_{\mu})\Big\}=0. \qquad (83)$$

3. The minimal form of equations

Let us derive the minimal form of the above system (67),(70),(73),(76),(83). From (83) it follows

$$\Psi_{\lambda}-\frac{1}{4}\gamma_{\lambda}(\gamma_{\mu}\Psi_{\mu})=\frac{i}{M}(\partial_{\lambda}-\frac{1}{4}\gamma_{\lambda}\hat{\partial})\Big(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)}\Big). \qquad (84)$$

Let us substitute this expression (84) in equations (67), (70), (73), (76). We will take into account the identities

$$\partial_{\lambda}(\Psi_{\lambda}-\frac{1}{4}\gamma_{\lambda}(\gamma_{\mu}\Psi_{\mu}))=\partial_{\lambda}\Psi_{\lambda}-\frac{1}{4}\hat{\partial}(\gamma_{\mu}\Psi_{\mu})=\frac{i}{M}(\Box-\frac{1}{4}\hat{\partial}\hat{\partial})(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)})$$
$$=\frac{i3}{4M}\Box(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)}),$$

so we derive

$$(M+\lambda_1\hat{\partial})\Psi^{(1)}+\frac{3\beta_1}{2M}\Box(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)})=0, \qquad (85)$$
$$(M+\lambda_2\hat{\partial})\Psi^{(2)}+\frac{3\beta_2}{2M}\Box(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)})=0, \qquad (86)$$
$$(M+\lambda_3\hat{\partial})\Psi^{(3)}+\frac{3\beta_3}{2M}\Box(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)})=0, \qquad (87)$$
$$(M+\lambda_4\hat{\partial})\Psi^{(4)}+\frac{3\beta_4}{2M}\Box(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)})=0. \qquad (88)$$

Let us act on Eq. (85) be operator

$$\beta_5(M+\lambda_2\hat{\partial})(M+\lambda_3\hat{\partial})(M+\lambda_4\hat{\partial});$$

on Eq. (86) – by

$$\beta_6(M+\lambda_1\hat{\partial})(M+\lambda_3\hat{\partial})(M+\lambda_4\hat{\partial});$$

on Eq. (87) by

$$\beta_7(M+\lambda_1\hat{\partial})(M+\lambda_2\hat{\partial})(M+\lambda_4\hat{\partial});$$

on Eq. (88) – by

$$\beta_8(M+\lambda_1\hat{\partial})(M+\lambda_2\hat{\partial})(M+\lambda_3\hat{\partial});$$

and sum the results – this gives

$$\begin{gathered}
\Big\{M^4+M^3(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\hat{\partial}+M^2(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)\Box \\
+M(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)\hat{\partial}\Box+\lambda_1\lambda_2\lambda_3\lambda_4\Box\Box\Big\}(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)}) \\
+\frac{3}{2M}\Big\{M^3(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8) \\
+M^2[(\lambda_2+\lambda_3+\lambda_4)\beta_1\beta_5+(\lambda_1+\lambda_3+\lambda_4)\beta_2\beta_6+(\lambda_1+\lambda_2+\lambda_4)\beta_3\beta_7 \\
+(\lambda_1+\lambda_2+\lambda_3)\beta_4\beta_8]\hat{\partial}+M[\beta_1\beta_5(\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4) \\
+\beta_2\beta_6(\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_3\lambda_4)+\beta_3\beta_7(\lambda_1\lambda_2+\lambda_1\lambda_4+\lambda_2\lambda_4)+\beta_4\beta_8(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)]\Box \\
+[\lambda_1\lambda_2(\lambda_4\beta_3\beta_7+\lambda_3\beta_4\beta_8)+\lambda_3\lambda_4(\lambda_2\beta_1\beta_5+\lambda_1\beta_2\beta_6)]\hat{\partial}\Box\Big\} \\
\times\Box(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)})=0,
\end{gathered}$$

or

$$\begin{gathered}
\Big\{M^4+M^3(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\hat{\partial}+M^2(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4) \\
+\frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\Box \\
+M\Big[(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)+\frac{3}{2}((\lambda_2+\lambda_3+\lambda_4)\beta_1\beta_5 \\
+(\lambda_1+\lambda_3+\lambda_4)\beta_2\beta_6+(\lambda_1+\lambda_2+\lambda_4)\beta_3\beta_7+(\lambda_1+\lambda_2+\lambda_3)\beta_4\beta_8)\Big]\hat{\partial}\Box \\
+\Big[\lambda_1\lambda_2\lambda_3\lambda_4+\frac{3}{2}(\beta_1\beta_5(\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4) \\
+\beta_2\beta_6(\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_3\lambda_4)+\beta_3\beta_7(\lambda_1\lambda_2+\lambda_1\lambda_4+\lambda_2\lambda_4)+\beta_4\beta_8(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3))\Big]\Box\Box \\
+[\lambda_1\lambda_2(\lambda_4\beta_3\beta_7+\lambda_3\beta_4\beta_8)+\lambda_3\lambda_4(\lambda_2\beta_1\beta_5+\lambda_1\beta_2\beta_6)]\hat{\partial}\Box\Box\Big\} \\
\times(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)})=0.
\end{gathered} \tag{89}$$

Taking in mind restrictions (16), (20), (28), (37), (41) for parameters λ_i, β_k, we derive

$$(M+\hat{\partial})\Big(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)}\Big)=0, \tag{90}$$

which is the minimal Dirac equation for a free p[article with spin 1/2, with respect to the following bispinor

$$\Phi=\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)}. \tag{91}$$

4. Equation in presence of electromagnetic fields

We start with the equations in presence of external electromagnetic fields ($D_\mu = \partial_\mu - ieA_\mu$

$$(M + \lambda_1 \hat{D})\Psi^{(1)} - i2\beta_1 \Big\{ D_\mu \Psi_\mu - \frac{1}{4}\hat{D}(\gamma_\mu \Psi_\mu) \Big\} = 0, \tag{92}$$

$$(M + \lambda_2 \hat{D})\Psi^{(2)} - i2\beta_2 \Big\{ D_\mu \Psi_\mu - \frac{1}{4}\hat{D}(\gamma_\mu \Psi_\mu) \Big\} = 0, \tag{93}$$

$$(M + \lambda_3 \hat{D})\Psi^{(3)} - i2\beta_3 \Big\{ D_\mu \Psi_\mu - \frac{1}{4}\hat{D}(\gamma_\mu \Psi_\mu) \Big\} = 0, \tag{94}$$

$$(M + \lambda_4 \hat{D})\Psi^{(4)} - i2\beta_4 \Big\{ D_\mu \Psi_\mu - \frac{1}{4}\hat{D}(\gamma_\mu \Psi_\mu) \Big\} = 0, \tag{95}$$

$$-i(D_\lambda - \frac{1}{4}\gamma_\lambda \hat{D})(\beta_5 \Psi^{(1)} + \beta_6 \Psi^{(2)} + \beta_7 \Psi^{(3)} + \beta_8 \Psi^{(4)}) + M\Big\{ \Psi_\lambda - \frac{1}{4}\gamma_\lambda(\gamma_\mu \Psi_\mu) \Big\} = 0. \tag{96}$$

From Eq. (96), it follows

$$\Psi_\lambda - \frac{1}{4}\gamma_\lambda(\gamma_\mu \Psi_\mu) = \frac{i}{M}(D_\lambda - \frac{1}{4}\gamma_\lambda \hat{D})(\beta_5 \Psi^{(1)} + \beta_6 \Psi^{(2)} + \beta_7 \Psi^{(3)} + \beta_8 \Psi^{(4)}).$$

Correspondingly, we obtain

$$D_\mu \Psi_\mu - \frac{1}{4}\hat{D}(\gamma_\mu \Psi_\mu) = \frac{i}{M}(D^2 - \frac{1}{4}\hat{D}\hat{D})(\beta_5 \Psi^{(1)} + \beta_6 \Psi^{(2)} + \beta_7 \Psi^{(3)} + \beta_8 \Psi^{(4)}). \tag{97}$$

Taking into account this identity (97) in equations (92)–(95), we derive

$$(M + \lambda_1 D)\Psi^{(1)} + \frac{2\beta_1}{M}(D^2 - \frac{1}{4}\hat{D}\hat{D})(\beta_5 \Psi^{(1)} + \beta_6 \Psi^{(2)} + \beta_7 \Psi^{(3)} + \beta_8 \Psi^{(4)}) = 0, \tag{98}$$

$$(M + \lambda_2 D)\Psi^{(2)} + \frac{2\beta_2}{M}(D^2 - \frac{1}{4}\hat{D}\hat{D})(\beta_5 \Psi^{(1)} + \beta_6 \Psi^{(2)} + \beta_7 \Psi^{(3)} + \beta_8 \Psi^{(4)}) = 0, \tag{99}$$

$$(M + \lambda_3 D)\Psi^{(3)} + \frac{2\beta_3}{M}(D^2 - \frac{1}{4}\hat{D}\hat{D})(\beta_5 \Psi^{(1)} + \beta_6 \Psi^{(2)} + \beta_7 \Psi^{(3)} + \beta_8 \Psi^{(4)}) = 0, \tag{100}$$

$$(M + \lambda_4 D)\Psi^{(4)} + \frac{2\beta_4}{M}(D^2 - \frac{1}{4}\hat{D}\hat{D})(\beta_5 \Psi^{(1)} + \beta_6 \Psi^{(2)} + \beta_7 \Psi^{(3)} + \beta_8 \Psi^{(4)}) = 0. \tag{101}$$

Let us act on Eq. (98) by operator

$$\beta_5(M + \lambda_2 \hat{D})(M + \lambda_3 \hat{D})(M + \lambda_4 \hat{D}),$$

on Eq. (99) – by

$$\beta_6(M + \lambda_1 \hat{D})(M + \lambda_3 \hat{D})(M + \lambda_4 \hat{D}),$$

in Eq. (100) – by

$$\beta_7(M + \lambda_1 \hat{D})(M + \lambda_2 \hat{D})(M + \lambda_4 \hat{D}),$$

on Eq. (101) – by

$$\beta_8(M + \lambda_1 \hat{D})(M + \lambda_2 \hat{D})(M + \lambda_3 \hat{D}),$$

and sum the results; so we derive

$$\Big\{ M^4 + M^3 \hat{D} + M^2(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4)\hat{D}\hat{D}$$

$$+M(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4)\hat{D}\hat{D}\hat{D}$$

$$+\lambda_1\lambda_2\lambda_3\lambda_4 \hat{D}\hat{D}\hat{D}\hat{D} \Big\}(\beta_5 \Psi^{(1)} + \beta_6 \Psi^{(2)} + \beta_7 \Psi^{(3)} + \beta_8 \Psi^{(4)})$$

$$+\frac{2}{M}\Big\{ \beta_1\beta_5[M^3 + M^2(\lambda_2 + \lambda_3 + \lambda_4)\hat{D} + M(\lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4)\hat{D}\hat{D} + \lambda_2\lambda_3\lambda_4 \hat{D}\hat{D}\hat{D}]$$

$$+\beta_2\beta_6[M^3 + M^2(\lambda_1 + \lambda_3 + \lambda_4)\hat{D} + M(\lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_3\lambda_4)\hat{D}\hat{D} + \lambda_1\lambda_3\lambda_4 \hat{D}\hat{D}\hat{D}]$$

$$+\beta_3\beta_7[M^3 + M^2(\lambda_1 + \lambda_2 + \lambda_4)\hat{D} + M(\lambda_1\lambda_2 + \lambda_1\lambda_4 + \lambda_2\lambda_4)\hat{D}\hat{D} + \lambda_1\lambda_2\lambda_4 \hat{D}\hat{D}\hat{D}]$$

$$+\beta_4\beta_8[M^3+M^2(\lambda_1+\lambda_2+\lambda_3)\hat{D}+M(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)\hat{D}\hat{D}+\lambda_1\lambda_2\lambda_3\hat{D}\hat{D}\hat{D}]\Big\}$$
$$\times(D^2-\frac{1}{4}\hat{D}\hat{D})\Big(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)}\Big)=0. \tag{102}$$

We will use the identity $\hat{D}\hat{D}=D^2-ieF_{[\mu\nu]}I_{[\mu\nu]}$, so that

$$D^2=\hat{D}\hat{D}+ieF_{[\mu\nu]}I_{[\mu\nu]}; \tag{103}$$

where $F_{[\mu\nu]}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the electromagnetic tensor, and $I_{[\mu\nu]}=\frac{1}{4}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$. Allowing for relations (103), we transform Eq. (102) to the form

$$\Big\{M^4+M^3\hat{D}+M^2[(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)$$
$$+\frac{3}{2}(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)]\hat{D}\hat{D}$$
$$+M[(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)$$
$$+\frac{3}{2}\beta_1\beta_5(\lambda_2+\lambda_3+\lambda_4)+\frac{3}{2}\beta_2\beta_6(\lambda_1+\lambda_3+\lambda_4)+\frac{3}{2}\beta_3\beta_7(\lambda_1+\lambda_2+\lambda_4)+\frac{3}{2}\beta_4\beta_8(\lambda_1+\lambda_2+\lambda_3)]\hat{D}\hat{D}\hat{D}$$
$$+\Big[\lambda_1\lambda_2\lambda_3\lambda_4+\frac{3}{2}\beta_1\beta_5(\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)+\frac{3}{2}\beta_2\beta_6(\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_3\lambda_4)$$
$$+\frac{3}{2}\beta_3\beta_7(\lambda_1\lambda_2+\lambda_1\lambda_4+\lambda_2\lambda_4)+\frac{3}{2}\beta_4\beta_8(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)\Big]\hat{D}\hat{D}\hat{D}\hat{D}$$
$$+\frac{3}{2M}[\beta_1\beta_5\lambda_2\lambda_3\lambda_4+\beta_2\beta_6\lambda_1\lambda_3\lambda_4+\beta_3\beta_7\lambda_1\lambda_2\lambda_4+\beta_4\beta_8\lambda_1\lambda_2\lambda_3]\hat{D}\hat{D}\hat{D}\hat{D}\hat{D}\Big\}(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)})$$
$$+\frac{2ie}{M}\Big\{M^3(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)F_{[\mu\nu]}I_{[\mu\nu]}$$
$$+M^2[\beta_1\beta_5(\lambda_2+\lambda_3+\lambda_4)+\frac{3}{2}\beta_2\beta_6(\lambda_1+\lambda_3+\lambda_4)+\frac{3}{2}\beta_3\beta_7(\lambda_1+\lambda_2+\lambda_4)+\frac{3}{2}\beta_4\beta_8(\lambda_1+\lambda_2+\lambda_3)]\hat{D}F_{[\mu\nu]}I_{[\mu\nu]}$$
$$+M[\beta_1\beta_5(\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)+\beta_2\beta_6(\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_3\lambda_4)$$
$$+\beta_3\beta_7(\lambda_1\lambda_2+\lambda_1\lambda_4+\lambda_2\lambda_4)+\beta_4\beta_8(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)]\hat{D}\hat{D}F_{[\mu\nu]}I_{[\mu\nu]}$$
$$+[\beta_1\beta_5\lambda_2\lambda_3\lambda_4+\beta_2\beta_6\lambda_1\lambda_3\lambda_4+\beta_3\beta_7\lambda_1\lambda_2\lambda_4+\beta_4\beta_8\lambda_1\lambda_2\lambda_3]\hat{D}\hat{D}\hat{D}F_{[\mu\nu]}I_{[\mu\nu]}\Big\}$$
$$\times\Big(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)}\Big)=0.$$

Taking in mind restrictions (20), (28), (41), (37), and (91), we derive

$$\Big\{M^4+M^3\hat{D}+2ieM^2(\beta_1\beta_5+\beta_2\beta_6+\beta_3\beta_7+\beta_4\beta_8)F_{[\mu\nu]}I_{[\mu\nu]}$$
$$+2ieM[\beta_1\beta_5(\lambda_2+\lambda_3+\lambda_4)+\beta_2\beta_6(\lambda_1+\lambda_3+\lambda_4)+\beta_3\beta_7(\lambda_1+\lambda_2+\lambda_4)+\beta_4\beta_8(\lambda_1+\lambda_2+\lambda_3)]\hat{D}F_{[\mu\nu]}I_{[\mu\nu]}$$
$$+2ie[\beta_1\beta_5(\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)+\beta_2\beta_6(\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_3\lambda_4)$$
$$+\beta_3\beta_7(\lambda_1\lambda_2+\lambda_1\lambda_4+\lambda_2\lambda_4)+\beta_4\beta_8(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)]\hat{D}\hat{D}F_{[\mu\nu]}I_{[\mu\nu]}\Big\}\Phi=0, \tag{104}$$

or differently

$$M+\hat{D}-ie\frac{4}{3}\Big[\frac{1}{M}(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)F_{[\mu\nu]}I_{[\mu\nu]}$$
$$+\frac{1}{M^2}(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)\hat{D}F_{[\mu\nu]}I_{[\mu\nu]}++\frac{1}{M^3}\lambda_1\lambda_2\lambda_3\lambda_4\hat{D}\hat{D}F_{\mu\nu}I_{[\mu\nu]}\Big]\Phi=0. \tag{105}$$

Allowing for the identities

$$\hat{D}\hat{D}=D^2-ieF_{[\rho\lambda]}I_{[\rho\lambda]},$$
$$\gamma_\mu\gamma_\nu\gamma_\rho=\delta_{\mu\nu}\gamma_\rho-\delta_{\mu\rho}\gamma_\nu+\delta_{\nu\rho}\gamma_\mu+\epsilon_{\mu\nu\rho\eta}\gamma_5\gamma_\eta,\quad \gamma_5\gamma_\mu\gamma_\nu=\delta_{\mu\nu}\gamma_5-\frac{1}{2}\epsilon_{\mu\nu\rho\lambda}\gamma_\rho\gamma_\lambda.$$

we get

$$\frac{1}{M^3}\lambda_1\lambda_2\lambda_3\lambda_4\Big\{D^2F_{[\mu\nu]}I_{[\mu\nu]}\Phi-ieF_{[\rho\lambda]}F_{[\mu\nu]}I_{[\rho\lambda]}I_{[\mu\nu]}\Phi\Big\}=\frac{1}{M^3}\lambda_1\lambda_2\lambda_3\lambda_4D^2F_{[\mu\nu]}I_{[\mu\nu]}\Phi,$$
$$-\frac{ie}{M^3}\lambda_1\lambda_2\lambda_3\lambda_4F_{[\rho\lambda]}F_{[\mu\nu]}I_{[\rho\lambda]}I_{[\mu\nu]}=\frac{1}{M^3}\lambda_1\lambda_2\lambda_3\lambda_4D^2F_{[\mu\nu]}I_{[\mu\nu]}\Phi,$$
$$-\frac{ie}{M^3}\lambda_1\lambda_2\lambda_3\lambda_4\Big\{F_{[\mu\nu]}I_{[\nu\rho]}\gamma_\mu\gamma_\rho+\frac{1}{4}\epsilon_{[\rho\lambda]}F_{[\mu\nu]}\gamma_5-\frac{1}{2}F_{[\mu\nu]}F_{[\nu\mu]}\Big\}\Phi.$$

Therefore, Eq. (124) may be presented as

$$\Big\{M + \gamma_\sigma D_\sigma - \frac{ie}{M}\frac{4}{3}\Big(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4\Big)\, F_{[\mu\nu]}I_{[\mu\nu]}$$
$$-\frac{ie}{M^2}\frac{4}{3}\Big(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\Big)\,\hat{D}F_{[\mu\nu]}I_{\mu\nu}$$
$$-\frac{ie}{M^3}\frac{4}{3}\lambda_1\lambda_2\lambda_3\lambda_4\Big(D^2F_{[\mu\nu]}I_{[\mu\nu]} + F_{[\mu\nu]}F_{[\nu\rho]}\gamma_\mu\gamma_\rho + \frac{1}{2}F_{[\mu\nu]}F_{[\mu\nu]} + \frac{1}{4}\epsilon_{\rho\lambda\mu\nu}F_{[\rho\lambda]}F_{[\mu\nu]}\gamma_5\Big)\Phi = 0. \tag{106}$$

Let us introduce the notations

$$\mu = \frac{4}{3}\Big(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4\Big),$$
$$\sigma = \frac{4}{3}\Big(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\Big), \quad \eta = \frac{4}{3}\,\lambda_1\lambda_2\lambda_3\lambda_4;$$

then the above equation reads

$$\Big\{M + \gamma_\rho D_\rho - \frac{ie}{M}\mu\; F_{\mu\beta}I_{\mu\beta} - \frac{ie}{M^2}\sigma\;\hat{D}F_{\mu\beta}I_{\mu\beta}$$
$$-\frac{ie}{M^3}\eta\;\Big(D^2F_{\alpha\beta}I_{\alpha\beta} + F_{\alpha\beta}F_{\beta\rho}\gamma_\alpha\gamma_\rho + \frac{1}{2}F_{\alpha\beta}F_{[\alpha\beta]} + \frac{1}{4}\epsilon_{\rho\lambda\alpha\beta}F_{\rho\lambda}F_{\alpha\beta}\gamma_5\Big)\Phi = 0. \tag{107}$$

This equation should be considered as related to a spin 1/2 particle which in addition to electric charge has three other electromagnetic characteristics μ, σ, γ. The μ corresponds to the anomalous magnetic moment; the σ corresponds to the polarizability; and the parameter γ should be associated with a certain intrinsic structure which assumes rather complicated additional interaction with the external electromagnetic field.

5. Equation in presence of gravitational fields

Assuming the use of relativistic interval in explicit real-valued form (Minkowski space with the signature $(+,-,-,-)$)

$$dS^2 = c^2dt^2 - dx^2 - dy^2 - dz^2, \quad (t,x,y,z) = (x^a), \quad a = 0,1,2,3$$

one should use the basic equation in the form

$$\Big\{i\gamma^\rho D_\rho - M + \frac{e\,\mu}{M}\;F_{\alpha\beta}J^{\alpha\beta} + \frac{e\,\sigma}{M^2}\;\hat{D}F_{\alpha\beta}J^{\alpha\beta}$$
$$+\frac{e\,\eta}{M^3}\;\Big(D^2F_{\alpha\beta}J^{\alpha\beta} + \gamma^\alpha F_{\alpha\beta}F^\beta{}_\rho\gamma^\rho + \frac{1}{2}F_{\alpha\beta}F^{\alpha\beta} + \frac{1}{4}\,\gamma_5\,\epsilon^{\rho\lambda\alpha\beta}F_{\rho\lambda}F_{\alpha\beta}\Big)\Psi = 0; \tag{108}$$

recall the notaions

$$D_\rho = \partial_\rho + ieA_\rho, \quad \hat{D} = \gamma^\rho D_\rho, \quad D^2 = g^{\alpha\beta}D_\alpha D_\beta, \quad J^{\alpha\beta} = \frac{1}{4}(\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha).$$

Let us extend this approach to curved space-time background. We should start with the modified system of the first order equations

$$(M + \lambda_1\hat{D})\Psi^{(1)} - 2i\beta_1\Big(D^\mu\Psi_\mu - \frac{1}{4}\hat{D}(\gamma^\mu\Psi_\mu)\Big) = 0, \tag{109}$$
$$(M + \lambda_2\hat{D})\Psi^{(2)} - 2i\beta_2\Big(D^\mu\Psi_\mu - \frac{1}{4}\hat{D}(\gamma^\mu\Psi_\mu)\Big) = 0, \tag{110}$$
$$(M + \lambda_3\hat{D})\Psi^{(3)} - 2i\beta_3\Big(D^\mu\Psi_\mu - \frac{1}{4}\hat{D}(\gamma^\mu\Psi_\mu)\Big) = 0, \tag{111}$$
$$(M + \lambda_4\hat{D})\Psi^{(4)} - 2i\beta_4\Big(D^\mu\Psi_\mu - \frac{1}{4}\hat{D}(\gamma^\mu\Psi_\mu)\Big) = 0, \tag{112}$$

$$-i(D_\lambda - \frac{1}{4}\gamma_\lambda\hat{D})(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}) + M\Big\{\Psi_\lambda - \frac{1}{4}\gamma_\lambda(\gamma^\mu\Psi_\mu)\Big\} = 0. \tag{113}$$

In this system, $\Psi^{(a)} (a = 1,2,3,4)$ are covariant bispinors, and Ψ_μ is a covariant vector-bispinor. We apply the generalized derivative D_μ, it takes into account the presence of electromagnetic fields and any curved space-time background; the symbol ∇_μ designates the covariant derivative; the symbol $\Gamma_\mu(x)$ stands for the bispinor connection; $\Gamma_\mu(x)$ represents the local Dirac matrices:

$$D_\mu = \nabla_\mu - ieA_\mu(x) + \Gamma_\mu(x), \quad \hat{D} = \gamma^\mu D_\mu = \gamma^\mu(x)\Big(\nabla_\mu - ieA_\mu + \Gamma_\mu\Big),$$

$$\begin{aligned} \hat{D}\hat{D} &= (\gamma^\alpha D_\alpha)(\gamma^\beta D_\beta) = D_\alpha \frac{\gamma^\alpha\gamma^\beta + \gamma^\beta\gamma^\alpha}{2} D_\beta + D_\alpha \frac{\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha}{2} D_\beta \\ &= g^{\alpha\beta}(x) D_\alpha D_\beta = D^\alpha D_\alpha + J^{\alpha\beta}(x)[D_\alpha, D_\beta]_- = D^2 + \sigma^{\alpha\beta}(x) M_{\alpha\beta}(x), \end{aligned} \tag{114}$$

$$D^2 = D^\alpha D_\alpha, \quad M_{\alpha\beta}(x) = [D_\alpha, D_\beta]_-, \quad J^{\alpha\beta}(x) = \frac{\gamma^\alpha(x)\gamma^\beta(x) - \gamma^\beta(x)\gamma^\alpha(x)}{4}.$$

From Eq. (113), it follows

$$\Psi_\lambda - \frac{1}{4}\gamma_\lambda(\gamma^\mu\Psi_\mu) = \frac{i}{M}\Big(D_\lambda - \frac{1}{4}\gamma_\lambda\hat{D}\Big)\Big(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}\Big).$$

Correspondingly, acting on this relation by D_λ, we obtain

$$D^\lambda\Psi_\lambda - \frac{1}{4}\hat{D}(\gamma^\mu\Psi_\mu) = \frac{i}{M}\Big(D^2 - \frac{1}{4}\hat{D}\hat{D}\Big)\Big(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}\Big). \tag{115}$$

Taking into account the last identity in equations (109)–(112), we derive

$$(M + \lambda_1\hat{D})\Psi^{(1)} + \frac{2\beta_1}{M}\Big(D^2 - \frac{1}{4}\hat{D}\hat{D}\Big)(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}) = 0, \tag{116}$$

$$(M + \lambda_2\hat{D})\Psi^{(2)} + \frac{2\beta_2}{M}\Big(D^2 - \frac{1}{4}\hat{D}\hat{D}\Big)(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}) = 0, \tag{117}$$

$$(M + \lambda_3\hat{D})\Psi^{(3)} + \frac{2\beta_3}{M}\Big(D^2 - \frac{1}{4}\hat{D}\hat{D}\Big)(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}) = 0, \tag{118}$$

$$(M + \lambda_4\hat{D})\Psi^{(4)} + \frac{2\beta_4}{M}\Big(D^2 - \frac{1}{4}\hat{D}\hat{D}\Big)(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}) = 0. \tag{119}$$

Let us act:

on Eq. (116) by operator

$$\beta_5(M + \lambda_2\hat{D})(M + \lambda_3\hat{D})(M + \lambda_4\hat{D}),$$

on Eq. (117) – by

$$\beta_6(M + \lambda_1\hat{D})(M + \lambda_3\hat{D})(M + \lambda_4\hat{D}),$$

in Eq. (118) – by

$$\beta_7(M + \lambda_1\hat{D})(M + \lambda_2\hat{D})(M + \lambda_4\hat{D}),$$

on Eq. (119) – by

$$\beta_8(M + \lambda_1\hat{D})(M + \lambda_2\hat{D})(M + \lambda_3\hat{D}),$$

and sum the results; so we derive

$$\begin{gathered} \Big\{M^4 + M^3\hat{D} + M^2(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4)\hat{D}\hat{D} \\ + M(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4)\hat{D}\hat{D}\hat{D} + \lambda_1\lambda_2\lambda_3\lambda_4\hat{D}\hat{D}\hat{D}\hat{D}\Big\}(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}) \\ + \frac{2}{M}\Big\{\beta_1\beta_5[M^3 + M^2(\lambda_2 + \lambda_3 + \lambda_4)\hat{D} + M(\lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4)\hat{D}\hat{D} + \lambda_2\lambda_3\lambda_4\hat{D}\hat{D}\hat{D}] \\ + \beta_2\beta_6[M^3 + M^2(\lambda_1 + \lambda_3 + \lambda_4)\hat{D} + M(\lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_3\lambda_4)\hat{D}\hat{D} + \lambda_1\lambda_3\lambda_4\hat{D}\hat{D}\hat{D}] \\ + \beta_3\beta_7[M^3 + M^2(\lambda_1 + \lambda_2 + \lambda_4)\hat{D} + M(\lambda_1\lambda_2 + \lambda_1\lambda_4 + \lambda_2\lambda_4)\hat{D}\hat{D} + \lambda_1\lambda_2\lambda_4\hat{D}\hat{D}\hat{D}] \\ + \beta_4\beta_8[M^3 + M^2(\lambda_1 + \lambda_2 + \lambda_3)\hat{D} + M(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)\hat{D}\hat{D} + \lambda_1\lambda_2\lambda_3\hat{D}\hat{D}\hat{D}]\Big\} \\ \times(D^2 - \frac{1}{4}\hat{D}\hat{D})\Big(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}\Big) = 0. \end{gathered} \tag{120}$$

We will use the identity (see (114))

$$\hat{D}\hat{D} = D^2 + J^{\alpha\beta}(x)M_{\alpha\beta}(x) \quad \Longrightarrow \quad D^2 = \hat{D}\hat{D} - M_{\alpha\beta}J^{\alpha\beta}; \tag{121}$$

instead of relation $D^2 = \hat{D}\hat{D} + ieF_{\alpha\beta}J^{\alpha\beta}$ in presence of only electromagnetic field. Therefore, we do not need to repeat the calculation in Minkovski space, it suffices the formal change

$$ieF_{\alpha\beta} \;\to\; -M_{\alpha\beta}.$$

Allowing for relations (120), we transform Eq. (120) to the form

$$\Big\{M^4 + M^3\hat{D} + M^2\Big(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4\Big)\hat{D}^2$$
$$+M\Big(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\Big)\hat{D}^3 + \lambda_1\lambda_2\lambda_3\lambda_4\hat{D}^4\Big\}\Big(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}\Big)$$
$$+\frac{2}{M}\Big\{\beta_1\beta_5\Big[M^3 + M^2(\lambda_2+\lambda_3+\lambda_4)\hat{D} + M(\lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4)\hat{D}^2 + \lambda_2\lambda_3\lambda_4\hat{D}^3\Big]$$
$$+\beta_2\beta_6\Big[M^3 + M^2(\lambda_1+\lambda_3+\lambda_4)\hat{D} + M(\lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_3\lambda_4)\hat{D}^2 + \lambda_1\lambda_3\lambda_4\hat{D}^3\Big]$$
$$+\beta_3\beta_7\Big[M^3 + M^2(\lambda_1+\lambda_2+\lambda_4)\hat{D} + M(\lambda_1\lambda_2 + \lambda_1\lambda_4 + \lambda_2\lambda_4)\hat{D}^2 + \lambda_1\lambda_2\lambda_4\hat{D}^3\Big]$$
$$+\beta_4\beta_8\Big[M^3 + M^2(\lambda_1+\lambda_2+\lambda_3)\hat{D} + M(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)\hat{D}^2 + \lambda_1\lambda_2\lambda_3\hat{D}^3\Big]\Big\}$$
$$\times(\frac{3}{4}\hat{D}^2 - \sigma_{\alpha\beta}J^{\alpha\beta})\Big(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}\Big) = 0. \tag{122}$$

Now, we are grouping the terms with respect to

$$\hat{D}, \quad \hat{D}^2, \quad \hat{D}^3, \quad \hat{D}^4, \quad \hat{D}^5;$$

in this way we obtain

$$\Big\{M^4 + M^3\hat{D} + M^2\Big[\Big(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4\Big)\frac{3}{2}\Big(\beta_1\beta_5 + \beta_2\beta_6 + \beta_3\beta_7 + \beta_4\beta_8\Big)\Big]\hat{D}^2$$
$$+M\Big[\Big(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\Big) + \frac{3}{2}\beta_1\beta_5\Big(\lambda_2+\lambda_3+\lambda_4\Big)$$
$$+\frac{3}{2}\beta_2\beta_6\Big(\lambda_1+\lambda_3+\lambda_4\Big) + \frac{3}{2}\beta_3\beta_7\Big(\lambda_1+\lambda_2+\lambda_4\Big) + \frac{3}{2}\beta_4\beta_8\Big(\lambda_1+\lambda_2+\lambda_3\Big)\Big]\hat{D}^3$$
$$+\Big[\lambda_1\lambda_2\lambda_3\lambda_4 + \frac{3}{2}\beta_1\beta_5\Big(\lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4\Big) + \frac{3}{2}\beta_2\beta_6\Big(\lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_3\lambda_4\Big)$$
$$+\frac{3}{2}\beta_3\beta_7\Big(\lambda_1\lambda_2 + \lambda_1\lambda_4 + \lambda_2\lambda_4\Big) + \frac{3}{2}\beta_4\beta_8\Big(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3\Big)\Big]\hat{D}^4$$
$$+\frac{3}{2M}\Big[\beta_1\beta_5\lambda_2\lambda_3\lambda_4 + \beta_2\beta_6\lambda_1\lambda_3\lambda_4 + \beta_3\beta_7\lambda_1\lambda_2\lambda_4 + \beta_4\beta_8\lambda_1\lambda_2\lambda_3\Big]\hat{D}^5\Big\}\Big(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}\Big)$$
$$-\frac{2}{M}\Big\{M^3\Big(\beta_1\beta_5 + \beta_2\beta_6 + \beta_3\beta_7 + \beta_4\beta_8\Big)M_{\alpha\beta}J^{\alpha\beta}$$
$$+M^2\Big[\beta_1\beta_5\Big(\lambda_2+\lambda_3+\lambda_4\Big) + \frac{3}{2}\beta_2\beta_6\Big(\lambda_1+\lambda_3+\lambda_4\Big) + \frac{3}{2}\beta_3\beta_7\Big(\lambda_1+\lambda_2+\lambda_4\Big) + \frac{3}{2}\beta_4\beta_8\Big(\lambda_1+\lambda_2+\lambda_3\Big)\Big]\hat{D}M_{\alpha\beta}J^{\alpha\beta}$$
$$+M\Big[\beta_1\beta_5\Big(\lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4\Big) + \beta_2\beta_6\Big(\lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_3\lambda_4\Big)$$
$$+\beta_3\beta_7\Big(\lambda_1\lambda_2 + \lambda_1\lambda_4 + \lambda_2\lambda_4\Big) + \beta_4\beta_8\Big(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3\Big)\Big]\hat{D}^2M_{\alpha\beta}J^{\alpha\beta}$$
$$+\Big[\beta_1\beta_5\lambda_2\lambda_3\lambda_4 + \beta_2\beta_6\lambda_1\lambda_3\lambda_4 + \beta_3\beta_7\lambda_1\lambda_2\lambda_4 + \beta_4\beta_8\lambda_1\lambda_2\lambda_3\Big]\hat{D}^3M_{\alpha\beta}J^{\alpha\beta}\Big\}$$
$$\times\Big(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}\Big) = 0.$$

Taking into account restrictions on parameters (20), (28), (41), (37), and also the notation (91)

$$\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)} = \Phi,$$

we derive

$$\Big\{M^4 + M^3\hat{D} - 2M^2\Big(\beta_1\beta_5 + \beta_2\beta_6 + \beta_3\beta_7 + \beta_4\beta_8\Big)M_{\alpha\beta}J^{\alpha\beta}$$
$$-2M\Big[\beta_1\beta_5(\lambda_2+\lambda_3+\lambda_4) + \beta_2\beta_6(\lambda_1+\lambda_3+\lambda_4) + \beta_3\beta_7(\lambda_1+\lambda_2+\lambda_4) + \beta_4\beta_8(\lambda_1+\lambda_2+\lambda_3)\Big]\hat{D}M_{\alpha\beta}J^{\alpha\beta}$$
$$-2\Big[\beta_1\beta_5(\lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4) + \beta_2\beta_6(\lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_3\lambda_4)$$
$$+\beta_3\beta_7(\lambda_1\lambda_2 + \lambda_1\lambda_4 + \lambda_2\lambda_4) + \beta_4\beta_8(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)\Big]\hat{D}\hat{D}M_{\alpha\beta}J^{\alpha\beta}\Big\}\Phi = 0, \tag{123}$$

or differently

$$\Big\{ M + \hat{D} + \frac{4}{3}\Big[\frac{1}{M}\Big(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4\Big) M_{\alpha\beta} J^{\alpha\beta}$$
$$+ \frac{1}{M^2}\Big(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\Big) \hat{D} M_{\alpha\beta} J^{\alpha\beta} + \frac{1}{M^3}\lambda_1\lambda_2\lambda_3\lambda_4 \hat{D}^2 M_{\alpha\beta} J^{\alpha\beta}\Big]\Big\}\Phi = 0.$$

Allowing for the identities $\hat{D}^2 = (D^2 + J^{\rho\sigma} M_{\rho\sigma})$, the last equation transforms to other form

$$\Big\{ M + \hat{D} + \frac{1}{M}\frac{4}{3}\Big(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4\Big) J^{\alpha\beta} M_{\alpha\beta}$$
$$+ \frac{1}{M^2}\frac{4}{3}\Big(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\Big) \hat{D} J^{\alpha\beta} M_{\alpha\beta} + \frac{1}{M^3}\frac{4}{3}\lambda_1\lambda_2\lambda_3\lambda_4 (D^2 + J^{\rho\sigma} M_{\rho\sigma}) J^{\alpha\beta} M_{\alpha\beta}\Big\}\Phi = 0,$$

or differently

$$\Big\{ M + \hat{D} + \frac{1}{M}\frac{4}{3}\Big(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4\Big) J^{\alpha\beta} M_{\alpha\beta}$$
$$+ \frac{1}{M^2}\frac{4}{3}\Big(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\Big) \hat{D} J^{\alpha\beta} M_{\alpha\beta}$$
$$+ \frac{1}{M^3}\frac{4}{3}\lambda_1\lambda_2\lambda_3\lambda_4 \Big(D^2 J^{\alpha\beta} M_{\alpha\beta} + J^{\rho\sigma} J^{\alpha\beta} M_{\rho\sigma} M_{\alpha\beta}\Big)\Big\}\Phi = 0. \tag{124}$$

Further, we will work with the short presentation of the above equation

$$\Big\{ M + \hat{D} + \frac{1}{M}\mu j^{\alpha\beta} M_{\alpha\beta} + \frac{1}{M^2}\sigma \hat{D} j^{\alpha\beta} M_{\alpha\beta}$$
$$+ \frac{1}{M^3}\eta\Big(D^2 j^{\alpha\beta} M_{\alpha\beta} + j^{\rho\sigma} j^{\alpha\beta} M_{\rho\sigma} M_{\alpha\beta}\Big)\Big\}\Phi = 0. \tag{125}$$

Let us detail the commutator $M_{\alpha\beta}$:

$$D_\alpha = \nabla_\alpha + ieA_\mu(x) + \Gamma_\alpha(x), \quad M_{\alpha\beta} = D_\alpha D_\beta - D_\beta D_\alpha;$$

it is (see in [3])

$$M_{\alpha\beta}\Psi = (D_\alpha D_\beta - D_\beta D_\alpha)\Psi = (ieF_{\alpha\beta} + \frac{1}{2} j^{\nu\rho} R_{\nu\rho\alpha\beta})\Psi. \tag{126}$$

Let us consider the term (see in [3])

$$\frac{\mu}{M} j^{\alpha\beta} M_{\alpha\beta}\Psi = \frac{\mu}{M} j^{\alpha\beta}\Big(ieF_{\alpha\beta} + \frac{1}{2} j^{\nu\rho} R_{\nu\rho\alpha\beta}\Big)\Psi$$
$$= \frac{\mu}{M}\Big(ieF_{\alpha\beta} j^{\alpha\beta} + \frac{1}{2} j^{\alpha\beta} j^{\nu\rho} R_{\nu\rho\alpha\beta}\Big)\Psi = \frac{\mu}{M}\Big(ieF_{\alpha\beta} j^{\alpha\beta} - \frac{1}{4} R(x)\Big)\Psi. \tag{127}$$

Consider the term

$$\frac{\sigma}{M^2}\ \hat{D} j^{\alpha\beta} M_{\alpha\beta}\Psi = \frac{1}{M^2}\sigma\ (\gamma^\rho D_\rho)\ \Big(ieF_{\alpha\beta} j^{\alpha\beta} - \frac{1}{4} R(x)\Big)\Psi. \tag{128}$$

Consider the term

$$\frac{\eta}{M^3}\ D^2\ j^{\alpha\beta} M_{\alpha\beta}\Psi = \frac{1}{M^3}\eta\ g^{\rho\sigma}(x) D_\rho D_\sigma\ \Big(ieF_{\alpha\beta} j^{\alpha\beta} - \frac{1}{4} R(x)\Big)\Psi. \tag{129}$$

Now we are to consider the term

$$A = \frac{\eta}{M^3}\ j^{\rho\sigma} j^{\alpha\beta} M_{\rho\sigma}\ M_{\alpha\beta} = \frac{\eta}{M^3}\ j^{\rho\sigma}\Big\{ (j^{\alpha\beta} M_{\rho\sigma} - M_{\rho\sigma} j^{\alpha\beta}) + M_{\rho\sigma} j^{\alpha\beta}\Big\} M_{\alpha\beta}$$
$$= \frac{\eta}{M^3}\ j^{\rho\sigma}\Big\{ \Big(j^{\alpha\beta}(ieF_{\rho\sigma} + \frac{1}{2} j^{\mu\nu} R_{\mu\nu\rho\sigma}) - (ieF_{\rho\sigma} + \frac{1}{2} j^{\mu\nu} R_{\mu\nu\rho\sigma}) j^{\alpha\beta}\Big) + M_{\rho\sigma} j^{\alpha\beta}\Big\} M_{\alpha\beta}$$
$$= \frac{\eta}{M^3}\ j^{\rho\sigma}\Big\{ \frac{1}{2}[j^{\alpha\beta}, j^{\mu\nu}]_- R_{\mu\nu\rho\sigma} + M_{\rho\sigma} j^{\alpha\beta}\Big\} M_{\alpha\beta}; \tag{130}$$

Thus, we get

$$A = \frac{\eta}{M^3}\ \Big\{ (j^{\rho\sigma} M_{\rho\sigma})(j^{\alpha\beta} M_{\alpha\beta}) + \frac{1}{2} j^{\rho\sigma}[j^{\alpha\beta}, j^{\mu\nu}]_- M_{\alpha\beta} R_{\mu\nu\rho\sigma}\Big\}. \tag{131}$$

Taking in mind the identity [3]

$$j^{\alpha\beta} M_{\alpha\beta} = (ieF_{\alpha\beta} j^{\alpha\beta} - \frac{1}{4}R),$$

we rewrite the last expression as follows

$$A = \frac{\eta}{M^3}\, j^{\rho\sigma} j^{\alpha\beta} M_{\rho\sigma} M_{\alpha\beta} = \frac{\eta}{M^3}\, \Big\{ \Big(ieF_{\rho\sigma} j^{\rho\sigma} - \frac{1}{4}R \Big)^2 + \frac{1}{2} j^{\rho\sigma} [j^{\alpha\beta}, j^{\mu\nu}]_-\, M_{\alpha\beta}\, R_{\mu\nu\rho\sigma} \Big\}. \tag{132}$$

We should detail the second term with commutator

$$B = \frac{1}{2} j^{\rho\sigma} [j^{\alpha\beta}, j^{\mu\nu}]_-\, R_{\mu\nu\rho\sigma}\, M_{\alpha\beta}$$

$$= \frac{1}{2} j^{\rho\sigma} \Big[(g^{\beta\mu} j^{\alpha\nu} - g^{\beta\nu} j^{\alpha\mu}) - (g^{\alpha\mu} j^{\beta\nu} - g^{\alpha\nu} j^{\beta\mu}) \Big] R_{\mu\nu\rho\sigma}\, M_{\alpha\beta}$$

$$= \frac{1}{2} j^{\rho\sigma} \Big[R^{\beta}{}_{\nu\rho\sigma} j^{\alpha\nu} - R_{\mu}{}^{\beta}{}_{\rho\sigma} j^{\alpha\mu} - R^{\alpha}{}_{\nu\rho\sigma} j^{\beta\nu} + R_{\mu}{}^{\alpha}{}_{\rho\sigma} j^{\beta\mu} \Big] M_{\alpha\beta}$$

$$= \frac{1}{2} j^{\rho\sigma} \Big[(R^{\beta}{}_{\nu\rho\sigma} j^{\alpha\nu} M_{\alpha\beta} - R^{\alpha}{}_{\nu\rho\sigma} j^{\beta\nu} M_{\alpha\beta}) + (R_{\mu}{}^{\alpha}{}_{\rho\sigma} j^{\beta\mu} M_{\alpha\beta} - R_{\mu}{}^{\beta}{}_{\rho\sigma} j^{\alpha\mu} M_{\alpha\beta}) \Big].$$

Making the needed change in mute indices, $\alpha \Longrightarrow \beta, \beta \Longrightarrow \alpha$, we get

$$B = \frac{1}{2} j^{\rho\sigma} \Big[(R^{\beta}{}_{\nu\rho\sigma} j^{\alpha\nu} M_{\alpha\beta} - R^{\beta}{}_{\nu\rho\sigma} j^{\alpha\nu} M_{\beta\alpha}) + (R_{\mu}{}^{\alpha}{}_{\rho\sigma} j^{\beta\mu} M_{\alpha\beta} - R_{\mu}{}^{\alpha}{}_{\rho\sigma} j^{\beta\mu} M_{\beta\alpha}) \Big]$$

$$= \frac{1}{2} j^{\rho\sigma} \Big[(R^{\beta}{}_{\nu\rho\sigma} j^{\alpha\nu} M_{\alpha\beta} + R^{\beta}{}_{\nu\rho\sigma} j^{\alpha\nu} M_{\alpha\beta}) + (R_{\mu}{}^{\alpha}{}_{\rho\sigma} j^{\beta\mu} M_{\alpha\beta} + R_{\mu}{}^{\alpha}{}_{\rho\sigma} j^{\beta\mu} M_{\alpha\beta}) \Big]$$

$$= \frac{1}{2} j^{\rho\sigma} \Big[2R^{\beta}{}_{\nu\rho\sigma} j^{\alpha\nu} M_{\alpha\beta} + 2R_{\mu}{}^{\alpha}{}_{\rho\sigma} j^{\beta\mu} M_{\alpha\beta} \Big] = R^{\beta}{}_{\nu\rho\sigma} j^{\rho\sigma} j^{\alpha\nu} M_{\alpha\beta} + R_{\mu}{}^{\alpha}{}_{\rho\sigma} j^{\rho\sigma} j^{\beta\mu} M_{\alpha\beta} = B.$$

Making the changes, $\mu \Longrightarrow \nu, \alpha \Longrightarrow \beta, \beta \Longrightarrow \alpha$, we get

$$B = R^{\beta}{}_{\nu\rho\sigma} j^{\rho\sigma} j^{\alpha\nu} M_{\alpha\beta} + R_{\mu}{}^{\alpha}{}_{\rho\sigma} j^{\rho\sigma} j^{\beta\mu} M_{\alpha\beta}$$

$$= R^{\beta}{}_{\nu\rho\sigma} j^{\rho\sigma} j^{\alpha\nu} M_{\alpha\beta} + R_{\nu}{}^{\beta}{}_{\rho\sigma} j^{\rho\sigma} j^{\alpha\nu} M_{\beta\alpha} = R^{\beta}{}_{\nu\rho\sigma} j^{\rho\sigma} j^{\alpha\nu} M_{\alpha\beta} + R^{\beta}{}_{\nu\rho\sigma} j^{\rho\sigma} j^{\alpha\nu} M_{\alpha\beta}$$

$$= 2R^{\beta}{}_{\nu\rho\sigma} j^{\rho\sigma} j^{\alpha\nu} M_{\alpha\beta} = B. \tag{133}$$

Allowing for expression for $M_{\alpha\beta}$, we derive

$$B = 2R^{\beta}{}_{\nu\rho\sigma} j^{\rho\sigma} j^{\alpha\nu} M_{\alpha\beta} = 2R^{\beta}{}_{\nu\rho\sigma} j^{\rho\sigma} j^{\alpha\nu} \Big(ieF_{\alpha\beta} + \frac{1}{2} j^{\epsilon\tau} R_{\epsilon\tau\alpha\beta} \Big)$$
$$= 2ieF_{\alpha\beta} R^{\beta}{}_{\nu\rho\sigma} (j^{\rho\sigma} j^{\alpha\nu}) + R^{\beta}{}_{\nu\rho\sigma} R_{\epsilon\tau\alpha\beta} j^{\rho\sigma} j^{\alpha\nu} j^{\epsilon\tau}. \tag{134}$$

Let us consider the product of two generators

$$j^{\rho\sigma} j^{\alpha\nu} = \frac{1}{16} (\gamma^{\rho}\gamma^{\sigma} - \gamma^{\sigma}\gamma^{\rho})(\gamma^{\alpha}\gamma^{\nu} - \gamma^{\nu}\gamma^{\alpha})$$

$$= \frac{1}{16} \Big[\gamma^{\rho} (\gamma^{\sigma}\gamma^{\alpha}\gamma^{\nu} - \gamma^{\sigma}\gamma^{\nu}\gamma^{\alpha}) - \gamma^{\sigma} (\gamma^{\rho}\gamma^{\alpha}\gamma^{\nu} - \gamma^{\rho}\gamma^{\nu}\gamma^{\alpha}) \Big]$$

$$= \frac{1}{16} \Big\{ \gamma^{\rho} \Big[(\gamma^{\sigma} g^{\alpha\nu} - \gamma^{\alpha} g^{\sigma\nu} + \gamma^{\nu} g^{\sigma\alpha} + i\gamma^5 \epsilon^{\sigma\alpha\nu\delta} \gamma_{\delta}) - (\gamma^{\sigma} g^{\nu\alpha} - \gamma^{\nu} g^{\sigma\alpha} + \gamma^{\alpha} g^{\nu\sigma} + i\gamma^5 \epsilon^{\sigma\nu\alpha\delta} \gamma_{\delta}) \Big]$$
$$- \gamma^{\sigma} \Big[(\gamma^{\rho} g^{\alpha\nu} - \gamma^{\alpha} g^{\rho\nu} + \gamma^{\nu} g^{\rho\alpha} + i\gamma^5 \epsilon^{\rho\alpha\nu\delta} \gamma_{\delta}) - (\gamma^{\rho} g^{\nu\alpha} - \gamma^{\nu} g^{\rho\alpha} + \gamma^{\alpha} g^{\rho\nu} + i\gamma^5 \epsilon^{\rho\nu\alpha\delta} \gamma_{\delta}) \Big] \Big\}$$

$$= \frac{1}{8} \Big\{ \gamma^{\rho} \Big[-\gamma^{\alpha} g^{\sigma\nu} + \gamma^{\nu} g^{\sigma\alpha} + i\gamma^5 \epsilon^{\sigma\alpha\nu\delta} \gamma_{\delta} \Big] - \gamma^{\sigma} \Big[-\gamma^{\alpha} g^{\rho\nu} + \gamma^{\nu} g^{\rho\alpha} + i\gamma^5 \epsilon^{\rho\alpha\nu\delta} \gamma_{\delta} \Big] \Big\}$$

Therefore, we derive

$$C = ieF_{\alpha\beta} R^{\beta}{}_{\nu\rho\sigma} j^{\rho\sigma} j^{\alpha\nu} = \frac{ie}{4} F_{\alpha\beta} \Big\{ -\gamma^{\rho}\gamma^{\alpha} g^{\sigma\nu} R^{\beta}{}_{\nu\rho\sigma} + \gamma^{\rho}\gamma^{\nu} g^{\sigma\alpha} R^{\beta}{}_{\nu\rho\sigma} + i\gamma^{\rho}\gamma^5 \epsilon^{\sigma\alpha\nu\delta} \gamma_{\delta} R^{\beta}{}_{\nu\rho\sigma}$$
$$+ \gamma^{\sigma}\gamma^{\alpha} g^{\rho\nu} R^{\beta}{}_{\nu\rho\sigma} - \gamma^{\sigma}\gamma^{\nu} g^{\rho\alpha} R^{\beta}{}_{\nu\rho\sigma} - i\gamma^{\sigma}\gamma^5 \epsilon^{\rho\alpha\nu\delta} \gamma_{\delta} R^{\beta}{}_{\nu\rho\sigma} \Big\}$$

$$= \frac{ie}{4} F_{\alpha\beta} \Big\{ -\gamma^{\rho}\gamma^{\alpha} R^{\beta\sigma}{}_{\rho\sigma} + \gamma^{\rho}\gamma^{\nu} R^{\beta}{}_{\nu\rho}{}^{\alpha} - i\gamma^{\rho}\gamma_{\delta}\gamma^5 \epsilon^{\sigma\alpha\nu\delta} R^{\beta}{}_{\nu\rho\sigma} + \gamma^{\sigma}\gamma^{\alpha} R^{\beta\rho}{}_{\rho\sigma} - \gamma^{\sigma}\gamma^{\nu} R^{\beta}{}_{\nu}{}^{\alpha}{}_{\sigma} + i\gamma^{\sigma}\gamma_{\delta}\gamma^5 \epsilon^{\rho\alpha\nu\delta} R^{\beta}{}_{\nu\rho\sigma} \Big\}.$$

Taking into account the symmetry properties of the curvature tensor, we derive

$$C = \frac{ie}{4} F_{\alpha\beta} \Big\{ -\gamma^{\rho}\gamma^{\alpha} R^{\sigma\beta}{}_{\sigma\rho} + (-\gamma^{\nu}\gamma^{\rho} + 2g^{\rho\nu}) R^{\beta}{}_{\nu\rho}{}^{\alpha} - i\gamma^{\rho}\gamma_{\delta}\gamma^5 \epsilon^{\sigma\alpha\nu\delta} R^{\beta}{}_{\nu\rho\sigma}$$

$$+\gamma^{\sigma}\gamma^{\alpha}R^{\beta\rho}{}_{\rho\sigma} + (\gamma^{\nu}\gamma^{\sigma} - 2g^{\nu\sigma})R^{\beta}{}_{\nu}{}^{\alpha}{}_{\sigma} - i\gamma^{\rho}\gamma_{\delta}\gamma^{5}\epsilon^{\sigma\alpha\nu\delta}R^{\beta}{}_{\nu\rho\sigma}\Big\}$$

$$= \frac{ie}{4}F_{\alpha\beta}\Big\{ -\gamma^{\rho}\gamma^{\alpha}R^{\beta}{}_{\rho} + (-\gamma^{\nu}\gamma^{\rho}R^{\beta}{}_{\nu\rho}{}^{\alpha} - 2R^{\alpha\beta}) - 2i\gamma^{\rho}\gamma_{\delta}\gamma^{5}\epsilon^{\sigma\alpha\nu\delta}R^{\beta}{}_{\nu\rho\sigma} - \gamma^{\rho}\gamma^{\alpha}R^{\beta}{}_{\rho} + (-\gamma^{\nu}\gamma^{\rho}R^{\beta}{}_{\nu\rho}{}^{\alpha} + 2R^{\alpha\beta})\Big\},$$

that is

$$C = \frac{ie}{2}F_{\alpha\beta}\Big\{ -\gamma^{\rho}\gamma^{\alpha}R^{\beta}{}_{\rho} - \gamma^{\nu}\gamma^{\rho}R^{\beta}{}_{\nu\rho}{}^{\alpha} - i\gamma^{\rho}\gamma_{\delta}\gamma^{5}\epsilon^{\sigma\alpha\nu\delta}R^{\beta}{}_{\nu\rho\sigma}\Big\}.$$

Using the identities

$$\gamma^{\rho}\gamma^{\alpha} = (2j^{\rho\alpha} + g^{\rho\alpha}), \quad \gamma^{\nu}\gamma^{\rho} = (2j^{\nu\rho} + g^{\nu\rho}), \quad \gamma^{\rho}\gamma_{\delta} = (2j^{\rho}{}_{\delta} + \delta^{\rho}_{\delta}),$$

we arrive at

$$C = \frac{ie}{2}F_{\alpha\beta}\Big\{ -(2j^{\rho\alpha}R^{\beta}{}_{\rho} + g^{\rho\alpha}R^{\beta}{}_{\rho}) - (2j^{\nu\rho}R^{\beta}{}_{\nu\rho}{}^{\alpha} + g^{\nu\rho}R^{\beta}{}_{\nu\rho}{}^{\alpha}) - i(2j^{\rho}{}_{\delta} + \delta^{\rho}_{\delta})\gamma^{5}\epsilon^{\sigma\alpha\nu\delta}R^{\beta}{}_{\nu\rho\sigma}\Big\}$$

$$= \frac{ie}{2}F_{\alpha\beta}\Big\{ -2j^{\rho\alpha}R^{\beta}{}_{\rho} - R^{\alpha\beta} - 2j^{\nu\rho}R^{\beta}{}_{\nu\rho}{}^{\alpha} + R^{\alpha\beta} - 2ij^{\rho}{}_{\delta}\gamma^{5}\epsilon^{\sigma\alpha\nu\delta}R^{\beta}{}_{\nu\rho\sigma}\Big\}$$
$$= \frac{ie}{2}F_{\alpha\beta}\Big\{ -2j^{\rho\alpha}R^{\beta}{}_{\rho} - 2j^{\nu\rho}R^{\beta}{}_{\nu\rho}{}^{\alpha} - 2i\gamma^{5}j^{\rho\delta}\epsilon^{\sigma\alpha\nu}{}_{\delta}R^{\beta}{}_{\nu\rho\sigma}\Big\} = C. \tag{135}$$

Collecting results together, we get

$$\frac{\eta}{M^{3}}\, j^{\rho\sigma}j^{\alpha\beta}M_{\rho\sigma}\, M_{\alpha\beta} = \frac{\eta}{M^{3}}\, \Big[\Big(ieF_{\rho\sigma}j^{\rho\sigma} - \frac{1}{4}R\Big)^{2}$$
$$+\frac{ie}{2}F_{\alpha\beta}\Big(-2j^{\rho\alpha}R^{\beta}{}_{\rho} - 2j^{\nu\rho}R^{\beta}{}_{\nu\rho}{}^{\alpha} - 2i\gamma^{5}j^{\rho\delta}\epsilon^{\sigma\alpha\nu}{}_{\delta}R^{\beta}{}_{\nu\rho\sigma}\Big) + R^{\beta}{}_{\nu\rho\sigma}R_{\epsilon\tau\alpha\beta}j^{\rho\sigma}j^{\alpha\nu}j^{\epsilon\tau}\Big]. \tag{136}$$

Therefore, the basic equation takes on the form

$$\Big\{(\gamma^{\sigma}D_{\sigma} + M) + \frac{\mu}{M}\Big(ieF_{\alpha\beta}j^{\alpha\beta} - \frac{1}{4}R\Big) + \frac{\sigma}{M^{2}}\,(\gamma^{\rho}D_{\rho})\,\Big(ieF_{\alpha\beta}j^{\alpha\beta} - \frac{1}{4}R\Big)$$
$$+\frac{\eta}{M^{3}}\Big[D^{\sigma}D_{\sigma}\Big(ieF_{\alpha\beta}j^{\alpha\beta} - \frac{1}{4}R\Big) + \Big(ieF_{\rho\sigma}j^{\rho\sigma} - \frac{1}{4}R\Big)^{2}$$
$$\frac{ie}{2}F_{\alpha\beta}\Big(2j^{\alpha\rho}R_{\rho}{}^{\beta} - 2j^{\nu\rho}R^{\beta}{}_{\nu\rho}{}^{\alpha} - 2i\gamma^{5}j^{\rho\delta}\epsilon_{\delta}{}^{\alpha\nu\sigma}R^{\beta}{}_{\nu\sigma\rho}\Big) + R^{\beta}{}_{\nu\rho\sigma}R_{\epsilon\tau\alpha\beta}\, j^{\rho\sigma}j^{\alpha\nu}j^{\epsilon\tau}\Big]\Big\}\Psi = 0. \tag{137}$$

The operator $D^{\sigma}D_{\sigma}$ is detailed as follows

$$D^{\sigma}D_{\sigma} = (\nabla^{\sigma} + \Gamma^{\sigma})(\nabla_{\sigma} + \Gamma_{\sigma}) = \nabla^{\sigma}\nabla_{\sigma} + (\nabla_{\sigma}\Gamma^{\sigma}) + \Gamma^{\sigma}\nabla_{\sigma} + \Gamma^{\sigma}\nabla_{\sigma} + \Gamma^{\sigma}\Gamma_{\sigma}$$

$$= \nabla^{\sigma}\nabla_{\sigma} + (\gamma_{\sigma}\Gamma^{\sigma} - \Gamma^{\sigma}\gamma_{\sigma}) + 2\Gamma^{\sigma}\nabla_{\sigma} + +\Gamma^{\sigma}\Gamma_{\sigma}.$$

The last term in (137) may be presented differently

$$R^{\beta}{}_{\nu\rho\sigma}R_{\epsilon\tau\alpha\beta}\, j^{\rho\sigma}j^{\alpha\nu}j^{\epsilon\tau} \equiv -(R^{\beta}{}_{\nu\rho\sigma}j^{\rho\sigma})j^{\nu\alpha}(R_{\alpha\beta\epsilon\tau}\, j^{\epsilon\tau}) \equiv -X^{\beta}{}_{\nu}\, j^{\nu\alpha}\, X_{\alpha\beta}. \tag{138}$$

It is convenient to transform the last term in (137) to tetrad form:

$$R^{\beta}{}_{\nu\rho\sigma}R_{\epsilon\tau\alpha\beta}\, j^{\rho\sigma}j^{\alpha\nu}j^{\epsilon\tau} = -\Big(R^{d}{}_{nab}(x)j^{ab}\Big)j^{nm}\Big(R_{mdkl}(x)j^{kl}\Big).$$

Therefore, the basic equation takes on the form

$$\Big\{\Big(\gamma^{\sigma}D_{\sigma} + M\Big) + \frac{\mu}{M}\Big(ieF_{\alpha\beta}j^{\alpha\beta} - \frac{1}{4}R\Big) + \frac{\sigma}{M^{2}}(\gamma^{\rho}D_{\rho})\Big(ieF_{\alpha\beta}j^{\alpha\beta} - \frac{1}{4}R\Big)$$

$$+\frac{\eta}{M^{3}}\Big[D^{\sigma}D_{\sigma}\Big(ieF_{\alpha\beta}j^{\alpha\beta} - \frac{1}{4}R\Big) + \Big(ieF_{\rho\sigma}j^{\rho\sigma} - \frac{1}{4}R\Big)^{2} +$$

$$+\frac{ie}{2}F_{\alpha\beta}\Big(2j^{\alpha\rho}R_{\rho}{}^{\beta} - 2j^{\nu\rho}R^{\beta}{}_{\nu\rho}{}^{\alpha} - 2i\gamma^{5}j^{\rho\delta}\epsilon_{\delta}{}^{\alpha\nu\sigma}R^{\beta}{}_{\nu\sigma\rho}\Big)$$

$$-\Big(R^{d}{}_{nab}(x)j^{ab}\Big)j^{nm}\Big(R_{mdkl}(x)j^{kl}\Big)\Big]\Big\}\Psi = 0. \tag{139}$$

Evidently, it is possible to detail additionally the term with the product of two Riemann tensors.

Conclusions

As seen, in the derived extended equation for spin 1/2 particle, there arises a number of additional geometrical interaction terms through the Ricci scalar $R(x)$ and tensor R_{ab}, and the Riemann curvature tensor $R_{mnkl}(x)$. The contribution of Ricci tensor and Riemann curvature tensor differ from zero only if the third parameter η vanishes.

References

[1] Pletyukhov, V.A. Relativistic wave equations and internal degrees of freedom / V.A. Pletyukhov, V. M. Red'kov, V. I. Strazhev // Belarussian Science, Minsk. – 2015. – 328 p.

[2] Elementary Particles with Internal Structure in External Fields. Vol I. General Theory; Vol II. Physical Problems / V.V. Kisel, E.M. Ovsiyuk, O.V. Veko, Y.A. Voynova, V. Balan, V.M. Red'kov. – New York: Nova Science Publishers Inc., 2018. – 402 p., 404 p.

[3] Red'kov V.M. Field particles in Riemannian space and the Lorentz group. – Belarussian Science, Minsk, 2009. 486 p. (in Russian).