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ERROR ANALYSIS MODEL FOR DUAL-AGENT COLLABORATIVE MONOCULAR RANGING SYSTEM BASED ON FIRST-ORDER ERROR PROPAGATION THEORY

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Abstract. In unknown environments, a single monocular camera struggles to achieve absolute distance measurement due to the lack of scale information. This paper proposes a method for absolute distance estimation of arbitrary targets based on the collaboration of two mobile agents. The method takes two agents as the basic implementation unit and is applicable to multi-agent systems, where any two agents with communication and collaboration capabilities can perform ranging using the same strategy. For this dual-agent collaborative monocular vision ranging system, a systematic error model is established based on first-order error propagation theory and the worst-case linear accumulation assumption, which traces the propagation of errors from pixel quantization to final distance estimation. The system ranging chain is decomposed into three serial stages: baseline calibration, viewing angle difference measurement, and triangulation distance calculation. Analytical expressions for the error in each stage are derived, and the relative error formula for the final distance is synthesized. The model reveals the key factors affecting ranging accuracy, providing a theoretical foundation for system design and parameter optimization.

Keywords: Error Propagation; Dual-Agent Collaboration; Multi-Agent Systems; Monocular Ranging; First-Order Approximation; Linear Accumulation.

Introduction

Utilizing dual-agent collaboration to achieve monocular vision ranging in unknown environments involves three serial stages: baseline calibration, viewing angle difference measurement, and triangulation distance calculation. Each stage is affected by sensor quantization errors, and the final ranging accuracy is determined by the propagation and accumulation of errors from each stage. To quantitatively evaluate the system performance boundaries, this paper adopts the first-order error propagation theory [1] and, under the worst-case scenario, models the linear accumulation of errors from each stage, deriving an analytical expression for the relative error of the final distance estimate, thereby providing theoretical guidance for system design and parameter optimization.

Error Propagation Model Establishment

1. First-order error propagation theory.

Assume the final output distance $AC = f\left[x_1, x_2, \cdots, x_n\right]$ of the system is a function of n independent measured quantities x_i. If each measured quantity x_i has an error Δx_i, and the errors are relatively small, the approximate first-order expansion of the output error (worst-case scenario, errors accumulating in the same direction) is

$$\Delta AC \approx \sum_{i=1}^{n} \left|\frac{\partial f}{\partial x_i}\right| \Delta x_i . \quad (1)$$

The following analysis will be based on this model to examine the linear accumulation process of errors from each stage.

2. System measurement chain and error source decomposition.

The system ranging process can be divided into three serial measurement stages, each introducing independent errors:

2.1. Baseline calibration stage: Input is the tag image width w, output is the baseline distance AB. The main error source is the quantization error Δw in pixel width detection.

2.2. Viewing angle difference measurement stage: Input is the pixel coordinate u, output is the viewing angles α and β. The main error source is the quantization error Δu in pixel coordinates. If the angular disparity exceeds the field of view, the chain tracking method is used for angle measurement [3]. Under chain tracking mode, this error accumulates linearly with the number of tracking steps k.

2.3. Triangulation calculation stage: Inputs are AB, α, β, output is the distance AC. Its error is determined jointly by the errors from the preceding two stages.

Error Modeling for Each Stage

1. Error in baseline distance AB.

According to the depth estimation method for monocular cameras based on prior information [2], the baseline distance is calculated as

$$AB = \frac{f \cdot L}{w}, \tag{2}$$

assuming the actual side length L of the tag and the camera's equivalent focal length f are precisely known, the main error originates from the measurement of the tag image width w (in pixels). Due to pixel coordinate discretization, the detected boundary position has a maximum quantization error of ± 0.5 pixels. Therefore, the maximum absolute error of the width is $\Delta w_{\max} = 1$ pixel. The maximum absolute error of the baseline distance is

$$\Delta AB_{\max} = \left| \frac{\partial AB}{\partial w} \right| \Delta w_{\max} = \frac{f \cdot L}{w^2}, \tag{3}$$

substituting equation (2) into equation (3) yields the error formula expressed in terms of the true baseline value

$$\Delta AB_{\max} = \frac{AB^2}{f \cdot L}. \tag{4}$$

2. Errors in viewing angles α and β.

The viewing angle is obtained through pixel-angle conversion $\theta = \Delta u \cdot \theta_p$, where $\theta_p = FOV_h / W$ is the angular resolution per pixel. FOV_h is the camera's horizontal field of view, and W is the image horizontal resolution.

2.1. Co-visibility case.

The viewing angle difference is calculated from a single pixel difference Δu. Since the maximum quantization error for each pixel coordinate is 0.5 pixels, the maximum error in the pixel difference Δu is 1 pixel. The corresponding maximum absolute error in the viewing angle is

$$\Delta\theta_{\max} = \theta_p. \tag{5}$$

2.2. Chain tracking case.

The total viewing angle is obtained by accumulating k independent angular increments. In the worst-case scenario, assuming the measurement error $\Delta\theta_i$ for each step reaches the maximum value θ_p and accumulates in the same direction, the total error accumulates linearly is

$$\Delta\alpha_{\max} = k \cdot \theta_p, \tag{6}$$

$$\Delta\beta_{\max} = k' \cdot \theta_p, \tag{7}$$

where k and k' are the number of tracking steps for agent A and agent B, respectively.

3. Error synthesis for distance AC.

According to the triangulation formula

$$AC = \frac{AB \cdot \sin(\beta)}{\sin(\alpha+\beta)}, \tag{8}$$

treating AB, α, β as independent variables, with errors $\Delta AB_{\max}$, $\Delta\alpha_{\max}$, $\Delta\beta_{\max}$ respectively. According to equation (1), the maximum absolute error in distance AC is

$$\Delta AC_{\max} = \left|\frac{\partial AC}{\partial AB}\right| \Delta AB_{\max} + \left|\frac{\partial AC}{\partial \alpha}\right| \Delta\alpha_{\max} + \left|\frac{\partial AC}{\partial \beta}\right| \Delta\beta_{\max}, \tag{9}$$

calculate the partial derivatives as

$$\frac{\partial AC}{\partial AB} = \frac{\sin(\beta)}{\sin(\alpha+\beta)}, \tag{10}$$

$$\frac{\partial AC}{\partial \alpha} = -AB \cdot \frac{\sin(\beta)\cos(\alpha+\beta)}{\sin^2(\alpha+\beta)}, \tag{11}$$

$$\frac{\partial AC}{\partial \beta} = AB \cdot \frac{\sin^2(\alpha)}{\sin^2(\alpha+\beta)}, \tag{12}$$

substituting equations (4), (6), (7), (10) – (12) into equation (9) yields the maximum absolute error in distance AC is

$$\Delta AC_{\max} = \frac{\sin(\beta)}{\sin(\alpha+\beta)} \cdot \frac{AB^2}{f \cdot L} + \frac{AB \cdot \sin(\beta)\left|\cos(\alpha+\beta)\right|}{\sin^2(\alpha+\beta)} \cdot k\theta_p + \frac{AB \cdot \sin(\alpha)}{\sin^2(\alpha+\beta)} \cdot k'\theta_p, \tag{13}$$

and the maximum relative error of AC is

$$\delta AC_{\max} = \frac{\Delta AC_{\max}}{AC} = \frac{AB}{f \cdot L} + \left|\cot(\alpha+\beta)\right| \cdot k\theta_p + \frac{\sin(\alpha)}{\sin(\beta)\sin(\alpha+\beta)} \cdot k'\theta_p. \tag{14}$$

Error Model Analysis

Equation (14) reveals three factors affecting the final ranging accuracy and their geometric significance:

1. Baseline measurement term $\frac{AB}{f \cdot L}$: The error is proportional to the baseline length AB and inversely proportional to the system intrinsic parameter $f \cdot L$.

2. Viewing angle α error term $\left|\cot(\alpha+\beta)\right| \cdot k\theta_p$: The error is modulated by the geometric factor $\cot(\alpha+\beta)$. This term is minimized when $\alpha+\beta \to 90^\circ$, and tends to infinity when $\alpha+\beta \to 0^\circ$ or 180°, indicating system instability.

3. Viewing angle β error term $\frac{\sin(\alpha)}{\sin(\beta)\sin(\alpha+\beta)} \cdot k'\theta_p$: The error is modulated by a complex geometric factor. This term is smaller when α is small or β is large.

The model clearly indicates directions for system design and parameter optimization: increasing the tag size L, increasing the camera focal length f, reducing the camera's horizontal field of view FOV_h or increasing the camera resolution W to lower θ_p, controlling the number of chain tracking steps k and k', and selecting observation configurations where $\alpha+\beta$ approaches 90° can all effectively improve ranging accuracy.

Conclusion

This paper addresses the dual-agent collaborative monocular vision ranging system. Based on the first-order error propagation theory and the worst-case linear accumulation assumption, a systematic error model is established. The measurement chain is decomposed into three stages: baseline calibration, viewing angle difference measurement, and triangulation distance calculation. Error expressions for each stage are derived separately, and an analytical formula for the relative error of the final distance estimate is synthesized. This model provides a theoretical foundation for parameter selection, observation configuration optimization, and performance boundary evaluation for ranging systems. The method takes two agents as the basic implementation unit and is applicable to multi-agent systems, where any two agents with communication and collaboration capabilities can perform ranging using the same strategy.

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