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DEVELOPMENT OF MATHEMATICAL TOOLS FOR IMAGE RESTORATION WITH DISTORTED DATA

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Abstract. This paper addresses image restoration in the presence of Gaussian noise, salt and pepper noise, occlusion and blur. A review of mathematical methods-from Fourier filters and statistical hypothesis testing to regression trees-is presented. The research aims to develop a unified framework for image quality assessment integrating statistical rigor with computational techniques.
Keywords: image; restoration; statistical hypothesis testing; Kolmogorov-Smirnov test; Gaussian noise; salt and pepper noise; blur; regression tree methods; occlusion; quality assessment.

Introduction

Images serve critical roles from medical diagnostics to forensic investigation, but their integrity is frequently compromised during acquisition, transmission, and storage. Restoration of images with missing or corrupted data represents a fundamental challenge. Unlike noise corruption, which perturbs pixel values while preserving structure, missing data scenarios involve complete absence of information in affected regions – common in medical imaging artifacts, satellite transmission errors, and historical photograph damage.

The novelty of this research lies in its comparison of mathematical methodologies for image quality assessment. While existing literature explores individual restoration techniques, a gap remains in comprehensive comparative analysis evaluating statistical hypothesis testing methods across diverse distortion types and sample sizes.

Main Part

Image recovery reconstructs missing or corrupted regions algorithmically. The degradation model is expressed as:

$$g(x,y) = M(x,y)\cdot f(x,y) + n(x,y), \quad (1)$$

where $f(x, y)$ – the original image; g(x, y) – the observed image; $M(x, y)$ – a binary mask indicating known ($M = 1$) and missing ($M = 0$) regions; $n(x, y)$ – additive noise.

Four distortion types are considered. Gaussian noise follows a normal distribution affecting all pixels, resulting from sensor noise. Salt and pepper noise manifests as random extreme pixel values (0 or 255) due to transmission errors. Occlusion represents complete loss of original information in affected regions. Blur reduces sharpness through convolution with a point spread function from camera motion or defocus.

Statistical methods use hypothesis testing to determine whether a distorted image maintains statistical similarity to the original. The Kolmogorov-Smirnov (*KS*) test is a non-parametric test comparing two probability distributions:

$$D = \sup_x |F_1(x) - F_2(x)|, \quad (2)$$

where F_1 and F_2 – empirical cumulative distribution functions.

If D exceeds a critical value,the null hypothesis that samples come from the same distribution is rejected.

The Anderson-Darling test gives more weight to distribution tails, detecting differences in extremes. The Shapiro-Wilk test for normality is effective for small to moderate sample sizes, verifying assumptions before applying parametric methods.

Other Approaches. Regression tree methods learn patterns from training data, capturing non-linear relationships between image features (mean, variance, skewness, kurtosis) and quality metrics. Fourier filter methods operate in the frequency domain, with the Wiener filter minimizing mean square error. Variational methods formulate recovery as optimization balancing data fidelity with regularization constraints.

Existing Software Analysis. Adobe Photoshop provides professional restoration but requires manual tuning and lacks statistical assessment. MATLAB offers comprehensive functions but needs programming expertise. OpenCV provides efficient implementations without automated quality assessment. Deep learning frameworks enable state-of-the-art restoration but require extensive training data and computational resources.

Research Objectives. This research develops a framework for image restoration with distorted data, addressing identified limitations. Tasks include: analysis of existing mathematical methods; development of a unified framework integrating multiple approaches; and systematic experimental validation across distortion types and sample sizes. Preliminary analysis indicates statistical hypothesis testing methods tolerate higher defect rates for Gaussian noise and blur than regression tree methods, providing interpretable results valuable for scientific applications.

Conclusion

This paper has presented an overview of image recovery, covering Gaussian noise, salt and pepper noise, occlusion, and blur. Review of Fourier filters, statistical hypothesis testing (*KS*, Anderson-Darling, Shapiro-Wilk tests), and regression trees revealed strengths and limitations. Analysis of existing software tools identified common shortcomings motivating unified framework development.

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