Nonlinear Phenomena in Complex Systems, vol. 28, no. 3 (2025), pp. 220 - 241

# Spin 2 Particle Theory, the Nonrelativistic Approximation

A. V. Ivashkevich,* A. V. Bury,* and V. M. Red'kov†
*B. I. Stepanov Institute of Physics of NAS of Belarus,*
*68 Nezavisimosti Ave., 220072 Minsk, BELARUS*

E. M. Ovsiyuk‡
*Mozyr State Pedagogical University named after I. P. Shamyakin*

V. V. Kisel§
*Belarus State University of Informatics and Radio-Electronics*
(Received 15 July, 2025)

The goal of the present paper is investigation of the nonrelativistic approximation in the 39-component theory for a spin 2 particle. We apply explicit expressions for four main matrices $\Gamma^a$ with dimension $39\times39$ in the relevant first order system of equations, written in Cartesian coordinates and in the presence of external an electromagnetic field. For distinguishing the large and small parts in the complete wave function, we use three projective operators constructed on the base of the minimal polynomial of the 7-th order for the matrix $\Gamma^0$. The relevant large and small components are found in explicit form. Among them we have found independent variables; in particular, among the large components there exist only five independent ones. We have derived the nonrelativistic equation for 5-component wave function; in which term describing interaction of the magnetic moment of the spin 2 particle with the external magnetic field is separated.

**PACS numbers:** 02.30.Gp, 02.40.Ky, 03.65Ge, 04.62.+v
**Keywords:** spin 2 particle, the non-relativistic approximation, external electromagnetic fields, projective operators
**DOI:** https://doi.org/10.5281/zenodo.17237053

## Introduction

After the investigation by Pauli and Fierz [1], [2], the theory of massive and massless fields with spin 2 has always attracted much attention [3]–[27]. Several key aspects and challenges of this theory have been explored over the years.

Most of the studies were performed in the framework of 2-nd order differential equations. It is known that many specific difficulties may be avoided if from the very beginning we start with 1-st order systems. Apparently, the first systematic study of the theory of spin 2 fields within that formalism was performed by F.I. Fedorov [4]. It turns out that this description requires a field function with 39 independent components. This theory was re-discovered by Regee in [5].

When studying the massless spin-2 field in a curved space-time, additional difficulties appear. For instance, unexpected constraints on space-time geometry arise to insure the gauge symmetry of the theory. In particular, the Ricci tensor $R_{\alpha\beta}$ and the Riemann tensor $R_{\alpha\beta\rho\sigma}$ must vanish [15]. To resolve this, a non-minimal interaction term involving the Riemann tensor can be introduced into the basic equations [20], allowing the constraints to be reduced to $R_{\alpha\beta}=0$. Another area of interest has been the problem of anomalous solutions in spin-2 theory [6, 7, 10].

A technical alternative for studying spin-2 fields, both massive and massless, involves formulating first-order systems. This approach, based on the Gel'fand–Yaglom formalism [3],

*E-mail: ivashkevich.alina@yandex.by
†E-mail: v.redkov@ifanbel.bas-net.by
‡E-mail: e.ovsiyuk@mail.ru
§E-mail: vasiliy-bspu@mail.ru

was first explored by Fedorov [4] and Regge [5]. Their papers demonstrated that a spin-2 particle requires a 39-component set of tensors for its description, it includes $\Phi, \Phi_k, \Phi_{(mn)}, \Phi_{[mn]k}$.

This formalism allows for exploration of new physical questions related to degrees of freedom. For instance, for the massless case, the 39-component matrix equation was solved in Minkowski space-time in [24], [25] using spherical and cylindrical coordinates. Six linearly independent solutions were found. By applying the Pauli–Fierz approach, adjusted to the tetrad formalism, the gauge solutions were constructed using exact solutions for the massless spin-1 field. This yielded four independent gauge solutions and two gauge-free solutions for the spin-2 field, as expected from physical reasoning.

Additionally, F.I. Fedorov initiated a more general theory for the spin-2 particle based on a 50-component set of tensors. This theory, in the presence of external electromagnetic fields, describes a spin-2 particle with an anomalous magnetic moment4 see in [12, 13, 21–23].

One notable aspect of this theory is its allowance for a new massless limit for the spin-2 field [21]. This is particularly significant because the minimal Pauli–Fierz theory does not possess gauge symmetry in curved space-times with $R_{\alpha\beta} = 0$. However, the generalized theory exhibits gauge symmetry under these conditions.

In the present paper, we will investigate the non-relativistic approximation in the basic 39-component theory.

Section 1 introduces the basic definitions and notations, including the structure of the 39-component matrix equation. Explicit expressions for the four key matrices $\Gamma^a$ of the equation, derived in [19], are assumed to be known. The system is formulated in Cartesian coordinates in the presence of external electromagnetic fields.

The non-relativistic approximation is performed by distinguishing between large and small components of the wave function using three projective operators derived from the seventh-order minimal polynomial for the $39 \times 39$ matrix $\Gamma^0$. The explicit structure of these components is determined; in Supplement A, we establish independent variables among large and small components.

In section 2, we perform the procedure of the non-relativistic approximation. The basic point consists in decomposition of the components of the complete wave function in large and small constituents and splitting all 39 equations in equations of different orders of smallness. At this we should separate the rest energy.

Besides, when performing the nonrelativistic approximation, we should assume the presence of terms of different smallness order; this permits in each equation to distinguish large and small terms.

As the result, we derive the nonrelativistic equation for the 5-component wave function, in which the term describing interaction of the magnetic moment of the spin 2 particle with external magnetic field is separated. This additional term is constructed with the use of the spin matrices and the components of the magnetic field.

## 1. Basic equation for a spin 2 particle, projective operators

We start with the matrix equation in Minkowski space [19, 24, 25]

$$(\Gamma^a D_a - M)\Psi(x) = 0, \quad \Gamma^a = \begin{vmatrix} 0 & G^a & 0 & 0 \\ \frac{1}{2}\Delta^a & 0 & -\frac{1}{3}K^a & 0 \\ 0 & \Lambda^a & 0 & \frac{1}{2}B^a \\ 0 & 0 & F^a & 0 \end{vmatrix}, \quad \Psi(x) = \begin{vmatrix} \Phi(x) \\ \Phi_l(x) \\ \Phi_{(mn)}(x) \\ \Phi_{[mn]l}(x) \end{vmatrix},$$

where $D_a = \partial_a + ieA_a$; we use the block matrices $G^1, \Delta^a, K^a, B^a, F^a$ (see in [19]); they dimensions are determined by the block columns in the complete field function; the field function consists of scalar, vector $\Phi_k(x)$, symmetric tensor $\Phi_{(mn)}(x)$ and 3-rank tensor $\Phi_{[mn]l}(x)$; it may be presented as the 39-dimension column

$$\Psi = \{\ \Phi;\ \Phi_l;\ \vec{f},\ \vec{c},\ \vec{d},\ f_0;\ \varphi_0,\ \varphi_1,\ \varphi_2,\ \varphi_3\ \} = \{H; H_1; H_2; H_3\}\,. \tag{1.1}$$

According to general theory, in order to perform the nonrelativistic approximation we should work with the matrix $\Gamma^0 = \Gamma_0 = \Gamma$ (its blocks are given below; we indicate their dimensions)

$$G_{1\times 4} = \left| \begin{array}{cccc} +1 & 0 & 0 & 0 \end{array} \right|, \Delta_{4\times 1} = \left| \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array} \right|, K_{4\times 10} = \left| \begin{array}{cccccccccc} 0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \end{array} \right|, \Lambda_{10\times 4} = \left| \begin{array}{cccc} \frac{1}{2}&0&0&0 \\ \frac{1}{2}&0&0&0 \\ \frac{1}{2}&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ \frac{3}{2}&0&0&0 \end{array} \right|,$$

$$B_{10\times 24} = \left| \begin{array}{cccccccccccccccccccccccc}
. & . & . & . & . & . & 3/2 & . & . & . & . & . & . & -1/2 & . & . & . & . & . & . & -1/2 & . & . & . \\
. & . & . & . & . & . & -1/2 & . & . & . & . & . & . & 3/2 & . & . & . & . & . & . & -1/2 & . & . & . \\
. & . & . & . & . & . & -1/2 & . & . & . & . & . & . & -1/2 & . & . & . & . & . & . & 3/2 & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & 1 & . & . & . & . \\
. & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . \\
. & . & . & . & . & . & . & 1 & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . \\
1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & 1/2 & . & . & . & . & . & . & 1/2 & . & . & . & . & . & . & 1/2 & . & . & .
\end{array} \right|,$$

$$
F_{24\times 10} = \begin{vmatrix}
. & . & . & . & . & . & 2/3 & . & . & . \\
. & . & . & . & . & . & . & 2/3 & . & . \\
. & . & . & . & . & . & . & . & 2/3 & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
1 & . & . & . & . & . & . & . & . & -1/3 \\
. & . & . & . & . & 1 & . & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & -1/3 & . \\
. & . & . & . & . & . & . & 1/3 & . & . \\
. & . & . & . & . & 1 & . & . & . & . \\
. & 1 & . & . & . & . & . & . & . & -1/3 \\
. & . & . & 1 & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & 1/3 & . \\
. & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & -1/3 & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . & -1/3 \\
. & . & . & . & . & . & . & -1/3 & . & . \\
. & . & . & . & . & . & 1/3 & . & . & . \\
. & . & . & . & . & . & . & . & . & .
\end{vmatrix} .
$$

The matrix $\Gamma$ satisfies the minimal equation (this is verified by direct calculation) $\Gamma^7 - \Gamma^5 = 0$; which permits us to introduce 3 projective operators

$$
P_+ = \frac{1}{2}\Gamma^5(\Gamma + I) = P_1, \quad P_- = \frac{1}{2}\Gamma^5(\Gamma - I) = P_2, \quad P_0 = I - \Gamma^6 = P_3 \tag{1.2}
$$

with the needed properties $P_i^2 = P_i, P_+ + P_- + P_0 = I, i = 1, 2, 3$. The components of the complete wave function are listed in accordance with the following notations (also see (1.1))

$$
H_3 = \Phi_{[mn]l} \Longrightarrow \begin{vmatrix} \Phi_{[01]l} \\ \Phi_{[02]l} \\ \Phi_{[03]l} \\ \Phi_{[23]l} \\ \Phi_{[31]l} \\ \Phi_{[12]l} \end{vmatrix} = \begin{vmatrix} E_{10} \\ E_{20} \\ E_{30} \\ B_{10} \\ B_{20} \\ B_{30} \end{vmatrix} = \varphi_0, \quad \begin{vmatrix} E_{11} \\ E_{21} \\ E_{31} \\ B_{11} \\ B_{21} \\ B_{31} \end{vmatrix} = \varphi_1, \quad \begin{vmatrix} E_{12} \\ E_{22} \\ E_{32} \\ B_{12} \\ B_{22} \\ B_{32} \end{vmatrix} = \varphi_2, \quad \begin{vmatrix} E_{13} \\ E_{23} \\ E_{3k} \\ B_{13} \\ B_{23} \\ B_{33} \end{vmatrix} = \varphi_3.
$$

We can find explicit form of three projective operators. Acting by these operators on the complete wave functions, we obtain the structure of three projective constituents and their sum $(\Psi_+ + \Psi_- + \Psi_0) = \Psi$ (we introduce notations for the variables $L_{\dots}$ referring to large components; the notations $S_{\dots}$ and $s_{\dots}$ for variables referring to small components:

In Supplement A, it is shown that among components there exist linear relations, they permits us to present the complete wave functions in terms of only independent large and small components.

## 2. The non-relativistic procedure

We should separate the rest energy by formal change (where $M$ is real valued and positive mass parameter))

$$D_0 \Longrightarrow (D_0 - iM).$$

Besides, when performing the nonrelativistic approximation, we should assume the presence of terms of different smallness order):

$$L \sim 1, \quad S \sim x, \; s \sim x, \; \frac{1}{M} D_i \sim x, \; \frac{D_0}{M} \sim x^2,$$

this permits in each of 39 equations distinguish large and small terms. They may be divided into three groups.

The group **I** consists of the constraints among $s_i$:

$$s_2 - s_1 = 0, \quad (\frac{s_1}{2} - \frac{s_{12}}{3}) - s_2 = 0,$$

$$-i\Big(\frac{1}{4}(-2s_{19} - S_{11} - S_{15})$$

$$+\frac{3}{4}(s_{19} - S_{11} - S_{15}) + \frac{s_2}{2}\Big) + i(s_6 + S_{11} + S_{15}) = 0,$$

$$-i\Big(\frac{1}{4}(-2s_{19}+S_{11})+\frac{3}{4}(s_{19}+S_{11})+\frac{s_2}{2}\Big)+i(s_6-S_{11}) = 0,$$

$$-i\Big(\frac{1}{4}(2s_{19}+S_{15})+\frac{3}{4}(s_{19}+S_{15})+\frac{s_2}{2}\Big)+i(s_6-S_{15}) = 0,$$

$$S_{14} = 0, \quad S_{13} = 0, \quad S_{10} = 0,$$

$$-i(\frac{1}{4}s_{19} + \frac{1}{4}s_{19} + \frac{1}{4}s_{19} + \frac{3s_2}{2}) + is_{12} = 0,$$

$$s_{16} = 0, \quad s_{17} = 0, \quad s_{18} = 0,$$

$$-i(s_6 - \frac{s_{12}}{3} + S_{11} + S_{15}) + i(s_{19} - S_{11} - S_{15}) = 0,$$

$$+iS_{10} + i(s_{20} + S_{10}) = 0,$$

$$iS_{13} + i(s_{21} + S_{13}) = 0, \; iS_{10} + i(-s_{20} + S_{10}) = 0,$$

$$-i(s_6 - \frac{s_{12}}{3} - S_{11}) + i(s_{19} + S_{11}) = 0,$$

$$iS_{14} + i(s_{27} + S_{14}) = 0, \; iS_{13} + i(-s_{21} + S_{13}) = 0,$$

$$iS_{14} + i(-s_{27} + S_{14}) = 0,$$

$$-i(s_6 - \frac{s_{12}}{3} - S_{15}) + i(s_{19} + S_{15}) = 0,$$

**II** permits to express small variables through the terms of the form $D_i L_n$:

$$\frac{D_1}{M}\frac{1}{3}(-L_{11} - L_{15}) + \frac{1}{3}\frac{D_2}{M}L_{10}$$

$$+\frac{1}{3}\frac{D_3}{M}L_{13} + i\frac{s_9}{3} + is_3 = 0,$$

$$\frac{D_2}{M}\frac{1}{3}L_{11} + \frac{1}{3}\frac{D_1}{M}L_{10}$$

$$+\frac{1}{3}\frac{D_3}{M}L_{14} + i\frac{s_{10}}{3} + is_4 = 0,$$

$$\frac{D_3}{M}\frac{1}{3}L_{15} + \frac{1}{3}\frac{D_1}{M}L_{13}$$

$$+\frac{1}{3}\frac{D_2}{M}L_{14} + i\frac{s_{11}}{3} + is_5 = 0,$$

$$\frac{D_2}{2M}L_{10} + \frac{D_3}{2M}L_{13} + \frac{D_1}{2M}(-L_{11} - L_{15})$$

$$-i(s_3 + \frac{s_{13}}{2}) + is_9 = 0,$$

$$\frac{D_1}{2M}L_{10} + \frac{D_2}{2M}L_{11} + \frac{D_3}{2M}L_{14}$$

$$-i(s_4 + \frac{s_{14}}{2}) + is_{10} = 0,$$

$$\frac{D_1}{M}\frac{1}{2}L_{13} + \frac{D_2}{M}\frac{1}{2}L_{14}$$

$$+\frac{D_3}{M}\frac{1}{2}L_{15} - i(s_5 + \frac{s_{15}}{2}) + is_{11} = 0;$$

$$\frac{D_1}{3M}(-L_{11} - L_{15}) + \frac{D_2}{3M}L_{10}$$

$$+\frac{D_3}{3M}L_{13}-\frac{2is_9}{3}+is_{13}=0;$$

$$\frac{D_2}{3M}L_{11}+\frac{D_1}{3M}L_{10}+\frac{D_3}{3M}L_{14}-\frac{2is_{10}}{3}+is_{14}=0,$$

$$\frac{D_3}{3M}L_{15}+\frac{D_1}{3M}L_{13}+\frac{D_2}{3M}L_{14}-\frac{2is_{11}}{3}+is_{15}=0,$$

$$-\frac{D_3}{M}L_{10}+\frac{D_2}{M}L_{13}+is_{22}=0,$$

$$\frac{D_3}{M}(\frac{1}{3}L_{15}-L_{11}-L_{15})-\frac{2}{3M}D_1L_{13}$$
$$+\frac{1}{3M}D_2L_{14}+\frac{1}{3}is_{11}+is_{23}=0,$$

$$\frac{1}{3M}D_2(2L_{11}+3L_{15})+\frac{2}{3M}D_1L_{10}$$
$$-\frac{1}{3M}D_3L_{14}-\frac{is_{10}}{3}+is_{24}=0,$$

$$\frac{1}{3M}D_3(-3L_{11}-L_{15})-\frac{1}{3M}D_1L_{13}$$
$$+\frac{2}{3M}D_2L_{14}-\frac{is_{11}}{3}+is_{28}=0,$$

$$\frac{D_3}{M}L_{10}-\frac{D_1}{M}L_{14}+is_{29}=0,$$

$$\frac{D_1}{M}(\frac{1}{3}(-L_{11}-L_{15})+L_{11})$$
$$-\frac{2D_2}{3M}L_{10}+\frac{D_3}{3M}L_{13}+\frac{is_9}{3}+is_{30}=0,$$

$$\frac{D_2}{M}(\frac{1}{3}L_{11}+L_{15})+\frac{1}{3M}D_1L_{10}$$
$$-\frac{2}{3M}D_3L_{14}+\frac{1}{3}is_{10}+is_{34}=0,$$

$$\frac{D_1}{3M}(L_{11}-2L_{15})-\frac{D_2}{3M}L_{10}$$
$$+\frac{D_3}{3M}L_{13}-\frac{is_9}{3}+is_{35}=0,$$

$$-\frac{D_2}{M}L_{13}+\frac{D_1}{M}L_{14}+is_{36}=0,$$

The group **III** contains the terms with the structure $\frac{D_0}{M}$:

$$\frac{D_0}{M}(-L_{11}-L_{15})$$
$$+\frac{D_3}{M}(-\frac{s_5}{2}+\frac{s_{15}}{4}-\frac{3s_{23}}{4}-\frac{s_{28}}{4})$$
$$+\frac{D_2}{M}(-\frac{s_4}{2}+\frac{s_{14}}{4}+\frac{3s_{24}}{4}+\frac{s_{34}}{4})$$
$$+\frac{D_1}{M}(\frac{3s_3}{2}+\frac{s_{13}}{4}+\frac{s_{30}}{4}-\frac{s_{35}}{4})=0,$$

$$\frac{D_0}{M}L_{11}+\frac{D_3}{M}(-\frac{s_5}{2}+\frac{s_{15}}{4}+\frac{s_{23}}{4}+\frac{3s_{28}}{4})$$
$$+\frac{D_2}{M}(\frac{3s_4}{2}+\frac{s_{14}}{4}-\frac{s_{24}}{4}+\frac{s_{34}}{4})$$
$$+\frac{D_1}{M}(-\frac{s_3}{2}+\frac{s_{13}}{4}-\frac{3s_{30}}{4}-\frac{s_{35}}{4})=0,$$

$$\frac{D_0}{M}L_{15}+\frac{D_3}{M}(\frac{3s_5}{2}+\frac{s_{15}}{4}+\frac{s_{23}}{4}-\frac{s_{28}}{4})$$
$$+\frac{D_2}{M}(-\frac{s_4}{2}+\frac{s_{14}}{4}-\frac{s_{24}}{4}-\frac{3s_{34}}{4})$$
$$+\frac{D_1}{M}(-\frac{s_3}{2}+\frac{s_{13}}{4}+\frac{s_{30}}{4}+\frac{3s_{35}}{4})=0,$$

$$\frac{D_0}{M}(\frac{1}{2}L_{14}+\frac{1}{2}L_{14})+\frac{D_2}{M}(s_5-\frac{s_{28}}{2})$$
$$+\frac{D_3}{M}(s_4+\frac{s_{34}}{2})+\frac{D_1}{M}(\frac{s_{29}}{2}-\frac{s_{36}}{2})=0,$$

$$\frac{D_0}{M}(\frac{1}{2}L_{13}+\frac{1}{2}L_{13})+\frac{D_1}{M}(s_5+\frac{s_{23}}{2})$$
$$+\frac{D_3}{M}(s_3-\frac{s_{35}}{2})+\frac{D_2}{M}(\frac{s_{36}}{2}-\frac{s_{22}}{2})=0,$$

$$\frac{D_0}{M}(\frac{1}{2}L_{10}+\frac{1}{2}L_{10})+\frac{D_1}{M}(s_4-\frac{s_{24}}{2})$$
$$+\frac{D_3}{M}(\frac{s_{22}}{2}-\frac{s_{29}}{2})+\frac{D_2}{M}(s_3+\frac{s_{30}}{2})=0,$$

$$\frac{D_0}{M}(-L_{11}-L_{15})-\frac{2}{3}\frac{D_1}{M}s_9+\frac{D_2}{M}\frac{s_{10}}{3}+\frac{D_3}{M}\frac{s_{11}}{3}=0,$$

$$\frac{D_0}{M}L_{10}-\frac{D_2}{M}s_9=0,\quad 24\quad \frac{D_0}{M}L_{13}-\frac{D_3}{M}s_9=0,$$

$$\frac{D_0}{M}L_{10}-\frac{D_1}{M}s_{10}=0,$$

$$\frac{D_0}{M}L_{11}+\frac{D_1}{M}\frac{s_9}{3}-\frac{D_2}{M}\frac{2s_{10}}{3}+\frac{D_3}{M}\frac{s_{11}}{3}=0,$$

$$\frac{D_0}{M}L_{14}-\frac{D_3}{M}s_{10}=0,\ \frac{D_0}{M}L_{13}-\frac{D_1}{M}s_{11}=0,$$

$$\frac{D_0}{M}L_{14}-\frac{D_2}{M}s_{11}=0,$$

$$\frac{D_0}{M}L_{15}+\frac{D_1}{M}\frac{s_9}{3}+\frac{D_2}{M}\frac{s_{10}}{3}-\frac{D_3}{M}\frac{2s_{11}}{3}=0.$$

We will ignore all equations from the first groups because they are not needed to derive equations with the non-relativistic structure. First we resolve equations of the group $II$ with respect to small variables:

$$s_3=0,\quad s_4=0,\quad s_5=0,$$

$$s_9 = \frac{i\left(D_2 L_{10} + D_3 L_{13} - D_1\left(L_{11} + L_{15}\right)\right)}{M},$$
$$s_{10} = \frac{i\left(D_1 L_{10} + D_2 L_{11} + D_3 L_{14}\right)}{M},$$
$$s_{11} = \frac{i\left(D_1 L_{13} + D_2 L_{14} + D_3 L_{15}\right)}{M},$$
$$s_{13} = \frac{i\left(D_2 L_{10} + D_3 L_{13} - D_1\left(L_{11} + L_{15}\right)\right)}{M},$$
$$s_{14} = \frac{i\left(D_1 L_{10} + D_2 L_{11} + D_3 L_{14}\right)}{M},$$
$$s_{15} = \frac{i\left(D_1 L_{13} + D_2 L_{14} + D_3 L_{15}\right)}{M},$$
$$s_{22} = \frac{i\left(D_2 L_{13} - D_3 L_{10}\right)}{M},$$
$$s_{23} = -\frac{i\left(D_1 L_{13} + D_3\left(L_{11} + L_{15}\right)\right)}{M},$$
$$s_{24} = \frac{i\left(D_1 L_{10} + D_2\left(L_{11} + L_{15}\right)\right)}{M},$$
$$s_{28} = \frac{i\left(D_2 L_{14} - D_3 L_{11}\right)}{M}, s_{29} = \frac{i\left(D_3 L_{10} - D_1 L_{14}\right)}{M},$$
$$s_{30} = \frac{i\left(D_1 L_{11} - D_2 L_{10}\right)}{M},$$
$$s_{34} = \frac{i\left(D_2 L_{15} - D_3 L_{14}\right)}{M},$$
$$s_{35} = \frac{i\left(2 D_3 L_{13} - 3 D_1 L_{15}\right)}{3M}, \ s_{36} = \frac{i\left(D_1 L_{14} - D_2 L_{13}\right)}{M}.$$

Then we substitute these expressions into equations from the group $III$; this leads to (we follow the order of the derivatives, and recollect the terms with respect to the large variables):

$$\frac{iD_2 D_1 L_{10}}{M^2} + \frac{\left(i\left(D_2 D_2 + D_3 D_3\right) - M D_0\right) L_{11}}{M^2}$$
$$+ \frac{i\left(12 D_3 D_1 + D_1 D_3\right) L_{13}}{12 M^2}$$
$$+ \frac{\left(i\left(D_2 D_2 + D_3 D_3\right) - M D_0\right) L_{15}}{M^2} = 0,$$
$$\frac{iD_1 D_2 L_{10}}{M^2} + \frac{\left(M D_0 - i\left(D_1 D_1 + D_3 D_3\right)\right) L_{11}}{M^2}$$
$$+ \frac{iD_1 D_3 L_{13}}{12 M^2} + \frac{iD_3 D_2 L_{14}}{M^2} = 0,$$
$$\frac{3iD_1 D_3 L_{13}}{4M^2} + \frac{iD_2 D_3 L_{14}}{M^2}$$
$$+ + \frac{\left(M D_0 - i\left(D_1 D_1 + D_2 D_2\right)\right) L_{15}}{M^2} = 0,$$
$$\frac{iD_1 D_3 L_{10}}{2M^2} + \frac{iD_2 D_3 L_{11}}{2M^2} + \frac{iD_1 D_2 L_{13}}{2M^2}$$
$$+ \frac{\left(2 M D_0 - i\left(2 D_1 D_1 + D_2 D_2 + D_3 D_3\right)\right) L_{14}}{2M^2}$$
$$+ \frac{iD_3 D_2 L_{15}}{2M^2} = 0,$$
$$\frac{iD_2 D_3 L_{10}}{2M^2} - \frac{iD_1 D_3 L_{11}}{2M^2}$$
$$+ \frac{\left(6 M D_0 - i\left(3 D_1 D_1 + 6 D_2 D_2 + 2 D_3 D_3\right)\right) L_{13}}{6M^2}$$
$$+ \frac{iD_2 D_1 L_{14}}{2M^2} + \frac{i\left(D_3 D_1 - D_1 D_3\right) L_{15}}{2M^2} = 0,$$
$$\frac{\left(2 M D_0 - i\left(D_1 D_1 + D_2 D_2 + 2 D_3 D_3\right)\right) L_{10}}{2M^2}$$
$$+ \frac{i\left(D_2 D_1 - D_1 D_2\right) L_{11}}{2M^2}$$
$$+ \frac{iD_3 D_2 L_{13}}{2M^2} + \frac{iD_3 D_1 L_{14}}{2M^2} - \frac{iD_1 D_2 L_{15}}{2M^2} = 0,$$
$$\frac{i\left(D_2 D_1 - 2 D_1 D_2\right) L_{10}}{3M^2}$$
$$+ \frac{\left(i\left(2 D_1 D_1 + D_2 D_2\right) - 3 M D_0\right) L_{11}}{3M^2}$$
$$+ \frac{i\left(D_3 D_1 - 2 D_1 D_3\right) L_{13}}{3M^2}$$
$$+ \frac{i\left(D_3 D_2 + D_2 D_3\right) L_{14}}{3M^2}$$
$$+ \frac{\left(i\left(2 D_1 D_1 + D_3 D_3\right) - 3 M D_0\right) L_{15}}{3M^2} = 0,$$
$$\frac{\left(M D_0 - iD_2 D_2\right) L_{10}}{M^2} + \frac{iD_2 D_1 L_{11}}{M^2}$$
$$- \frac{iD_2 D_3 L_{13}}{M^2} + \frac{iD_2 D_1 L_{15}}{M^2} = 0,$$
$$- \frac{iD_3 D_2 L_{10}}{M^2} + \frac{iD_3 D_1 L_{11}}{M^2}$$
$$+ \frac{\left(M D_0 - iD_3 D_3\right) L_{13}}{M^2} + \frac{iD_3 D_1 L_{15}}{M^2} = 0,$$
$$\frac{\left(M D_0 - iD_1 D_1\right) L_{10}}{M^2} - \frac{iD_1 D_2 L_{11}}{M^2} - \frac{iD_1 D_3 L_{14}}{M^2} = 0,$$
$$\frac{i\left(D_1 D_2 - 2 D_2 D_1\right) L_{10}}{3M^2}$$
$$+ \frac{\left(3 M D_0 - i\left(D_1 D_1 + 2 D_2 D_2\right)\right) L_{11}}{3M^2}$$
$$+ \frac{i\left(D_3 D_1 + D_1 D_3\right) L_{13}}{3M^2}$$
$$+ \frac{i\left(D_3 D_2 - 2 D_2 D_3\right) L_{14}}{3M^2}$$

$$-\frac{i\,(D_1D_1-D_3D_3)\,L_{15}}{3M^2}=0,$$

$$-\frac{iD_3D_1L_{10}}{M^2}-\frac{iD_3D_2L_{11}}{M^2}+\frac{(MD_0-iD_3D_3)\,L_{14}}{M^2}=0,$$

$$\frac{(MD_0-iD_1D_1)\,L_{13}}{M^2}$$

$$-\frac{iD_1D_2L_{14}}{M^2}-\frac{iD_1D_3L_{15}}{M^2}=0,$$

$$-\frac{iD_2D_1L_{13}}{M^2}+\frac{(MD_0-iD_2D_2)\,L_{14}}{M^2}-\frac{iD_2D_3L_{15}}{M^2}=0,$$

$$\frac{i\,(D_2D_1+D_1D_2)\,L_{10}}{3M^2}$$

$$-\frac{i\,(D_1D_1-D_2D_2)\,L_{11}}{3M^2}+\frac{i\,(D_1D_3-2D_3D_1)\,L_{13}}{3M^2}$$

$$+\frac{i\,(D_2D_3-2D_3D_2)\,L_{14}}{3M^2}$$

$$+\frac{(3MD_0-i\,(D_1D_1+2D_3D_3))\,L_{15}}{3M^2}=0.$$

Let us separate 15 equations with the nonrelativistic structure (it is convenient to numerate them)):

with the use of more short notations

$$L_{10}=L_1,\quad L_{11}=L_2,\quad L_{13}=L_3,\quad L_{14}=L_4,\quad L_{15}=L_5, \tag{2.1}$$

$$1)\quad -2D_2D_1L_1M-2D_2^2L_2M-2D_3^2L_2M-2D_3D_1L_3M$$
$$-\frac{1}{6}D_1D_3L_3M-2D_2^2L_5M-2D_3^2L_5M-2iD_0L_2M^2-2iD_0L_5M^2=0,$$

$$2)\quad -2D_1D_2L_1M+2D_1^2L_2M+2D_3^2L_2M-\frac{1}{6}D_1D_3L_3M-2D_3D_2L_4M+2iD_0L_2M^2=0,$$

$$3)\quad -\frac{3}{2}D_1D_3L_3M-2D_2D_3L_4M+2D_1^2L_5M+2D_2^2L_5M+2iD_0L_5M^2=0,$$

$$4)\quad -D_1D_3L_1M-D_2D_3L_2M-D_1D_2L_3M+2D_1^2L_4M+D_2^2L_4M+D_3^2L_4M-D_3D_2L_5M+2iD_0L_4M^2=0,$$

$$5)\quad -D_2D_3L_1M+D_1D_3L_2M+D_1^2L_3M+2D_2^2L_3M$$
$$+\frac{2}{3}D_3^2L_3M-D_2D_1L_4M-D_3D_1L_5M+D_1D_3L_5M+2iD_0L_3M^2=0,$$

$$6)\quad D_1^2L_1M+D_2^2L_1M+2D_3^2L_1M-D_2D_1L_2M+D_1D_2L_2M-D_3D_2L_3M$$
$$-D_3D_1L_4M+D_1D_2L_5M+2iD_0L_1M^2=0,$$

$$7)\quad -\frac{2}{3}D_2D_1L_1M+\frac{4}{3}D_1D_2L_1M-\frac{4}{3}D_1^2L_2M-\frac{2}{3}D_2^2L_2M-\frac{2}{3}D_3D_1L_3M+\frac{4}{3}D_1D_3L_3M-\frac{2}{3}D_3D_2L_4M$$
$$-\frac{2}{3}D_2D_3L_4M-\frac{4}{3}D_1^2L_5M-\frac{2}{3}D_3^2L_5M-2iD_0L_2M^2-2iD_0L_5M^2=0,$$

$$8)\quad 2D_2^2L_1M-2D_2D_1L_2M+2D_2D_3L_3M-2D_2D_1L_5M+2iD_0L_1M^2=0,$$

$$9)\quad 2D_3D_2L_1M-2D_3D_1L_2M+2D_3^2L_3M-2D_3D_1L_5M+2iD_0L_3M^2=0,$$

$$10)\quad 2D_1^2L_1M+2D_1D_2L_2M+2D_1D_3L_4M+2iD_0L_1M^2=0,$$

$$11)\quad (D_0M-\frac{1}{3}i(D_1^2+2D_2^2))L_2+\frac{1}{3}i(D_1D_2-2D_2D_1)L_1$$
$$+\frac{1}{3}i(D_3D_1+D_1D_3)L_3+\frac{1}{3}i(D_3D_2-2D_2D_3)L_4-\frac{1}{3}i(D_1^2-D_3^2)L_5=0,$$

$$11)\quad \frac{4}{3}D_2D_1L_1M-\frac{2}{3}D_1D_2L_1M+\frac{2}{3}D_1^2L_2M+\frac{4}{3}D_2^2L_2M-\frac{2}{3}D_3D_1L_3M-\frac{2}{3}D_1D_3L_3M$$

$$-\frac{2}{3}D_3D_2L_4M+\frac{4}{3}D_2D_3L_4M+\frac{2}{3}D_1^2L_5M-\frac{2}{3}D_3^2L_5M+2iD_0L_2M^2=0,$$

$$12)\quad 2D_3D_1L_1M+2D_3D_2L_2M+2D_3^2L_4M+2iD_0L_4M^2=0,$$

$$13)\quad 2D_1^2L_3M+2D_1D_2L_4M+2D_1D_3L_5M+2iD_0L_3M^2=0,$$

$$14)\quad 2D_2D_1L_3M+2D_2^2L_4M+2D_2D_3L_5M+2iD_0L_4M^2=0,$$

$$15)\quad -\frac{2}{3}D_2D_1L_1M-\frac{2}{3}D_1D_2L_1M+\frac{2}{3}D_1^2L_2M-\frac{2}{3}D_2^2L_2M+\frac{4}{3}D_3D_1L_3M-\frac{2}{3}D_1D_3L_3M$$
$$+\frac{4}{3}D_3D_2L_4M-\frac{2}{3}D_2D_3L_4M+\frac{2}{3}D_1^2L_5M+\frac{4}{3}D_3^2L_5M+2iD_0L_5M^2=0.$$

Further, let us take into account the following identities (note that $F_{ij}=\partial_iA_j-\partial_jA_i$)

$$D_iD_j=\frac{1}{2}(D_iD_j+D_jD_i)+\frac{1}{2}(D_iD_j-D_jD_i)=\frac{1}{2}(D_iD_j+D_jD_i)+ieF_{ij}\equiv D_{(ij)}+ieF_{ij}; \tag{2.2}$$

so in all equations should arise only the following terms

$$\begin{gathered}2iMD_0L_k,\quad D_{(11)}=D_1^2L_k,\quad D_{(22)}=D_2^2L_k,\quad D_{(33)}=D_3^2L_k,\\ D_{(23)}L_k,\quad D_{(32)}L_k,\quad D_{(12)}L_k, F_{23}L_k,\quad F_{31}L_k,\quad F_{12}L_k.\end{gathered} \tag{2.3}$$

In this way, we obtain (below we will omit parentheses in $D_{(ij)}$)

$$1)\quad -2iD_0L_2M^2-2iD_0L_5M^2-2D_{12}L_1M-2D_{22}L_2M-2D_{33}L_2M$$
$$-\frac{13}{6}D_{31}L_3M-2D_{22}L_5M-2D_{33}L_5M+2ieF_{12}L_1M-\frac{11}{6}ieF_{31}L_3M=0,$$

$$2)\quad 2iD_0L_2M^2-2D_{12}L_1M+2D_{11}L_2M+2D_{33}L_2M-\frac{1}{6}D_{31}L_3M-2D_{23}L_4M-2ieF_{12}L_1M$$
$$+\frac{1}{6}ieF_{31}L_3M+2ieF_{23}L_4M=0,$$

$$3)\quad 2iD_0L_5M^2-\frac{3}{2}D_{31}L_3M-2D_{23}L_4M+2D_{11}L_5M+2D_{22}L_5M+\frac{3}{2}ieF_{31}L_3M-2ieF_{23}L_4M=0,$$

$$4)\quad 2iD_0L_4M^2-D_{31}L_1M-D_{23}L_2M-D_{12}L_3M+2D_{11}L_4M+D_{22}L_4M+D_{33}L_4M-D_{23}L_5M$$
$$+ieF_{31}L_1M-ieF_{23}L_2M-ieF_{12}L_3M+ieF_{23}L_5M=0,$$

$$5)\quad 2iD_0L_3M^2-D_{23}L_1M+D_{31}L_2M+D_{11}L_3M+2D_{22}L_3M+\frac{2}{3}D_{33}L_3M-D_{12}L_4M$$
$$-ieF_{23}L_1M-ieF_{31}L_2M+ieF_{12}L_4M-2ieF_{31}L_5M=0,$$

$$6)\quad 2iD_0L_1M^2+D_{11}L_1M+D_{22}L_1M+2D_{33}L_1M-D_{23}L_3M-D_{31}L_4M+D_{12}L_5M$$
$$+2ieF_{12}L_2M+ieF_{23}L_3M-ieF_{31}L_4M+ieF_{12}L_5M=0,$$

$$7)\quad -2iD_0L_2M^2-2iD_0L_5M^2+\frac{2}{3}D_{12}L_1M-\frac{4}{3}D_{11}L_2M-\frac{2}{3}D_{22}L_2M+\frac{2}{3}D_{31}L_3M-\frac{4}{3}D_{23}L_4M$$
$$-\frac{4}{3}D_{11}L_5M-\frac{2}{3}D_{33}L_5M+2ieF_{12}L_1M-2ieF_{31}L_3M=0,$$

$$8)\quad 2iD_0L_1M^2+2D_{22}L_1M-2D_{12}L_2M+2D_{23}L_3M-2D_{12}L_5M$$
$$+2ieF_{12}L_2M+2ieF_{23}L_3M+2ieF_{12}L_5M=0,$$

$$9)\quad 2iD_0L_3M^2+2D_{23}L_1M-2D_{31}L_2M+2D_{33}L_3M-2D_{31}L_5M-2ieF_{23}L_1M-2ieF_{31}L_2M-2ieF_{31}L_5M=0,$$

$$10)\qquad 2iD_0L_1M^2 + 2D_{11}L_1M + 2D_{12}L_2M + 2D_{31}L_4M + 2ieF_{12}L_2M - 2ieF_{31}L_4M = 0,$$

$$11)\qquad 2iD_0L_2M^2 + \frac{2}{3}D_{12}L_1M + \frac{2}{3}D_{11}L_2M + \frac{4}{3}D_{22}L_2M - \frac{4}{3}D_{31}L_3M + \frac{2}{3}D_{23}L_4M + \frac{2}{3}D_{11}L_5M$$
$$-\frac{2}{3}D_{33}L_5M - 2ieF_{12}L_1M + 2ieF_{23}L_4M = 0,$$

$$12)\qquad 2iD_0L_4M^2 + 2D_{31}L_1M + 2D_{23}L_2M + 2D_{33}L_4M + 2ieF_{31}L_1M - 2ieF_{23}L_2M = 0,$$

$$13)\qquad 2iD_0L_3M^2 + 2D_{11}L_3M + 2D_{12}L_4M + 2D_{31}L_5M + 2ieF_{12}L_4M - 2ieF_{31}L_5M = 0,$$

$$14)\qquad 2iD_0L_4M^2 + 2D_{12}L_3M + 2D_{22}L_4M + 2D_{23}L_5M - 2ieF_{12}L_3M + 2ieF_{23}L_5M = 0,$$

$$15)\qquad 2iD_0L_5M^2 - \frac{4}{3}D_{12}L_1M + \frac{2}{3}D_{11}L_2M - \frac{2}{3}D_{22}L_2M + \frac{2}{3}D_{31}L_3M$$
$$+\frac{2}{3}D_{23}L_4M + \frac{2}{3}D_{11}L_5M + \frac{4}{3}D_{33}L_5M + 2ieF_{31}L_3M - 2ieF_{23}L_4M = 0.$$

Let us combine the above equations so that to get equations with the non-relativistic structure (they contain the terms $D_0L_1, , ..., D_0L_5$), and some constraints for large functions which do not contain the derivative $D_0$:

In this way, we obtain the five equations with the needed structure

$$10),\quad 2iD_0L_1M + 2D_{11}L_1 + 2D_{12}L_2 + 2D_{31}L_4 + 2ieF_{12}L_2 - 2ieF_{31}L_4 = 0,$$

$$2)+11),\quad 4iD_0L_2 - \frac{4}{3}D_{12}L_1 + \frac{8}{3}D_{11}L_2 + \frac{4}{3}D_{22}L_2 + 2D_{33}L_2 - \frac{3}{2}D_{31}L_3 - \frac{4}{3}D_{23}L_4$$

$$+\frac{2}{3}D_{11}L_5 - \frac{2}{3}D_{33}L_5 - 4ieF_{12}L_1 + \frac{1}{6}ieF_{31}L_3 + 4ieF_{23}L_4 = 0,$$

$$13),\quad 2iD_0L_3M + 2D_{11}L_3 + 2D_{12}L_4 + 2D_{31}L_5 + 2ieF_{12}L_4 - 2ieF_{31}L_5 = 0,$$

$$14),\quad 2iD_0L_4M + 2D_{12}L_3 + 2D_{22}L_4 + 2D_{23}L_5 - 2ieF_{12}L_3 + 2ieF_{23}L_5 = 0,$$

$$3),\quad 2iD_0L_5M - \frac{3}{2}D_{31}L_3 - 2D_{23}L_4 + 2D_{11}L_5 + 2D_{22}L_5 + \frac{3}{2}ieF_{31}L_3 - 2ieF_{23}L_4 = 0.$$

Besides, there arise differential constraints which do not contain the derivative $D_0$:

$1) + 7) + 2 + 11 + 2 \times 3)$

$$-\frac{8}{3}D_{12}L_1 + \frac{4}{3}D_{11}L_2 - \frac{4}{3}D_{22}L_2 - 6D_{31}L_3 - \frac{20}{3}D_{23}L_4\frac{10}{3}D_{11}L_5 + 2D_{22}L_5 - \frac{10}{3}D_{33}L_5 - \frac{2}{3}ieF_{31}L_3 = 0,$$

$1) - 7)$

$$-\frac{8}{3}D_{12}L_1 + \frac{4}{3}D_{11}L_2 - \frac{4}{3}D_{22}L_2 - 2D_{33}L_2 - \frac{17}{6}D_{31}L_3 + \frac{4}{3}D_{23}L_4\frac{4}{3}D_{11}L_5 - 2D_{22}L_5 - \frac{4}{3}D_{33}L_5 + \frac{1}{6}ieF_{31}L_3 = 0,$$

$6) - 10)$

$$-D_{11}L_1 + D_{22}L_1 + 2D_{33}L_1 - 2D_{12}L_2 - D_{23}L_3 - 3D_{31}L_4 + D_{12}L_5 + ieF_{23}L_3 + ieF_{31}L_4 + ieF_{12}L_5 = 0,$$

8) − 10)

$$-2D_{11}L_1 + 2D_{22}L_1 - 4D_{12}L_2 + 2D_{23}L_3 - 2D_{31}L_4 - 2D_{12}L_5 + 2ieF_{23}L_3 + 2ieF_{31}L_4 + 2ieF_{12}L_5 = 0,$$

2) − 11)

$$-\frac{8}{3}D_{12}L_1 + \frac{4}{3}D_{11}L_2 - \frac{4}{3}D_{22}L_2 + 2D_{33}L_2 + \frac{7}{6}D_{31}L_3 - \frac{8}{3}D_{23}L_4 - \frac{2}{3}D_{11}L_5 + \frac{2}{3}D_{33}L_5 + \frac{1}{6}ieF_{31}L_3 = 0,$$

5(−13)

$$-D_{23}L_1 + D_{31}L_2 - D_{11}L_3 + 2D_{22}L_3 + \frac{2}{3}D_{33}L_3 - 3D_{12}L_4 - 2D_{31}L_5 - ieF_{23}L_1 - ieF_{31}L_2 - ieF_{12}L_4 = 0;$$

9) − 13)

$$2D_{23}L_1 - 2D_{31}L_2 - 2D_{11}L_3 + 2D_{33}L_3 - 2D_{12}L_4 - 4D_{31}L_5 - 2ieF_{23}L_1 - 2ieF_{31}L_2 - 2ieF_{12}L_4 = 0,$$

4) − 14)

$$-D_{31}L_1 - D_{23}L_2 - 3D_{12}L_3 + 2D_{11}L_4 - D_{22}L_4 + D_{33}L_4 - 3D_{23}L_5 + ieF_{31}L_1 - ieF_{23}L_2 + ieF_{12}L_3 - ieF_{23}L_5 = 0,$$

12) − 14)

$$2D_{31}L_1 + 2D_{23}L_2 - 2D_{12}L_3 - 2D_{22}L_4 + 2D_{33}L_4 - 2D_{23}L_5 + 2ieF_{31}L_1 - 2ieF_{23}L_2 + 2ieF_{12}L_3 - 2ieF_{23}L_5 = 0,$$

15) − 3)

$$-\frac{4}{3}D_{12}L_1 + \frac{2}{3}D_{11}L_2 - \frac{2}{3}D_{22}L_2 + \frac{13}{6}D_{31}L_3 + \frac{8}{3}D_{23}L_4 - \frac{4}{3}D_{11}L_5 - 2D_{22}L_5 + \frac{4}{3}D_{33}L_5 + \frac{1}{2}ieF_{31}L_3 = 0.$$

The last system of constraints may be presented in matrix form

$$A_{(10\times 15)} \times \left| \begin{array}{c} D_{12}L_1 \\ D_{12}L_2 \\ D_{12}L_3 \\ D_{12}L_4 \\ D_{12}L_5 \\ D_{31}L_1 \\ D_{31}L_2 \\ D_{31}L_3 \\ D_{31}L_4 \\ D_{31}L_5 \\ D_{23}L_1 \\ D_{23}L_2 \\ D_{23}L_3 \\ D_{23}L_4 \\ D_{23}L_5 \end{array} \right| = Y_{(10\times 1)},$$

where

$$
Y = \left| \begin{array}{c}
-\frac{4}{3}D_{11}L_2 + \frac{4D_{22}L_2}{3} - \frac{10D_{11}L_5}{3} - 2D_{22}L_5 + \frac{10D_{33}L_5}{3} + \frac{2}{3}ieF_{31}L_3 \\
-\frac{4}{3}D_{11}L_2 + \frac{4D_{22}L_2}{3} + 2D_{33}L_2 - \frac{4D_{11}L_5}{3} + 2D_{22}L_5 + \frac{4D_{33}L_5}{3} - \frac{1}{6}ieF_{31}L_3 \\
D_{11}L_1 - D_{22}L_1 - 2D_{33}L_1 - ieF_{23}L_3 - ieF_{31}L_4 - ieF_{12}L_5 \\
2D_{11}L_1 - 2D_{22}L_1 - 2ieF_{23}L_3 - 2ieF_{31}L_4 - 2ieF_{12}L_5 \\
-\frac{4}{3}D_{11}L_2 + \frac{4D_{22}L_2}{3} - 2D_{33}L_2 + \frac{2D_{11}L_5}{3} - \frac{2D_{33}L_5}{3} - \frac{1}{6}ieF_{31}L_3 \\
D_{11}L_3 - 2D_{22}L_3 - \frac{2D_{33}L_3}{3} + ieF_{23}L_1 + ieF_{31}L_2 + ieF_{12}L_4 \\
2D_{11}L_3 - 2D_{33}L_3 + 2ieF_{23}L_1 + 2ieF_{31}L_2 + 2ieF_{12}L_4 \\
-2D_{11}L_4 + D_{22}L_4 - D_{33}L_4 - ieF_{31}L_1 + ieF_{23}L_2 - ieF_{12}L_3 + ieF_{23}L_5 \\
2D_{22}L_4 - 2D_{33}L_4 - 2ieF_{31}L_1 + 2ieF_{23}L_2 - 2ieF_{12}L_3 + 2ieF_{23}L_5 \\
-\frac{2}{3}D_{11}L_2 + \frac{2D_{22}L_2}{3} + \frac{4D_{11}L_5}{3} + 2D_{22}L_5 - \frac{4D_{33}L_5}{3} - \frac{1}{2}ieF_{31}L_3
\end{array} \right| ,
$$

$$
A = \left| \begin{array}{ccccccccccccccc}
-\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 & -\frac{20}{3} & 0 \\
-\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{17}{6} & 0 & 0 & 0 & 0 & 0 & \frac{4}{3} & 0 \\
0 & -2 & 0 & 0 & 1 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & -4 & 0 & 0 & -2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 2 & 0 & 0 \\
-\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{6} & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & 0 \\
0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 & -2 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2 & 0 & 0 & -2 & 0 & 0 & -4 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & -3 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -3 \\
0 & 0 & -2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -2 \\
-\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{13}{6} & 0 & 0 & 0 & 0 & 0 & \frac{8}{3} & 0
\end{array} \right| .
$$

The rank of the matrix $A_{(10\times15)}$ equals 9; when eliminating the first row, the rank is not changed. Therefore, the first row should be decomposed into the terms of remaining 9 rows:

$$x_1 = b_2x_2 + b_3x_3 + ... + b_{10}x_{10}.$$

We find the relevant coefficients

$$b_2 = 1, b_3 = 0, b_4 = 0, b_5 = 1, b_6 = 0,$$

$$b_7 = 0, b_8 = 0, b_9 = 0, b_{10} = -2.$$

We can verify that the same is valid for row of the right side $y_1 = b_2y_2 + b_3y_3 + ... + b_{10}y_{10}$. Therefore, the first equation in the system may be removed, so we arrive at 9 equations

$$
A \times \begin{vmatrix} D_{12}L_1 \\ D_{12}L_2 \\ D_{12}L_3 \\ D_{12}L_4 \\ D_{12}L_5 \\ D_{31}L_1 \\ D_{31}L_2 \\ D_{31}L_3 \\ D_{31}L_4 \\ D_{31}L_5 \\ D_{23}L_1 \\ D_{23}L_2 \\ D_{23}L_3 \\ D_{23}L_4 \\ D_{23}L_5 \end{vmatrix} = Y, Y = \begin{vmatrix} -\frac{4}{3}D_{11}L_2 + \frac{4D_{22}L_2}{3} + 2D_{33}L_2 - \frac{4D_{11}L_5}{3} + 2D_{22}L_5 + \frac{4D_{33}L_5}{3} - \frac{1}{6}ieF_{31}L_3 \\ D_{11}L_1 - D_{22}L_1 - 2D_{33}L_1 - ieF_{23}L_3 - ieF_{31}L_4 - ieF_{12}L_5 \\ 2D_{11}L_1 - 2D_{22}L_1 - 2ieF_{23}L_3 - 2ieF_{31}L_4 - 2ieF_{12}L_5 \\ -\frac{4}{3}D_{11}L_2 + \frac{4D_{22}L_2}{3} - 2D_{33}L_2 + \frac{2D_{11}L_5}{3} - \frac{2D_{33}L_5}{3} - \frac{1}{6}ieF_{31}L_3 \\ D_{11}L_3 - 2D_{22}L_3 - \frac{2D_{33}L_3}{3} + ieF_{23}L_1 + ieF_{31}L_2 + ieF_{12}L_4 \\ 2D_{11}L_3 - 2D_{33}L_3 + 2ieF_{23}L_1 + 2ieF_{31}L_2 + 2ieF_{12}L_4 \\ -2D_{11}L_4 + D_{22}L_4 - D_{33}L_4 - ieF_{31}L_1 + ieF_{23}L_2 - ieF_{12}L_3 + ieF_{23}L_5 \\ 2D_{22}L_4 - 2D_{33}L_4 - 2ieF_{31}L_1 + 2ieF_{23}L_2 - 2ieF_{12}L_3 + 2ieF_{23}L_5 \\ -\frac{2}{3}D_{11}L_2 + \frac{2D_{22}L_2}{3} + \frac{4D_{11}L_5}{3} + 2D_{22}L_5 - \frac{4D_{33}L_5}{3} - \frac{1}{2}ieF_{31}L_3 \end{vmatrix},
$$

$$
A = \begin{vmatrix}
-\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{17}{6} & 0 & 0 & 0 & 0 & 0 & \frac{4}{3} & 0 \\
0 & -2 & 0 & 0 & 1 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & -4 & 0 & 0 & -2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 2 & 0 & 0 \\
-\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{6} & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & 0 \\
0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 & -2 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2 & 0 & 0 & -2 & 0 & 0 & -4 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & -3 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -3 \\
0 & 0 & -2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -2 \\
-\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{13}{6} & 0 & 0 & 0 & 0 & 0 & \frac{8}{3} & 0
\end{vmatrix}.
$$

In the matrix $A_{9\times 15}$, we should find the sub-matrix $A_{9\times 9}$ which determinant does not non-vanish; it is verified that we can preserve only the columns $1, 2, 3, 4, 5, 6, 7, 8, 14$:

$$
A_{9\times 9} = \begin{vmatrix}
-\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{17}{6} & \frac{4}{3} \\
0 & -2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & -4 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\
-\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{6} & -\frac{8}{3} \\
0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -2 & 0 & 0 & -2 & 0 & 0 \\
0 & 0 & -3 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 2 & 0 & 0 & 0 \\
-\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{13}{6} & \frac{8}{3}
\end{vmatrix}, \quad \det A = -\frac{274432}{9};
$$

Thus, we arrive at the non-homogenous equation

$$
\begin{vmatrix}
-\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{17}{6} & \frac{4}{3} \\
0 & -2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & -4 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\
-\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{6} & -\frac{8}{3} \\
0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -2 & 0 & 0 & -2 & 0 & 0 \\
0 & 0 & -3 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 2 & 0 & 0 & 0 \\
-\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{13}{6} & \frac{8}{3}
\end{vmatrix} \times \begin{vmatrix} D_{12}L_1 \\ D_{12}L_2 \\ D_{12}L_3 \\ D_{12}L_4 \\ D_{12}L_5 \\ D_{31}L_1 \\ D_{31}L_2 \\ D_{31}L_3 \\ D_{23}L_4 \end{vmatrix}
$$

$$= \left| \begin{array}{c} -\frac{4}{3}D_{11}L_2 + \frac{4D_{22}L_2}{3} + 2D_{33}L_2 - \frac{4D_{11}L_5}{3} + 2D_{22}L_5 + \frac{4D_{33}L_5}{3} - \frac{1}{6}ieF_{31}L_3 \\ D_{11}L_1 - D_{22}L_1 - 2D_{33}L_1 - ieF_{23}L_3 - ieF_{31}L_4 - ieF_{12}L_5 \\ 2D_{11}L_1 - 2D_{22}L_1 - 2ieF_{23}L_3 - 2ieF_{31}L_4 - 2ieF_{12}L_5 \\ -\frac{4}{3}D_{11}L_2 + \frac{4D_{22}L_2}{3} - 2D_{33}L_2 + \frac{2D_{11}L_5}{3} - \frac{2D_{33}L_5}{3} - \frac{1}{6}ieF_{31}L_3 \\ D_{11}L_3 - 2D_{22}L_3 - \frac{2D_{33}L_3}{3} + ieF_{23}L_1 + ieF_{31}L_2 + ieF_{12}L_4 \\ 2D_{11}L_3 - 2D_{33}L_3 + 2ieF_{23}L_1 + 2ieF_{31}L_2 + 2ieF_{12}L_4 \\ -2D_{11}L_4 + D_{22}L_4 - D_{33}L_4 - ieF_{31}L_1 + ieF_{23}L_2 - ieF_{12}L_3 + ieF_{23}L_5 \\ 2D_{22}L_4 - 2D_{33}L_4 - 2ieF_{31}L_1 + 2ieF_{23}L_2 - 2ieF_{12}L_3 + 2ieF_{23}L_5 \\ -\frac{2}{3}D_{11}L_2 + \frac{2D_{22}L_2}{3} + \frac{4D_{11}L_5}{3} + 2D_{22}L_5 - \frac{4D_{33}L_5}{3} - \frac{1}{2}ieF_{31}L_3 \end{array} \right|$$

$$+ \left| \begin{array}{c} 0 \\ 3 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right| D_{31}L_4 + \left| \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 4 \\ 0 \\ 0 \\ 0 \end{array} \right| D_{31}L_5 + \left| \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -2 \\ 0 \\ 0 \\ 0 \end{array} \right| D_{23}L_1 + \left| \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -2 \\ 0 \end{array} \right| D_{23}L_2 + \left| \begin{array}{c} 0 \\ 1 \\ -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right| D_{23}L_3 + \left| \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 3 \\ 2 \\ 0 \end{array} \right| D_{23}L_5.$$

Its solution has the form

$$\begin{aligned} D_{12}L_1 &= \frac{D_{11}L_2}{2} - \frac{D_{22}L_2}{2} + \frac{7D_{33}L_2}{134} - \frac{7D_{11}L_5}{134} - \frac{D_{22}L_5}{2} + \frac{7D_{33}L_5}{134} + \frac{7}{67}ieF_{31}L_3, \\ D_{12}L_2 &= -\frac{1}{2}D_{11}L_1 + \frac{D_{22}L_1}{2} + \frac{D_{33}L_1}{2} - D_{31}L_4 + \frac{1}{2}ieF_{23}L_3 + \frac{1}{2}ieF_{31}L_4 + \frac{1}{2}ieF_{12}L_5, \\ D_{12}L_3 &= \frac{D_{11}L_4}{2} - \frac{D_{22}L_4}{2} + \frac{D_{33}L_4}{2} - D_{23}L_5 + \frac{1}{2}ieF_{31}L_1 - \frac{1}{2}ieF_{23}L_2 + \frac{1}{2}ieF_{12}L_3 - \frac{1}{2}ieF_{23}L_5, \\ D_{12}L_4 &= -\frac{D_{11}L_3}{2} + \frac{D_{22}L_3}{2} + \frac{5D_{33}L_3}{12} - D_{31}L_5 - \frac{1}{2}ieF_{23}L_1 - \frac{1}{2}ieF_{31}L_2 - \frac{1}{2}ieF_{12}L_4, \\ D_{12}L_5 &= -D_{33}L_1 + D_{23}L_3 + D_{31}L_4, \\ D_{31}L_1 &= -D_{23}L_2 + \frac{D_{11}L_4}{2} + \frac{D_{22}L_4}{2} - \frac{D_{33}L_4}{2} - \frac{1}{2}ieF_{31}L_1 + \frac{1}{2}ieF_{23}L_2 - \frac{1}{2}ieF_{12}L_3 + \frac{1}{2}ieF_{23}L_5, \\ D_{31}L_2 &= D_{23}L_1 - \frac{D_{11}L_3}{2} - \frac{D_{22}L_3}{2} + \frac{7D_{33}L_3}{12} - D_{31}L_5 - \frac{1}{2}ieF_{23}L_1 - \frac{1}{2}ieF_{31}L_2 - \frac{1}{2}ieF_{12}L_4, \\ D_{31}L_3 &= -\frac{36}{67}D_{33}L_2 + \frac{36D_{11}L_5}{67} - \frac{36D_{33}L_5}{67} - \frac{5}{67}ieF_{31}L_3, \\ D_{23}L_4 &= \frac{31D_{33}L_2}{67} + \frac{5D_{11}L_5}{134} + \frac{D_{22}L_5}{2} - \frac{5D_{33}L_5}{134} - \frac{5}{67}ieF_{31}L_3. \end{aligned} \tag{2.4}$$

We should take them into account in the above 5 equations; as a result, we obtain

$$
\begin{aligned}
2iMD_0L_1 &= -(D_{11}+D_{22}+D_{33})L_1 + ie\Big\{-F_{23}L_3 + F_{31}L_4 + F_{12}(-2L_2 - L_5)\Big\},\\
2iMD_0L_2 &= -(D_{11}+D_{22}+D_{33})L_2 + ie\Big\{-2F_{23}L_4 + 2F_{12}L_1\Big\},\\
2iMD_0L_3 &= -(D_{11}+D_{22}+D_{33})L_3 + ie\Big\{F_{23}L_1 + F_{31}(L_2 + 2L_5) - F_{12}L_4\Big\},\\
2iMD_0L_4 &= -(D_{11}+D_{22}+D_{33})L_4 + ie\Big\{F_{23}(L_2 - L_5) - F_{31}L_1 + F_{12}L_3\Big\},\\
2iMD_0L_5 &= -(D_{11}+D_{22}+D_{33})L_5 + ie\Big\{2F_{23}L_4 - 2F_{31}L_3\Big\}.
\end{aligned}
\tag{2.5}
$$

Let us present this system in the matrix form

$$
iD_0\Psi = \frac{1}{2M}(D_{11}+D_{22}+D_{33})\Psi + \frac{ie}{2M}(F_{23}S_1 + F_{31}S_2 + F_{12}S_3)\Psi, \tag{2.6}
$$

$$
\Psi = \begin{vmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \end{vmatrix}, \quad S_1 = \begin{vmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 & 0 \end{vmatrix}, S_2 = \begin{vmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \end{vmatrix}, \quad S_3 = \begin{vmatrix} 0 & -2 & 0 & 0 & -1 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}.
$$

The needed commutators are valid

$$
S_1S_2 - S_2S_1 = S_3, \text{ and so on;}
$$

so the matrices $S_i$ are 3 projections of spin for a spin 2 particle.

## 3. Conclusions

We began with the well-established 39-component relativistic matrix equation for a spin-2 particle interacting with external electromagnetic fields.

Through a non-relativistic approximation, we decomposed the wave function into three parts: one dominant component and two smaller ones. This process revealed five independent large variables, all remaining are small ones.

To simplify the problem, we eliminated the variables associated with the scalar and symmetrical two rank tensor, leading to a relativistic system of second-order equations for the 11 components of the comlete wave function. We then expressed these 11 variables as linear combinations of the large and small components, aligning with the non-relativistic framework.

Following the general method, we separated the rest energy from the wave function and established the orders of smallness for various terms in the equations. By systematically performing the necessary calculations, we derived a system of five coupled equations for the large variables. These equations are presented in matrix form, exhibiting a non-relativistic structure. Crucially, this system includes an interaction term representing the coupling with the external magnetic field through the three spin projections.

The Pauli like equation (2.6) for spin 2 particle should be generalized to the curved space-times models.

## Appendix A. Independent large and small variables

$$
\Psi_+ = \begin{vmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
\frac{1}{6}\left(2E_{11} - E_{22} - E_{33} + 2f_1 - f_2 - f_3\right) \\
\frac{1}{6}\left(-E_{11} + 2E_{22} - E_{33} - f_1 + 2f_2 - f_3\right) \\
\frac{1}{6}\left(-E_{11} - E_{22} + 2E_{33} - f_1 - f_2 + 2f_3\right) \\
\frac{1}{4}\left(2c_1 + E_{23} + E_{32}\right) \\
\frac{1}{4}\left(2c_2 + E_{13} + E_{31}\right) \\
\frac{1}{4}\left(2c_3 + E_{12} + E_{21}\right) \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\frac{1}{6}\left(2E_{11} - E_{22} - E_{33} + 2f_1 - f_2 - f_3\right) \\
\frac{1}{4}\left(2c_3 + E_{12} + E_{21}\right) \\
\frac{1}{4}\left(2c_2 + E_{13} + E_{31}\right) \\
0 \\
0 \\
0 \\
\frac{1}{4}\left(2c_3 + E_{12} + E_{21}\right) \\
\frac{1}{6}\left(-E_{11} + 2E_{22} - E_{33} - f_1 + 2f_2 - f_3\right) \\
\frac{1}{4}\left(2c_1 + E_{23} + E_{32}\right) \\
0 \\
0 \\
0 \\
\frac{1}{4}\left(2c_2 + E_{13} + E_{31}\right) \\
\frac{1}{4}\left(2c_1 + E_{23} + E_{32}\right) \\
\frac{1}{6}\left(-E_{11} - E_{22} + 2E_{33} - f_1 - f_2 + 2f_3\right) \\
0 \\
0 \\
0
\end{vmatrix}
= \begin{vmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ L_7 \\ L_8 \\ L_9 \\ 0 \\ 0 \\ 0 \\ L_{10} \\ L_{11} \\ L_{12} \\ 0 \\ 0 \\ 0 \\ L_{13} \\ L_{14} \\ L_{15} \\ 0 \\ 0 \\ 0
\end{vmatrix},
$$

$$\Psi_- = \left| \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6}\left(-2E_{11}+E_{22}+E_{33}+2f_1-f_2-f_3\right) \\ \frac{1}{6}\left(E_{11}-2E_{22}+E_{33}-f_1+2f_2-f_3\right) \\ \frac{1}{6}\left(E_{11}+E_{22}-2E_{33}-f_1-f_2+2f_3\right) \\ \frac{1}{4}\left(2c_1-E_{23}-E_{32}\right) \\ \frac{1}{4}\left(2c_2-E_{13}-E_{31}\right) \\ \frac{1}{4}\left(2c_3-E_{12}-E_{21}\right) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6}\left(2E_{11}-E_{22}-E_{33}-2f_1+f_2+f_3\right) \\ \frac{1}{4}\left(-2c_3+E_{12}+E_{21}\right) \\ \frac{1}{4}\left(-2c_2+E_{13}+E_{31}\right) \\ 0 \\ 0 \\ 0 \\ \frac{1}{4}\left(-2c_3+E_{12}+E_{21}\right) \\ \frac{1}{6}\left(-E_{11}+2E_{22}-E_{33}+f_1-2f_2+f_3\right) \\ \frac{1}{4}\left(-2c_1+E_{23}+E_{32}\right) \\ 0 \\ 0 \\ 0 \\ \frac{1}{4}\left(-2c_2+E_{13}+E_{31}\right) \\ \frac{1}{4}\left(-2c_1+E_{23}+E_{32}\right) \\ \frac{1}{6}\left(-E_{11}-E_{22}+2E_{33}+f_1+f_2-2f_3\right) \\ 0 \\ 0 \\ 0 \end{array} \right| = \left| \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ S_7 \\ S_8 \\ S_9 \\ 0 \\ 0 \\ 0 \\ S_{10} \\ S_{11} \\ S_{12} \\ 0 \\ 0 \\ 0 \\ S_{13} \\ S_{14} \\ S_{15} \\ 0 \\ 0 \\ 0 \end{array} \right| ,$$

$$\Psi_0 = P_3\Psi = \left| \begin{array}{c} \Phi \\ \Phi_0 \\ \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \frac{1}{3}(f_1+f_2+f_3) \\ \frac{1}{3}(f_1+f_2+f_3) \\ \frac{1}{3}(f_1+f_2+f_3) \\ 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \\ d_3 \\ f_0 \\ E_{10} \\ E_{20} \\ E_{30} \\ B_{10} \\ B_{20} \\ B_{30} \\ \frac{1}{3}(E_{11}+E_{22}+E_{33}) \\ \frac{1}{2}(E_{21}-E_{12}) \\ \frac{1}{2}(E_{31}-E_{13}) \\ B_{11} \\ B_{21} \\ B_{31} \\ \frac{1}{2}(E_{12}-E_{21}) \\ \frac{1}{3}(E_{11}+E_{22}+E_{33}) \\ \frac{1}{2}(E_{32}-E_{23}) \\ B_{12} \\ B_{22} \\ B_{32} \\ \frac{1}{2}(E_{13}-E_{31}) \\ \frac{1}{2}(E_{23}-E_{32}) \\ \frac{1}{3}(E_{11}+E_{22}+E_{33}) \\ B_{13} \\ B_{23} \\ B_{33} \end{array} \right| = \left| \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ 0 \\ 0 \\ 0 \\ s_9 \\ s_{10} \\ s_{11} \\ s_{12} \\ s_{13} \\ s_{14} \\ s_{15} \\ s_{16} \\ s_{17} \\ s_{18} \\ s_{19} \\ s_{20} \\ s_{21} \\ s_{22} \\ s_{23} \\ s_{24} \\ s_{25} \\ s_{26} \\ s_{27} \\ s_{28} \\ s_{29} \\ s_{30} \\ s_{31} \\ s_{32} \\ s_{33} \\ s_{34} \\ s_{35} \\ s_{36} \end{array} \right| , \quad \Psi = \left| \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ -L_{11}-L_{15}+S_{11}+S_{15}+s_6 \\ L_{11}-S_{11}+s_6 \\ L_{15}-S_{15}+s_6 \\ L_{14}-S_{14} \\ L_{13}-S_{13} \\ L_{10}-S_{10} \\ s_9 \\ s_{10} \\ s_{11} \\ s_{12} \\ s_{13} \\ s_{14} \\ s_{15} \\ s_{16} \\ s_{17} \\ s_{18} \\ -L_{11}-L_{15}-S_{11}-S_{15}+s_{19} \\ L_{10}+S_{10}+s_{20} \\ L_{13}+S_{13}+s_{21} \\ s_{22} \\ s_{23} \\ s_{24} \\ L_{10}+S_{10}-s_{20} \\ L_{11}+S_{11}+s_{19} \\ L_{14}+S_{14}+s_{27} \\ s_{28} \\ s_{29} \\ s_{30} \\ L_{13}+S_{13}-s_{21} \\ L_{14}+S_{14}-s_{27} \\ L_{15}+S_{15}+s_{19} \\ s_{34} \\ s_{35} \\ s_{36} \end{array} \right| .$$

It is convenient to write down expressions for separate components:

$$\Psi_+,$$

$$L_1 = \frac{1}{6}\left(2E_{11} - E_{22} - E_{33} + 2f_1 - f_2 - f_3\right),$$

$$L_2 = \frac{1}{6}\left(-E_{11} + 2E_{22} - E_{33} - f_1 + 2f_2 - f_3\right),$$

$$L_3 = \frac{1}{6}\left(-E_{11} - E_{22} + 2E_{33} - f_1 - f_2 + 2f_3\right),$$

$$L_4 = \frac{1}{4}\left(2c_1 + E_{23} + E_{32}\right),$$

$$L_5 = \frac{1}{4}\left(2c_2 + E_{13} + E_{31}\right),$$

$$L_6 = \frac{1}{4}\left(2c_3 + E_{12} + E_{21}\right),$$

$$L_7 = \frac{1}{6}\left(2E_{11} - E_{22} - E_{33} + 2f_1 - f_2 - f_3\right),$$

$$L_8 = \frac{1}{4}\left(2c_3 + E_{12} + E_{21}\right),$$

$$L_9 = \frac{1}{4}(2c_2 + E_{13} + E_{31}), L_{10} = \frac{1}{4}(2c_3 + E_{12} + E_{21}),$$

$$L_{11} = \frac{1}{6}(-E_{11} + 2E_{22} - E_{33} - f_1 + 2f_2 - f_3),$$

$$L_{12} = \frac{1}{4}(2c_1 + E_{23} + E_{32}),$$

$$L_{13} = \frac{1}{4}(2c_2 + E_{13} + E_{31}),$$

$$L_{14} = \frac{1}{4}(2c_1 + E_{23} + E_{32}),$$

$$L_{15} = \frac{1}{6}(-E_{11} - E_{22} + 2E_{33} - f_1 - f_2 + 2f_3);$$

$$\Psi_-,$$

$$S_1 = \frac{1}{6}(-2E_{11} + E_{22} + E_{33} + 2f_1 - f_2 - f_3),$$

$$S_2 = \frac{1}{6}(E_{11} - 2E_{22} + E_{33} - f_1 + 2f_2 - f_3),$$

$$S_3 = \frac{1}{6}(E_{11} + E_{22} - 2E_{33} - f_1 - f_2 + 2f_3),$$

$$S_4 = \frac{1}{4}(2c_1 - E_{23} - E_{32}), S_5 = \frac{1}{4}(2c_2 - E_{13} - E_{31}),$$

$$S_6 = \frac{1}{4}(2c_3 - E_{12} - E_{21}),$$

$$S_7 = \frac{1}{6}(2E_{11} - E_{22} - E_{33} - 2f_1 + f_2 + f_3),$$

$$S_8 = \frac{1}{4}(-2c_3 + E_{12} + E_{21}),$$

$$S_9 = \frac{1}{4}(-2c_2 + E_{13} + E_{31}), S_{10} = \frac{1}{4}(-2c_3 + E_{12} + E_{21}),$$

$$S_{11} = \frac{1}{6}(-E_{11} + 2E_{22} - E_{33} + f_1 - 2f_2 + f_3),$$

$$S_{12} = \frac{1}{4}(-2c_1 + E_{23} + E_{32}), S_{13} = \frac{1}{4}(-2c_2 + E_{13} + E_{31}),$$

$$S_{14} = \frac{1}{4}(-2c_1 + E_{23} + E_{32}),$$

$$S_{15} = \frac{1}{6}(-E_{11} - E_{22} + 2E_{33} + f_1 + f_2 - 2f_3);$$

$$\Psi_0,$$

$$s_1 = \Phi, \quad s_2 = \Phi_0,$$

$$s_3 = \Phi_1, \; s_4 = \Phi_2, \; s_5 = \Phi_3,$$

$$s_6 = \frac{1}{3}(f_1 + f_2 + f_3), s_7 = \frac{1}{3}(f_1 + f_2 + f_3),$$

$$s_8 = \frac{1}{3}(f_1 + f_2 + f_3),$$

$$s_9 = d_1, \; s_{10} = d_2, \; s_{11} = d_3, \; s_{12} = f_0,$$

$$s_{13} = E_{10}, \; s_{14} = E_{20}, \; s_{15} = E_{30},$$

$$s_{16} = B_{10}, \; s_{17} = B_{20}, \; s_{18} = B_{30},$$

$$s_{19} = \frac{1}{3}(E_{11} + E_{22} + E_{33}),$$

$$s_{20} = \frac{1}{2}(E_{21} - E_{12}),$$

$$s_{21} = \frac{1}{2}(E_{31} - E_{13}),$$

$$s_{22} = B_{11}, \; s_{23} = B_{21}, \; s_{24} = B_{31},$$

$$s_{25} = \frac{1}{2}(E_{12} - E_{21}),$$

$$s_{26} = \frac{1}{3}(E_{11} + E_{22} + E_{33}),$$

$$s_{27} = \frac{1}{2}(E_{32} - E_{23}),$$

$$s_{28} = B_{12}, \; s_{29} = B_{22}, \; s_{30} = B_{32},$$

$$s_{31} = \frac{1}{2}(E_{13} - E_{31}),$$

$$s_{32} = \frac{1}{2}(E_{23} - E_{32}),$$

$$s_{33} = \frac{1}{3}(E_{11} + E_{22} + E_{33}),$$

$$s_{34} = B_{13}, \; s_{35} = B_{23}, \; s_{36} = B_{33}.$$

We can find independent variables in all three sets. First consider the large components $L_i$

$$L_1 = \frac{1}{6}(2E_{11} - E_{22} - E_{33} + 2f_1 - f_2 - f_3), \quad L_2 = \frac{1}{6}(-E_{11} + 2E_{22} - E_{33} - f_1 + 2f_2 - f_3),$$

$$L_3 = \frac{1}{6}(-E_{11} - E_{22} + 2E_{33} - f_1 - f_2 + 2f_3), \quad L_4 = \frac{1}{4}(2c_1 + E_{23} + E_{32}),$$

$$L_5 = \frac{1}{4}\left(2c_2 + E_{13} + E_{31}\right), \quad L_6 = \frac{1}{4}\left(2c_3 + E_{12} + E_{21}\right),$$

$$L_7 = \frac{1}{6}\left(2E_{11} - E_{22} - E_{33} + 2f_1 - f_2 - f_3\right), \quad L_8 = \frac{1}{4}\left(2c_3 + E_{12} + E_{21}\right),$$

$$L_9 = \frac{1}{4}\left(2c_2 + E_{13} + E_{31}\right), \quad L_{10} = \frac{1}{4}\left(2c_3 + E_{12} + E_{21}\right),$$

$$L_{11} = \frac{1}{6}\left(-E_{11} + 2E_{22} - E_{33} - f_1 + 2f_2 - f_3\right), \quad L_{12} = \frac{1}{4}\left(2c_1 + E_{23} + E_{32}\right),$$

$$L_{13} = \frac{1}{4}\left(2c_2 + E_{13} + E_{31}\right), \quad L_{14} = \frac{1}{4}\left(2c_1 + E_{23} + E_{32}\right),$$

$$L_{15} = \frac{1}{6}\left(-E_{11} - E_{22} + 2E_{33} - f_1 - f_2 + 2f_3\right).$$

Making up the matrix of the system, we find its rank, it turns out to be equal to 5. Eliminating the rows 1,...,9 and 12, we get the matrix with the same rank; the variables $L_{10}, L_{11}, L_{13}, L_{14}, L_{15}$ can be taken as independent: let us use the notations $L_{10} = B_1, L_{11} = B_2, L_{13} = B_3, L_{14} = B_4, L_{15} = B_5$; then we get

$$L_1 = L_7 = -L_{11} - L_{15} = -B_2 - B_5, \quad L_2 = L_{11} = B_2, \quad L_3 = L_{15} = B_5, \quad L_4 = L_{14} = B_4,$$

$$L_5 = L_9 = L_{13} = B_3, \quad L_6 = L_8 = L_{10} = B_1, \quad L_{12} = L_4 = L_{14} = B_4.$$

Similarly, for small components $S_i$ we get

$$S_1 = \frac{1}{6}\left(-2E_{11} + E_{22} + E_{33} + 2f_1 - f_2 - f_3\right), \quad S_2 = \frac{1}{6}\left(E_{11} - 2E_{22} + E_{33} - f_1 + 2f_2 - f_3\right),$$

$$S_3 = \frac{1}{6}\left(E_{11} + E_{22} - 2E_{33} - f_1 - f_2 + 2f_3\right), \quad S_4 = \frac{1}{4}\left(2c_1 - E_{23} - E_{32}\right),$$

$$S_5 = \frac{1}{4}\left(2c_2 - E_{13} - E_{31}\right), \quad S_6 = \frac{1}{4}\left(2c_3 - E_{12} - E_{21}\right),$$

$$S_7 = \frac{1}{6}\left(2E_{11} - E_{22} - E_{33} - 2f_1 + f_2 + f_3\right), \quad S_8 = \frac{1}{4}\left(-2c_3 + E_{12} + E_{21}\right),$$

$$S_9 = \frac{1}{4}\left(-2c_2 + E_{13} + E_{31}\right), \quad S_{10} = \frac{1}{4}\left(-2c_3 + E_{12} + E_{21}\right), S_{12} = \frac{1}{4}\left(-2c_1 + E_{23} + E_{32}\right),$$

$$S_{11} = \frac{1}{6}\left(-E_{11} + 2E_{22} - E_{33} + f_1 - 2f_2 + f_3\right), \quad S_{13} = \frac{1}{4}\left(-2c_2 + E_{13} + E_{31}\right),$$

$$S_{14} = \frac{1}{4}\left(-2c_1 + E_{23} + E_{32}\right), \ S_{15} = \frac{1}{6}\left(-E_{11} - E_{22} + 2E_{33} + f_1 + f_2 - 2f_3\right).$$

We make up the matrix of the system, its rank equals 5. Eliminating the rows $1 - -9, 12$, we get the matrix with the same rank. The variables $S_{10}, S_{11}, S_{13}, S_{14}, S_{15}$ are taken as independent. We find expressions for remaining components through independent ones

$$S_1 = S_{11} + S_{15}, \quad S_2 = -S_{11}, \quad S_3 = -S_{15}, \quad S_4 = -S_{14}, \quad S_5 = -S_{13},$$

$$S_6 = -S_{10}, \quad S_7 = -S_1 = -S_{11} - S_{15}, \quad S_8 = -S_6 = S_{10}, \quad S_9 = -S_5 = S_{13}, \quad S_{12} = -S_4 = S_{14}.$$

Now consider the small components $s_i$:

$$s_1 = \Phi, \quad s_2 = \Phi_0, \quad s_3 = \Phi_1, \quad s_4 = \Phi_2, \quad s_5 = \Phi_3,$$

$$s_6 = \frac{1}{3}(f_1+f_2+f_3), s_7 = \frac{1}{3}(f_1+f_2+f_3), s_8 = \frac{1}{3}(f_1+f_2+f_3), s_9 = d_1, s_{10} = d_2, s_{11} = d_3, s_{12} = f_0,$$
$$s_{13} = E_{10}, s_{14} = E_{20}, s_{15} = E_{30}, s_{16} = B_{10}, s_{17} = B_{20}, s_{18} = B_{30},$$
$$s_{19} = \frac{1}{3}(E_{11}+E_{22}+E_{33}), s_{20} = \frac{1}{2}(E_{21}-E_{12}), s_{21} = \frac{1}{2}(E_{31}-E_{13}),$$
$$s_{22} = B_{11},\ s_{23} = B_{21},\ s_{24} = B_{31},$$
$$s_{25} = \frac{1}{2}(E_{12}-E_{21}),\ s_{26} = \frac{1}{3}(E_{11}+E_{22}+E_{33}),\ s_{27} = \frac{1}{2}(E_{32}-E_{23}),$$
$$s_{28} = B_{12},\ s_{29} = B_{22},\ s_{30} = B_{32},$$
$$s_{31} = \frac{1}{2}(E_{13}-E_{31}),\ s_{32} = \frac{1}{2}(E_{23}-E_{32}),\ s_{33} = \frac{1}{3}(E_{11}+E_{22}+E_{33}),$$
$$s_{34} = B_{13},\ s_{35} = B_{23},\ s_{36} = B_{33}.$$

here we have 29 independent variables; besides there exist relations

$$s_6 = s_7 = s_8, \quad s_{19} = s_{26} = s_{33}, \quad s_{20} = -s_{25}, \quad s_{21} = -s_{31}, \quad s_{27} = -s_{32}.$$

Let us collect all constraints together:

$$\Psi_+, \quad L_{10}, \quad L_{11}, \quad L_{13}, \quad L_{14}, \quad L_{15}$$
$$L_1 = -L_{11} - L_{15}, \quad L_2 = L_{11}, \quad L_3 = L_{15}, \quad L_4 = L_{14}, \quad L_5 = L_{13},$$
$$L_6 = L_{10}, \quad L_7 = -L_{11} - L_{15}, \quad L_8 = L_{10}, \quad L_9 = L_{13}, \quad L_{12} = L_{14};$$
$$\Psi_-, \quad S_{10}, \quad S_{11}, \quad S_{13}, \quad S_{14}, \quad S_{15}$$
$$S_1 = S_{11} + S_{15}, \quad S_2 = -S_{11}, \quad S_3 = -S_{15}, \quad S_4 = -S_{14}, \quad S_5 = -S_{13},$$
$$S_6 = -S_{10}, \quad S_7 = -S_{11} - S_{15}, \quad S_8 = S_{10}, \quad S_9 = S_{13}, \quad S_{12} = S_{14};$$
$$\Psi_0, \quad s_1, \quad s_2, \quad s_3, \quad s_4, \quad s_5, \quad s_6, \quad s_9,$$
$$s_{10}, \quad s_{11}, \quad s_{12}, \quad s_{13}, \quad s_{14}, \quad s_{15}, \quad s_{16}, \quad s_{17}, \quad s_{18}, \quad s_{19}, \quad s_{20}, \quad s_{21}, \quad s_{22},$$
$$s_{23}, \quad s_{24}, \quad s_{27}, \quad s_{28}, \quad s_{29}, \quad s_{30}, \quad s_{34}, \quad s_{35}, \quad s_{36},$$
$$s_7 = s_6, \quad s_8 = s_6, \quad s_{25} = -s_{20}, \quad s_{26} = s_{19}, \quad s_{31} = -s_{21}, \quad s_{32} = -s_{27}, \quad s_{33} = s_{19}.$$

## References


[1] W. Pauli, M. Fierz, Über relativistische Feldleichungen von Teilchen mit beliebigem Spin im elektromagnetishen Feld, Helv. Phys. Acta, **12**, 297-300 (1939).

[2] M. Fierz, W. Pauli, On relativistic wave equations for particles of arbitrary spin in an electromagnetic field, Proc. Roy. Soc. London, **A173**, 211-232 1939).

[3] I.M. Gel'fand, A.M. Yaglom, General relativistically invariant equations and infinite-dimensional representations of the Lorentz group, Journal of Experimental and Theoretical Physics, **18(8)**, 703-733 (1948).

[4] F.I. Fedorov, To the theory of particles with spin

2, Proceedings of Belarus State University, Ser. Phys.-Math, **12**, 156-173 (1951).

[5] T. Regge, On properties of the particle with spin 2. Nuovo Cimento, **5(2)**, 325-326 (1957).

[6] G. Velo, D. Zwanziger, Noncausality and other defects of interaction Lagrangians for particles with spin one and higher, Phys. Rev., **188(5)**, 2218–2222 (1969).

[7] G. Velo, Anomalous behavior of a massive spin two charged particle in an external electromagnetic field, Nucl. Phys., **B43**, 389-401 (1972).

[8] C.R. Hagen, Minimal electromagnetic coupling of spin-two fields, Phys. Rev., **D6(4)**, 984-987 (1972).

[9] J.F. Reilly, Minimal coupling in spin-2 field, Nucl. Phys., **B1**, 356-364 (1974.).

[10] P.M. Mathews, M. Seetharaman, J. Prabhakaran, Inconsistencies in the symmetric tensor field description of spin-2 particles in an external homogeneous magnetic field, Phys. Rev., **D14**, 1021-1031(1976).

[11] W. Cox, First-order formulation of massive spin-2 field theories, J. Phys., **A15**, 253-268 (1982).

[12] V.V. Kisel, On relativistic wave equations for a spin 2 particle, Proceedings of the National Academy of Sciences of Belarus, Physics and Mathematics Series, **5**, 94-99 (1986).

[13] A.A. Bogush, V.V. Kisel, On description of anomalous magnetic moment of a massive particle with spin 2 in the theory of relativistic wave equations, Izvestia Vuzov. SSSR. Fizika, **31(3)**, 11-16 (1988).

[14] I.L. Buchbinder, D.M. Gitman, V.A. Krykhtin, V.D. Pershin, Equations of motion for massive spin 2 field coupled to gravity, Nucl. Phys., **B584**, 615-640 (2000).

[15] V.M. Red'kov, N.G. Tokarevskaya, V.V. Kisel, Graviton in a curved space-time background and gauge symmetry, Nonlinear Phenomena in Complex Systems, **6(3)**, 772-778 (2003).

[16] A.A. Bogush, V.V. Kisel, N.G. Tokarevskaya, V.M. Red'kov, On equations for a spin 2 particle in external electromagnetic and gravitational fields, Proceedings of the National Academy of Sciences of Belarus, Physics and Mathematics Series, **1**, 62-67 (2003).

[17] L. Bernard, C. Deffayet, A. Schmidt-May and M. von Strauss, Linear spin-2 fields in most general backgrounds, Phys. Rev., **D93**. 084020 (2016).

[18] Charles Mazuet,a Mikhail S. Volkov, Massive spin-2 field in arbitrary spacetimes, the detailed derivation, Phys. Rev., **D96**, 124023 (2017).

[19] A. Ivashkevich, A. Buryy, E. Ovsiyuk, V. Balan, V. Kisel, V. Red'kov, On the matrix equation for a spin 2 particle in pseudo-Riemannian space-time, tetrad method, Proceedings of Balkan Society of Geometers, **28**, 1-23 (2021).

[20] V.M. Red'kov, N.G. Tokarevskaya, V.V. Kisel, Graviton in a curved space-time background and gauge symmetry, Nonlinear Phenomena in Complex Systems, **6(3)**, 772-778 (2003).

[21] I.G. Dudko, O.A. Semenyuk, V.V Kisel, V.M. Red'kov, Spin 2 particle with anomalous magnetic moment in Riemann space-time, restriction to massless case, gauge symmetry, Nonlinear Phenomena in Complex Systems, **25(3)**, 286-296 (2022).

[22] A.V. Ivashkevich, A.V. Bury, V.M. Red'kov, V.V. Kisel, On new form of the 50-component theory for spin 2 particle with anomalous magnetic moment in the basis of tensors of 2-nd and 3-rd ranks, Nonlinear Dynamics and Applications, **29**, 289-330 (2023).

[23] A.V. Ivashkevich, V.M. Red'kov, A.M.Ishkhanyan, Massless spin 2 field in 50-component approach, exact solutions with cylindrical symmetry, eliminating the gauge degrees of freedom, Proceedings of the National Academy of Sciences of Belarus. Physics and Mathematics series, **60(2)**, 132-145 (2024).

[24] A.V. Ivashkevich, A.V. Chichurin, A.M. Ishkhanyan, V.M. Red'kov, Massless spin 2 field solutions with spherical symmetry: eliminating gauge degrees of freedom, Modern Physics Letters, **A39**, Paper 2450065 [22 pages] (2024).

[25] A.V. Ivashkevich, A.V. Bury, V.M. Red'kov, E.M. Ovsiyk, V.V. Kisel, Solutions with cylindrical symmetry for graviton field in linear approximation, the gauge degrees of freedom, Nonlinear Phenomena in Complex Systems, **27(3)**, 256-277 2024.

[26] A.V. Ivashkevich, V.M. Red'kov, A.M. Ishkhanyan, Nonrelativistic approximation in the theory of a spin 2 particle with anomalous magnetic moment Axioms, **14(1)**, 35 (2025).

[27] Ivashkevich, Alina. A. Ivashkevich, V. Red'kov, A. Ishkhanyan, Spin-2 particle in Coulomb field: non-relativistic approximation, Symmetry, **17(7)**, 1075 [29 pages] (2025).